# CSE 373
# Data Structures & Algorithms

Lecture 09

Binary Heaps (Part II)

# The Midterm

- Friday, October 23, 12:30, in class
- Closed books, closed notes
- Topics covered:
  - Asymptotic complexity, Big-O, stacks, queues, trees, AVL trees, B trees

# Six Questions on the Midterm

1. Compute asymptotic complexities of programs
2. Write programs with better asymptotic complexities
3. Big O notation basics
4. Stacks and Queues
5. Trees: general concepts
6. Search trees (AVL, B): insertion, deletion

Speed and familiarity with the material are critical

# Building a Heap

- At every point, the new item may need to percolate all the way through the heap
- Adding the items one at a time is O(n log n) in the worst case (what is the worst case?)
- Today we get clever and do it in O(n)

# BuildHeap: Floyd's Method

| 12 | 5 | 11 | 3 | 10 | 6 | 9 | 4 | 8 | 1 | 7 | 2 |
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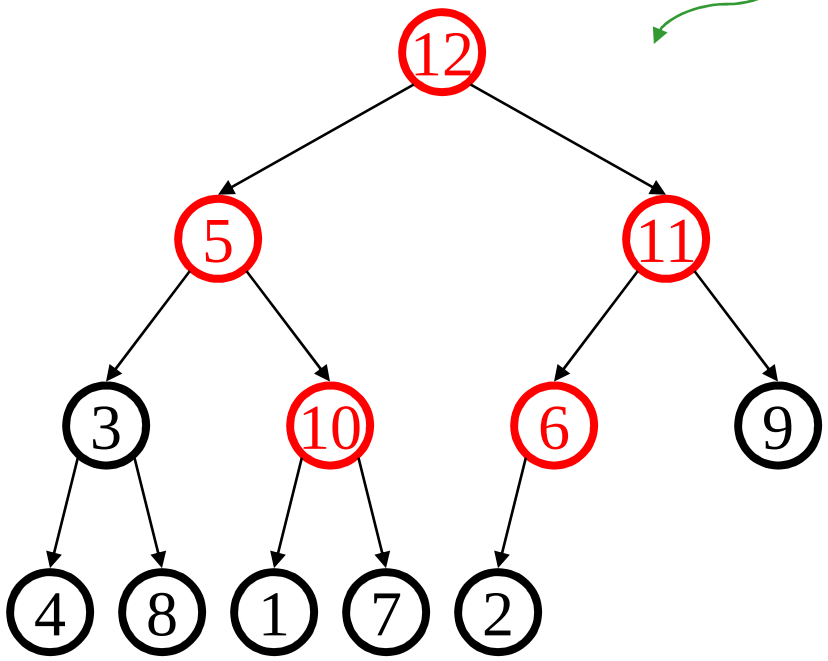
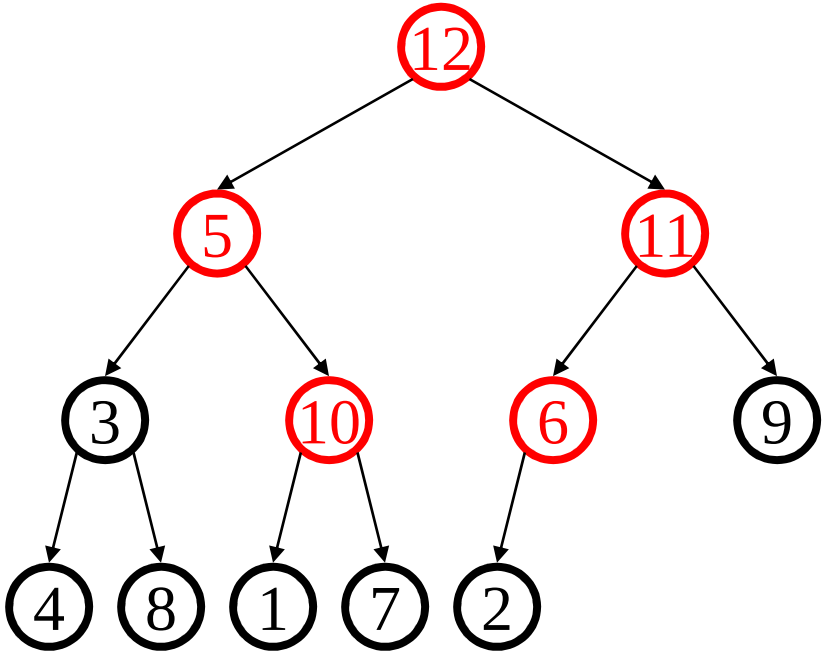
Add elements arbitrarily to form a complete tree.
Pretend it's a heap and fix the heap-order property!

