Trees

CSE 373
Data Structures
Winter 2007

Readings

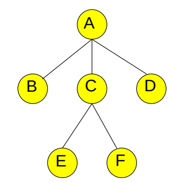
- Reading
  - Chapter 4

Why Do We Need Trees?

- Lists, Stacks, and Queues are linear relationships
- Information often contains hierarchical relationships
  - File directories or folders
  - Moves in a game
  - Hierarchies in organizations
- Can build a tree to support fast searching

Tree Jargon

- root
- nodes and edges (aka vertices and arcs)
- leaves
- parent, children, siblings
- ancestors, descendants
- subtrees
- path, path length
- height, depth



Definition and Tree Trivia

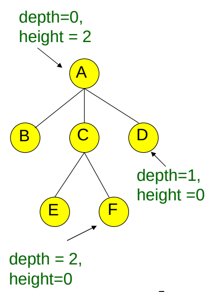
- A tree is a set of nodes,i.e., either
  - it's an empty set of nodes, or
  - it has one node called the root from which zero or more trees (subtrees) descend
- A tree with N nodes always has N-1 edges (prove it by induction)
- A node has a single parent
- Two nodes in a tree have at most one path between them

More definitions

- Leaf (aka external) node: node without children
- Internal node: a node that is not a leaf
- Siblings: two nodes with the same parent

More Tree Jargon

- **Length** of a path = number of edges
- **Depth** of a node N = length of path from root to N
- **Height** of node N = length of longest path from N to a leaf
- **Depth of tree** = depth of deepest node
- **Height of tree** = height of root



Paths

- Can a non-zero path from node N reach node N again?
  - No. Trees can never have cycles (loops)
- Does depth (height) of nodes in a non-zero path increase or decrease?
  - Depth always increases in a non-zero path
  - Height always decreases in a non-zero path

More jargon…..

- If there is a path from node u to node v, u is an ancestor of v
- Yes but… path in which direction? Better to say:
  - Recursive definition: u is an ancestor of v if u= v or u is an ancestor of the parent of v
- Similar definition for descendent

Tree Operations

- The usual (`size(), isEmpty()`...
- Accessor methods
  - `root();` error if the tree is empty
  - `parent(v);`error if v is the root
  - `children(v);`returns an iterable collection (i.e.,ordered list) of children
- Queries (`isRoot()` etc…)
- How about iterators (or positions?)

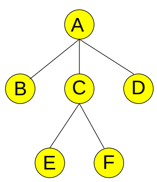
Implementation of Trees (1)

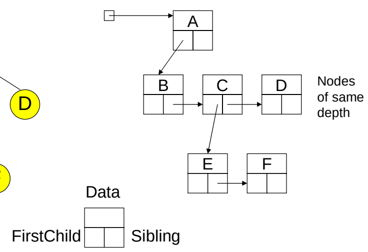
- One possible pointer-based implementation
  - tree nodes with value and a pointer to each child
  - but how many pointers should we allocate space for?
  - OK if we use a pointer to a "collection" of children
  - But how should the "collection" be implemented? (doubly linked list?)
  - Should there be a parent link or not?

Implementation of Trees (2)

- A more flexible pointer-based implementation
  - 1st Child / Next Sibling List Representation
  - Each node has 2 pointers: one to its first child and one to next sibling
  - Can handle arbitrary number of children
  - Having a parent link is an orthogonal decision

Arbitrary Branching





Binary Trees

- Every node has at most two children
  - › Most popular tree in computer science
    - (But n-way branching common in databases, file structures; e.g., B-trees)
- Given N nodes, what is the minimum depth of a binary tree?
  - › At depth d, you can have N = 2^d to N = 2^{d+1}-1 nodes

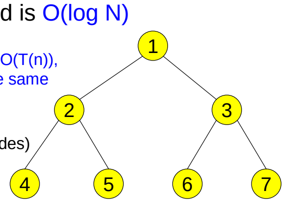
$$2^d \le N \le 2^{d+1}-1 \text{ implies } d_{min} = \lfloor \log_2 N \rfloor$$

Minimum depth vs node count

- At depth d, you can have N = 2^d to 2^{d+1}-1 nodes
- minimum depth d is O(log N)

T(n) = Θ(f(n)) means
T(n) = O(f(n)) and f(n) = O(T(n)),
i.e. T(n) and f(n) have the same
growth rate, so d = Θ(n)

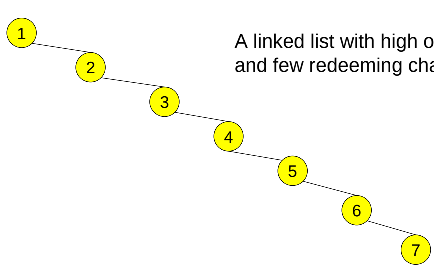
d=2
N=2^2 to 2^3-1 (i.e, 4 to 7 nodes)



Maximum depth vs node count

- What is the maximum depth of a binary tree?
  - › Degenerate case: Tree is a linked list!
  - › Maximum depth = N-1
- Goal: Would like to keep depth at around log N to get better performance than linked list for operations like Find

A degenerate tree

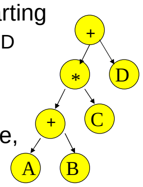


A linked list with high overhead and few redeeming characteristics

Traversing Binary Trees

- The definitions of the traversals are recursive definitions. For example:
  - › Visit the root
  - › Visit the left subtree (i.e., visit the tree whose root is the left child) and do this recursively
  - › Visit the right subtree (i.e., visit the tree whose root is the right child) and do this recursively
- Traversal definitions can be extended to general (non-binary) trees

Traversing Binary Trees



- Preorder: Node, then Children (starting with the left) recursively + * + A B C D
- Inorder: Left child recursively, Node, Right child recursively A + B * C + D
- Postorder: Children recursively, then Node A B + C * D +