B-Trees

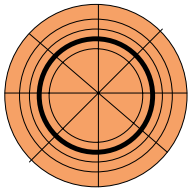
CSE 373
Data Structures
Winter 2007

Readings

- Reading Chapter 4
  - Section 4.7

Data Layout on Disk

- Track: one ring
- Sector: one pie-shaped piece.
- Block: intersection of a track and a sector.



Disk Block Access Time

Seek time = combination of
Time for the disk head to move to the correct track.
Time for the beginning of the correct sector to spin round to the head. (Some authors use "latency" as the term for this component, or they use latency to refer to all of what we are calling seek time.)

Transfer time =
Time to read or write the data.
(Approximately the time for the sector to spin by the head).

For a 7200 RPM hard disk with 8 millisec seek time, average access time for a block is about 12 millisec.

Considerations for Disk Based Dictionary Structures

Use a disk-based method when the dictionary is too big to fit in RAM at once.

Minimize the expected or worst-case number of disk accesses for the essential operations (put, get, remove).

Keep space requirements reasonable -- O(n).

Methods based on binary trees, such as AVL search trees, are not optimal for disk-based representations. The number of disk accesses can be greatly reduced by using *m*-way search trees.

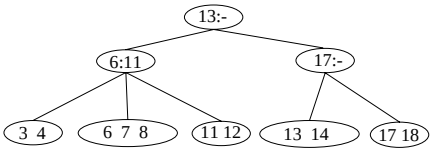
B-trees

- Invented in 1972 by

Rudolf Bayer (-) and Ed McCreight(-)

Beyond Binary Search Trees: Multi-Way Trees

- Example: B-tree of order 3 has 2 or 3 children per node



- Search for 8

B-Trees

B-Trees are multi-way search trees commonly used in database systems or other applications where data is stored externally on disks and keeping the tree shallow is important.

A B-Tree of order M has the following properties:

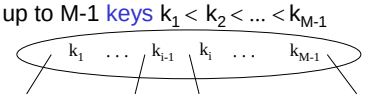
1. The root is either a leaf or has between 2 and M children.
2. All nonleaf nodes (except the root) have between $\lceil M/2 \rceil$ and M children.
3. All leaves are at the same depth.

All data records are stored at the leaves.
Internal nodes have "keys" guiding to the leaves.
Leaves store between $\lceil M/2 \rceil$ and M data records.

B-Tree Details

Each (non-leaf) internal node of a B-tree has:

- Between $\lceil M/2 \rceil$ and M children.
- up to M-1 keys $k_1 < k_2 < ... < k_{M-1}$

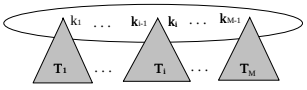


Keys are ordered so that:
$k_1 < k_2 < ... < k_{M-1}$

B-tree alternate definitions

- There are several definitions
- What was in the previous slide is the original def.
- Different textbooks have slightly different ones

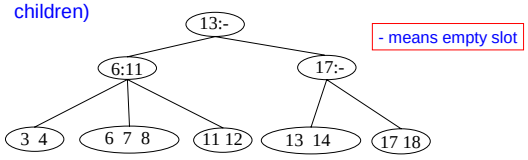
Properties of B-Trees



Children of each internal node are "between" the items in that node.
Suppose subtree T_i is the *i*th child of the node:
all keys in T_i must be between keys k_{i-1} and k_i
i.e. $k_{i-1} \le T_i < k_i$
k_{i-1} is the smallest key in T_i
All keys in first subtree $T_1 < k_1$
All keys in last subtree $T_M \ge k_{M-1}$

Example: Searching in B-trees

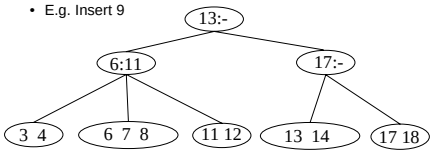
- B-tree of order 3: also known as 2-3 tree (2 to 3 children)



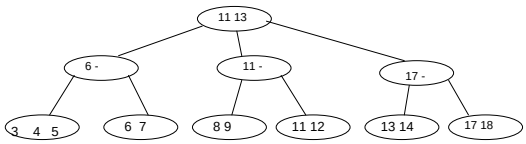
- Examples: Search for 9, 14, 12
- Note: If leaf nodes are connected as a Linked List, B-tree is called a B+ tree – Allows sorted list to be accessed easily

Inserting into B-Trees

- Insert X: Do a Find on X and find appropriate leaf node
  - If leaf node is not full, fill in empty slot with X
    - E.g. Insert 5
  - If leaf node is full, split leaf node and adjust parents up to root node
    - E.g. Insert 9

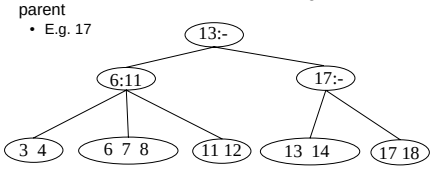


After insert of 5 and 9



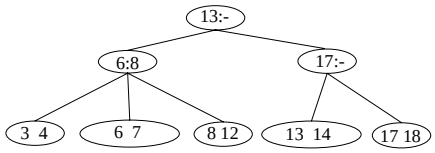
Deleting From B-Trees

- Delete X : Do a find and remove from leaf
  - Leaf underflows – borrow from a neighbor
    - E.g. 11
  - Leaf underflows and can't borrow – merge nodes, delete parent
    - E.g. 17

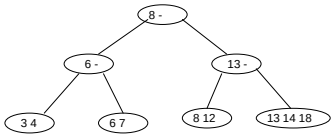


Deleting case 1

"8" was borrowed from neighbor. Note the change in the parent



Deleting Case 2



Run Time Analysis of B-Tree Operations

- For a B-Tree of order M
  - Each internal node has up to M-1 keys to search
  - Each internal node has between $\lceil M/2 \rceil$ and M children
  - Depth of B-Tree storing N items is $O(\log_{\lceil M/2 \rceil} N)$
- Find: Run time is:
  - O(log M) to binary search which branch to take at each node. But M is small compared to N.
  - Total time to find an item is O(depth*log M) = O(log N)

Summary of Search Trees

- Problem with Binary Search Trees: Must keep tree balanced to allow fast access to stored items
- AVL trees: Insert/Delete operations keep tree balanced
- Splay trees: Repeated Find operations produce balanced trees
- Multi-way search trees (e.g. B-Trees): More than two children
  - › per node allows shallow trees; all leaves are at the same depth
  - › keeping tree balanced at all times