Sorting (Part I)

CSE 373

Data Structures

Lecture 13

Reading

- Reading
  - › Sections 7.1-7.3 and 7.5

Sorting

- Input
  - › an array A of data records (Note: we have seen how to sort when elements are in linked lists: Mergesort)
  - › a key value in each data record
  - › a comparison function which imposes a consistent ordering on the keys (e.g., integers)
- Output
  - › reorganize the elements of A such that
    - For any i and j, if $i < j$ then $A[i] \leq A[j]$

Consistent Ordering

- The comparison function must provided a consistent *ordering* on the set of possible keys
  - › You can compare any two keys and get back an indication of a < b, a > b, or a = b
  - › The comparison functions must be consistent
    - If `compare(a,b)` says a<b, then `compare(b,a)` must say b>a
    - If `compare(a,b)` says a=b, then `compare(b,a)` must say b=a
    - If `compare(a,b)` says a=b, then `equals(a,b) and equals(b,a)` must say a=b

Why Sort?

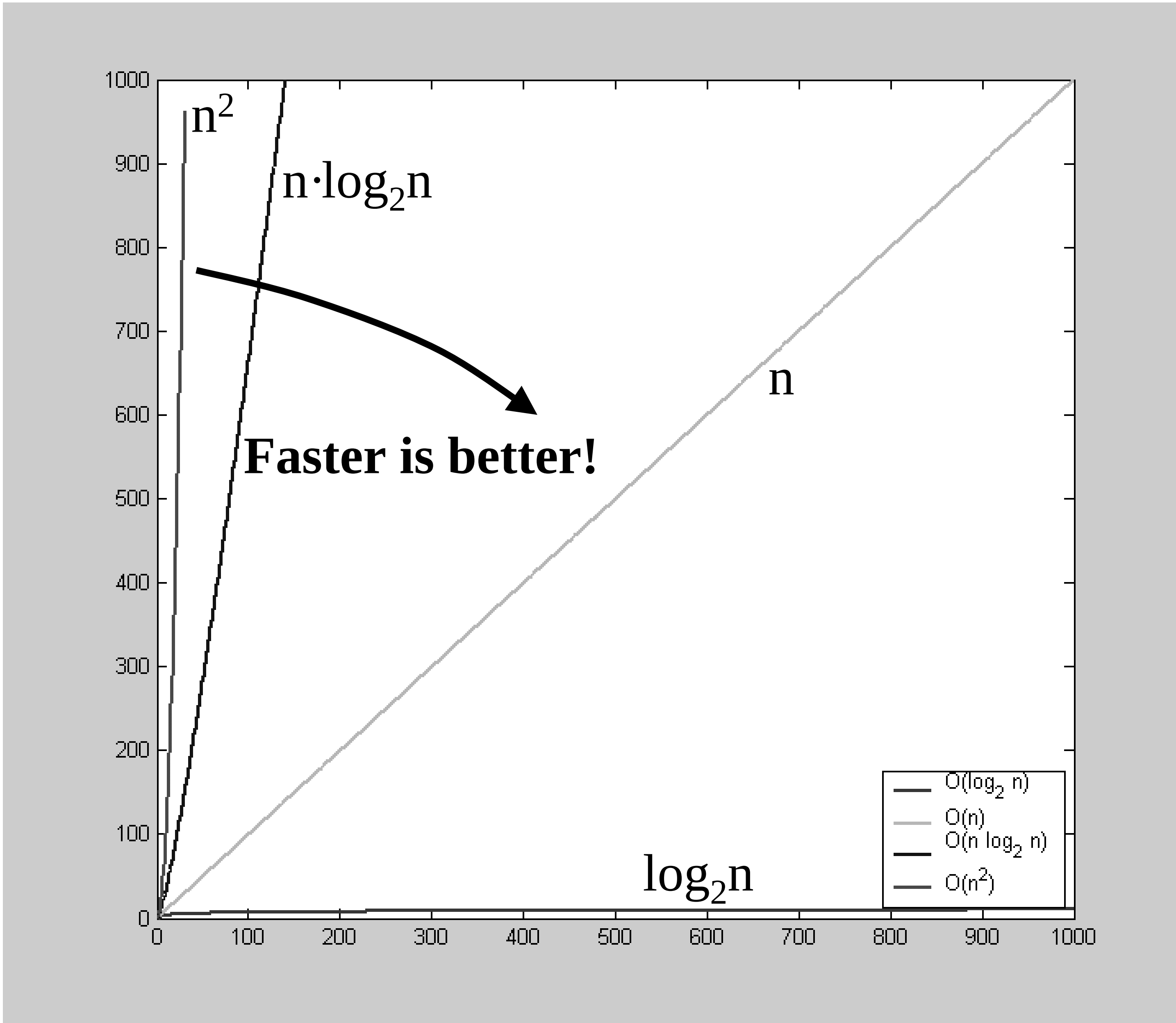
- Sorting algorithms are among the most frequently used algorithms in computer science
- Allows binary search of an N-element array in O(log N) time
- Allows O(1) time access to *k*th largest element in the array for any *k*
- Allows easy detection of any duplicates

Space

- How much space does the sorting algorithm require in order to sort the collection of items?
  - › Is copying needed? O(n) additional space
  - › In-place sorting – no copying – O(1) additional space
  - › Somewhere in between for “temporary”, e.g. O(logn) space
  - › External memory sorting – data so large that does not fit in memory

Time

- How fast is the algorithm?
  - The definition of a sorted array A says that for any i<j, A[i] < A[j]
  - This means that you need to at least check on each element at the very minimum, I.e., at least O(N)
  - And you could end up checking each element against every other element, which is $O(N^2)$
  - The big question is: How close to O(N) can you get?