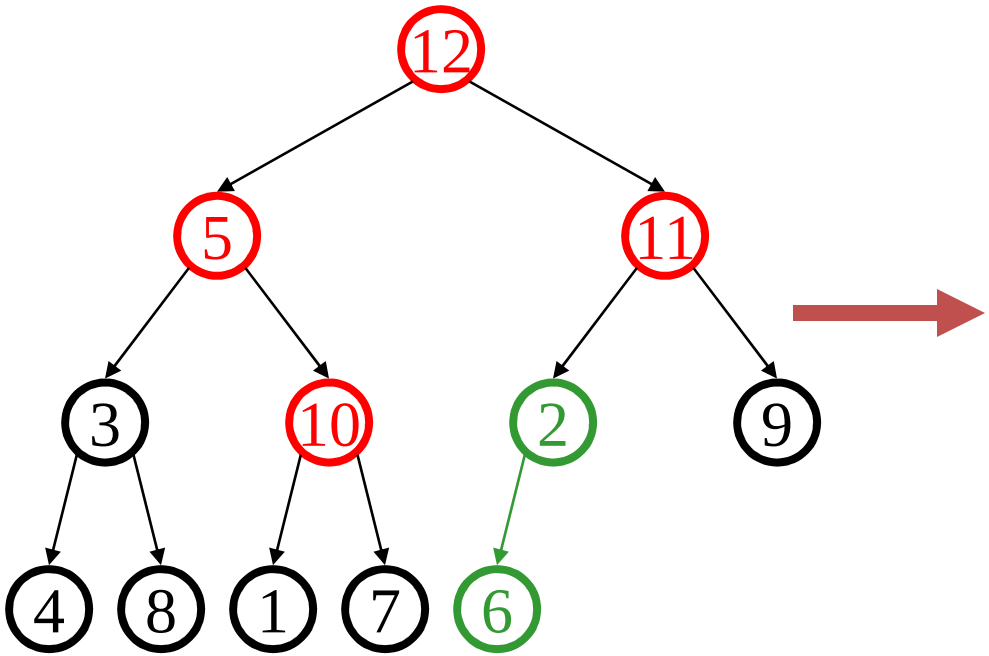


# Buildheap pseudocode

```
private void buildHeap() {
    for ( int i = currentSize/2; i > 0; i-- ) {
     percolateDown( i );
    }
}
```

