

In-class activity on abstract data types for CSE 373.

Using Mathematical Functions to Describe the Methods of Abstract Data Types

A stack of integers can be described by the following ADT:

Data: a list of integers, i.e., a function $f_s : \{0, 1, 2, \dots, n-1\} \rightarrow \{0, 1, \dots\}$, where n is the number of elements in the list.

Let us refer to the set of all such functions f_s by the name STACKS. Also, let us use the name INTEGERS for the set $\{0, 1, \dots\}$.

Methods:

In this description, let us assume that the top element of the stack is in the first (leftmost) position in the list. Then the methods can be described as follows:

PUSH. $f_{\text{push}} : \text{INTEGERS} \times \text{STACKS} \rightarrow \text{STACKS}$.

$$f_{\text{push}} : (e, [e_0, e_1, \dots, e_{n-1}]) \mapsto [e, e_0, e_1, \dots, e_{n-1}]$$

POP. $f_{\text{pop}} : \text{STACKS} \rightarrow \text{INTEGERS} \times \text{STACKS}$.

$$f_{\text{pop}} : [e_0, e_1, \dots, e_{n-1}] \mapsto (e_0, [e_1, \dots, e_{n-1}])$$

Now consider a queue data structure for integers. An abstract data type for queues will contain two methods: ENQUEUE, which puts an element onto the end of a list, and DEQUEUE, which removes the first element of the list.

Give the mathematical descriptions for each of these two operations.