# BuildHeap: Floyd's Method

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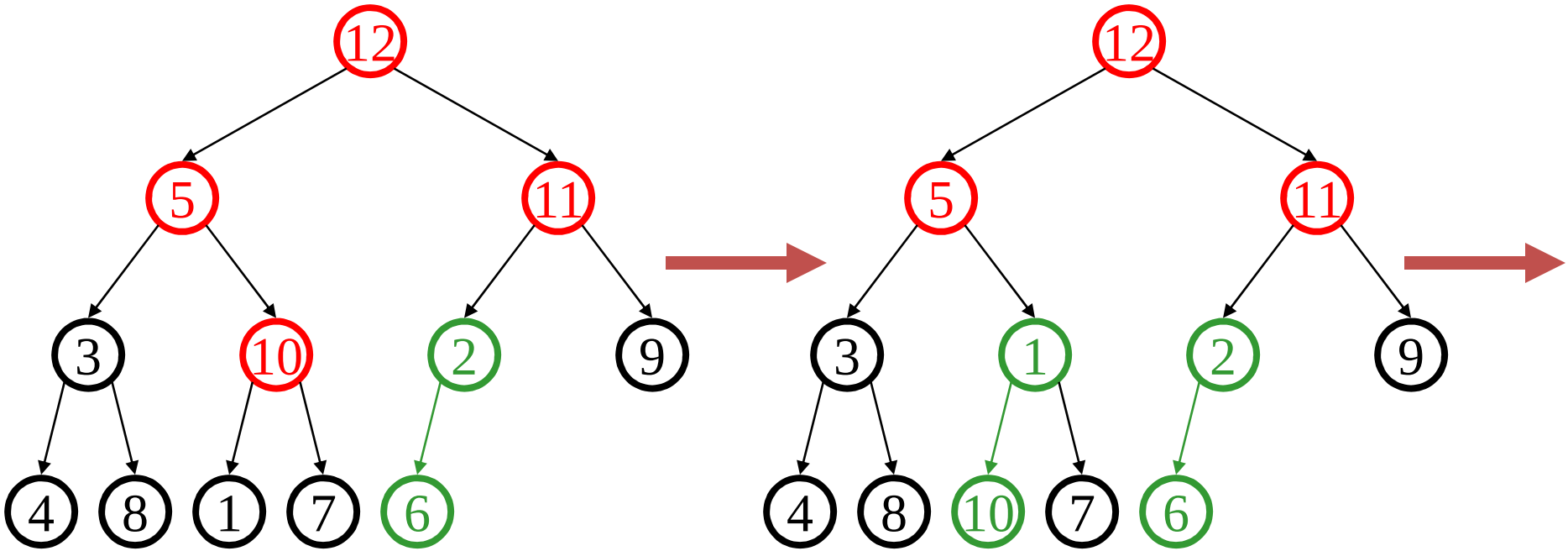
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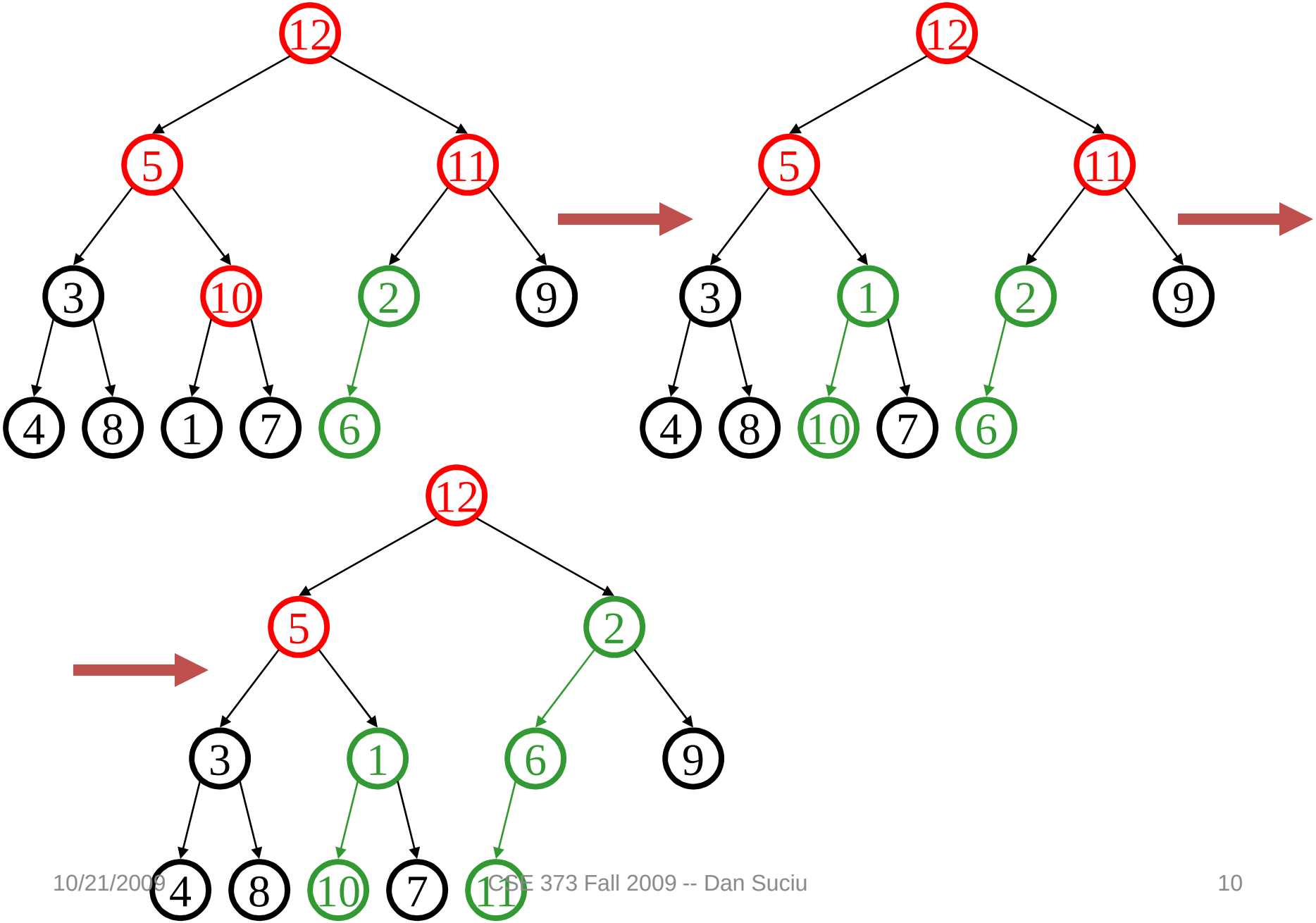
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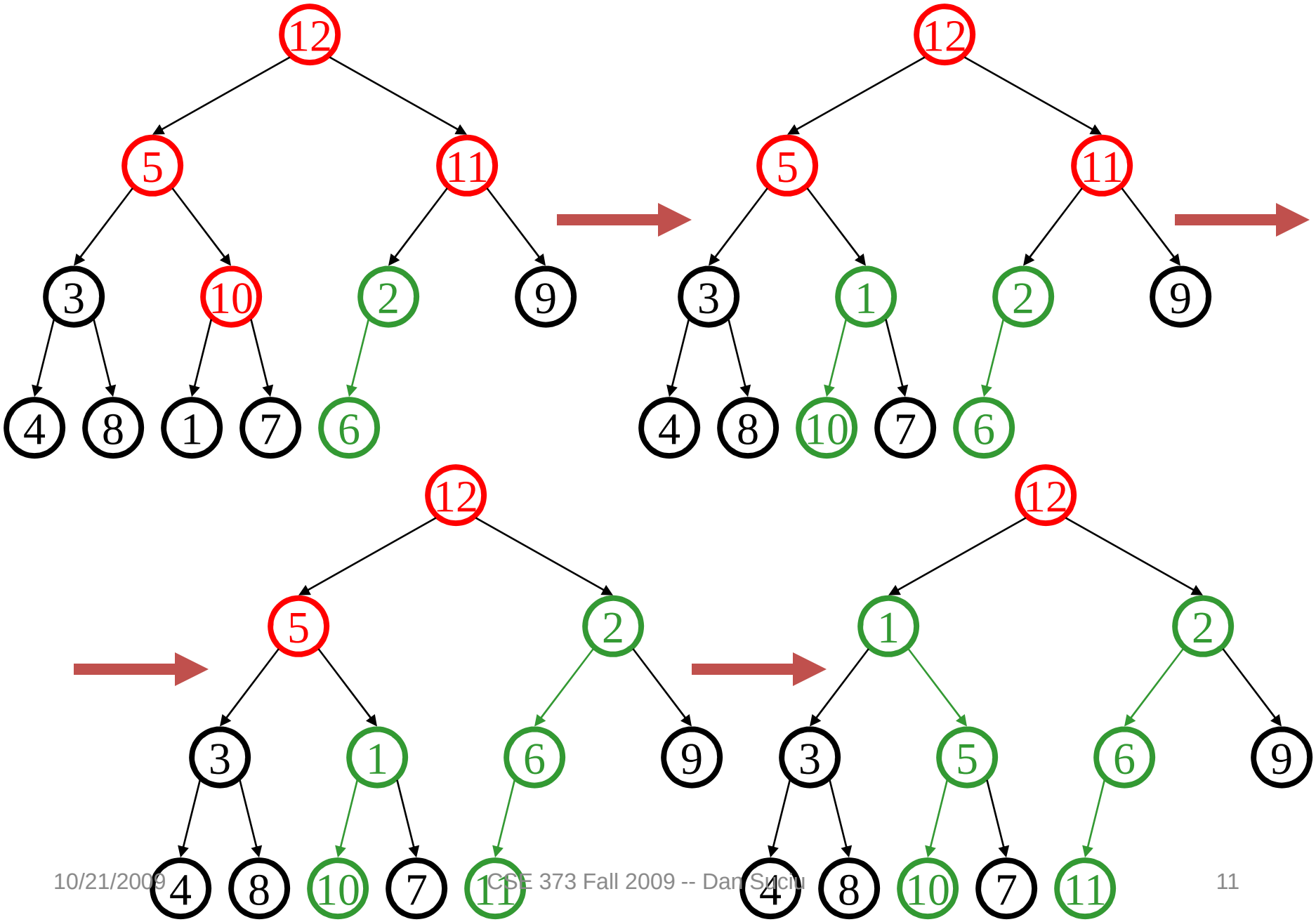
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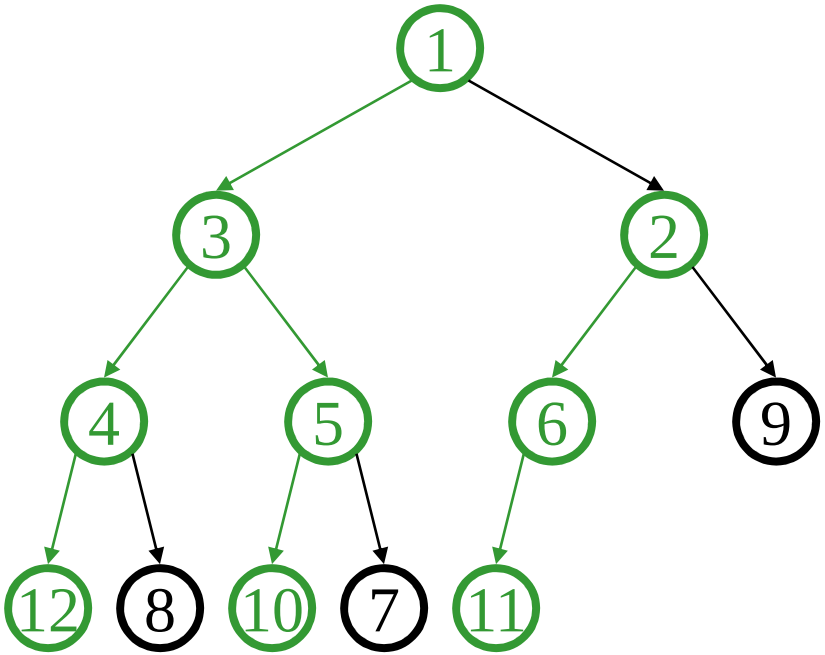
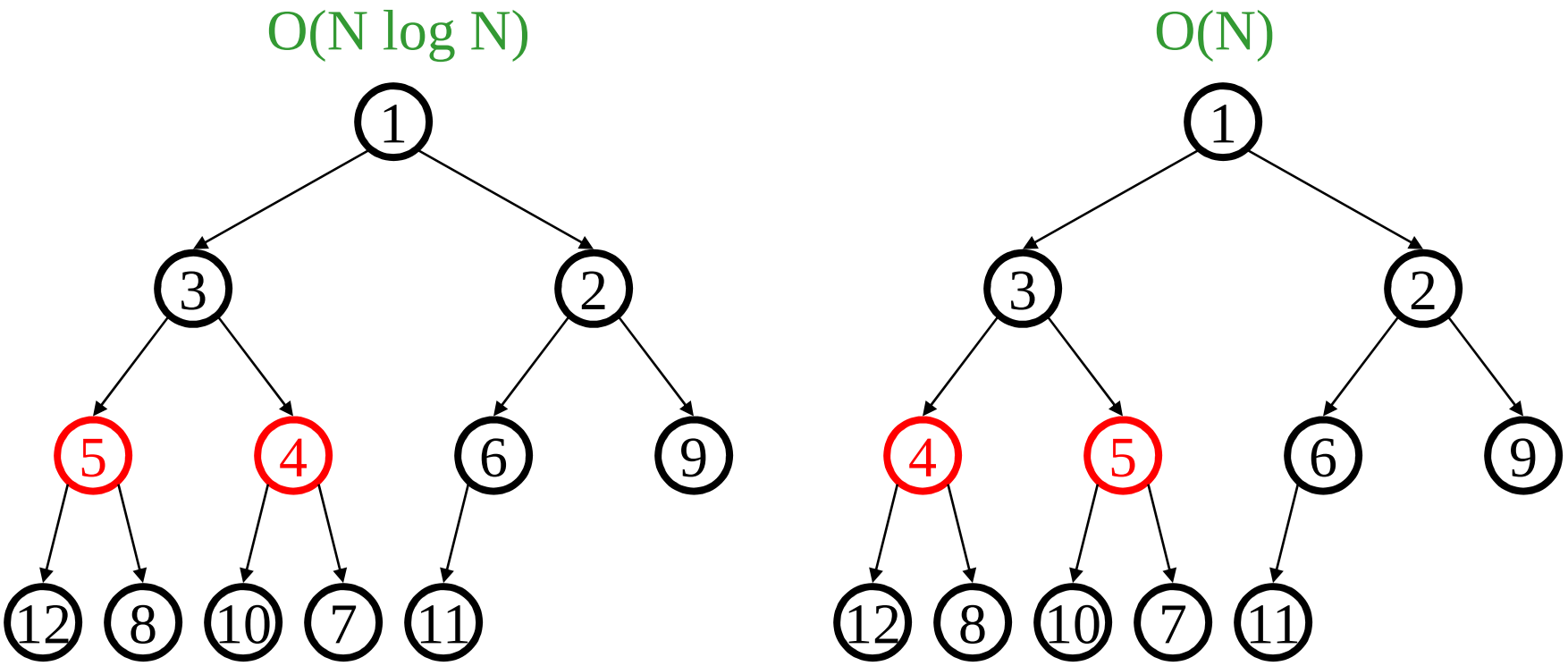
# Finally…



# Note they're not the same




But that doesn't matter, they're both heaps

Floyd's method runs in time O(n); read the proof in Ch. 6.3.4

# Facts about Heaps

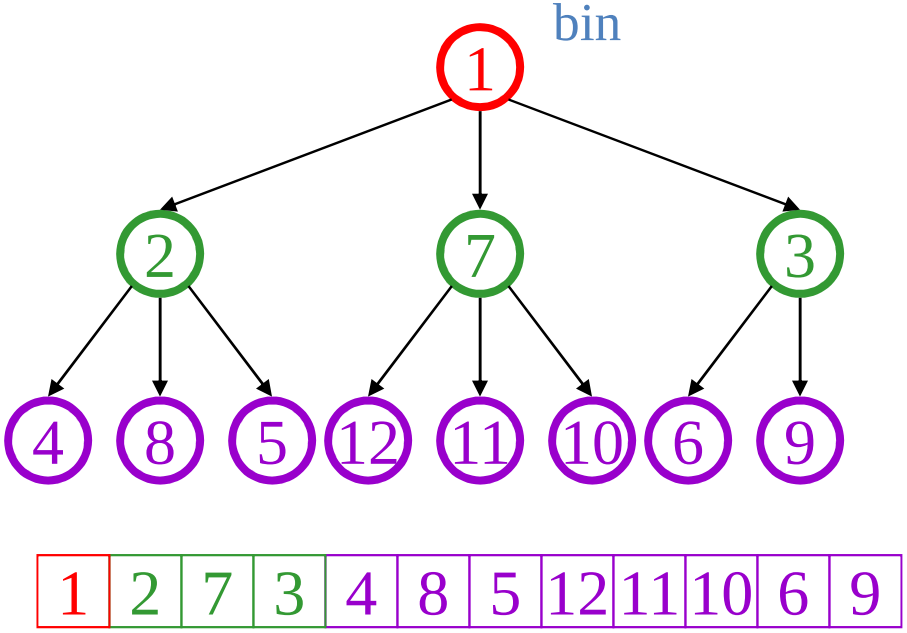
- Observations:
  - Inserts are at least as common as deleteMins
  - Finding a child/parent index is a multiply/divide by two
  - Each percolate step looks at only two new nodes
    - Operations jump widely through the heap
- Realities:
  - Division/multiplication by powers of two are equally fast
  - With huge data sets, disk accesses dominate
  - Looking at only two new pieces of data: bad for cache!

# Extension: *d*-Heaps

How does height compare to bin heap? (less)

This example has height 2, vs. 3 for bin

- Each node has *d* children
- Still representable by array
- Good choices for *d*:
  - choose a power of two
  - fit one set of children in a cache line/memory page/disk block



# Operations on *d*-Heap

- Insert: runtime =

depth of tree decreases, $O(\log_d n)$ worst

- deleteMin: runtime =

percolateDown requires comparison to find min, $O(d \log_d n)$, worst/ave

Does this help insert or deleteMin more?