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100
200
300
400
500
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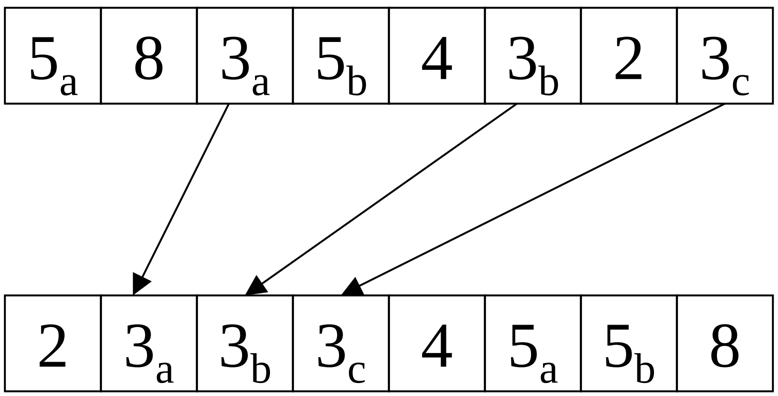
700
800
900
1000
n2
n·log2n
n
log2n
Faster is better!
O(log2 n)
O(n)
O(n log2 n)
O(n2)

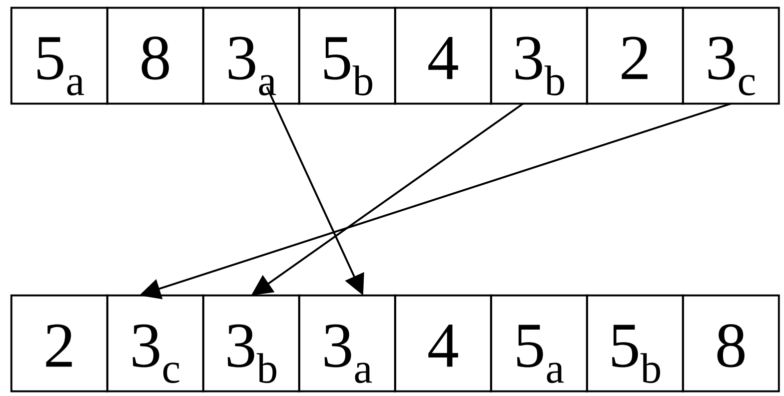
Stability

- Stability: Does it rearrange the order of input data records which have the same key value (duplicates)?
  - E.g. Phone book sorted by name. Now sort by county – is the list still sorted by name within each county?
  - Extremely important property for databases
  - A stable sorting algorithm is one which does not rearrange the order of duplicate keys

Example



Stable Sort



Unstable Sort

Bubble Sort

- "Bubble" elements to to their proper place in the array by comparing elements i and i+1, and swapping if A[i] > A[i+1]
  - › Bubble every element towards its correct position
    - last position has the largest element
    - then bubble every element except the last one towards its correct position
    - then repeat until done or until the end of the quarter, whichever comes first ...

Bubblesort

```
bubble(A[1..n]: integer array, n : integer): {
  i, j : integer;
  for i = 1 to n-1 do
    for j = 2 to n-i+1 do
       if A[j-1] > A[j] then SWAP(A[j-1],A[j]);
  }

SWAP(a,b) :  {
  t :integer;
  t:=a; a:=b; b:=t;
}
```

Put the largest element in its place

larger value? → 2 3 8 8

1	2	3	8	7	9	10	12	23	18	15	16	17	14

swap

1	2	3	7	8	9	10	12	23	18	15	16	17	14

9 10 12 23 23

1	2	3	7	8	9	10	12	23	18	15	16	17	14

swap

1	2	3	7	8	9	10	12	18	23	15	16	17	14

swap

1	2	3	7	8	9	10	12	18	15	23	16	17	14

swap

1	2	3	7	8	9	10	12	18	15	16	23	17	14

swap

1	2	3	7	8	9	10	12	18	15	16	17	23	14

swap

1	2	3	7	8	9	10	12	18	15	16	17	14	**23**

Put 2nd largest element in its place

larger value? 2 3 7 8 9 10 12 18 18

1	2	3	7	8	9	10	12	18	15	16	17	14	23

swap

1	2	3	7	8	9	10	12	15	18	16	17	14	23

swap

1	2	3	7	8	9	10	12	15	16	18	17	14	23

swap

1	2	3	7	8	9	10	12	15	16	17	18	14	23

swap

1	2	3	7	8	9	10	12	15	16	17	14	18	23

Two elements done, only n-2 more to go ...

Bubble Sort: **Just Say No**

- “Bubble” elements to to their proper place in the array by comparing elements i and i+1, and swapping if A[i] > A[i+1]
- We bubblize for i=1 to n (i.e, n times)
- Each bubblization is a loop that makes n-i comparisons
- This is $O(n^2)$

Insertion Sort

- What if first *k* elements of array are already sorted?
  - <u>4, 7, 12,</u> 5, 19, 16
- We can shift the tail of the sorted elements list down and then *insert* next element into proper position and we get k+1 sorted elements
  - <u>4, 5, 7, 12,</u> 19, 16

Insertion Sort

```
InsertionSort(A[1..N]: integer array, N: integer) {
   i, j, temp: integer ;
   for i = 2 to N {
     temp := A[i];
     j := i-1;
     while j > 1 and A[j-1] > temp {
           A[j] := A[j-1]; j := j–1;}
     A[j] = temp;
   }
}
```

- Is Insertion sort in place? Stable? Running time = ?
- Have we used this before?

Example

1	2	3	8	7	9	10	12	23	18	15	16	17	14
1	2	3	7	8	9	10	12	23	18	15	16	17	14
1	2	3	7	8	9	10	12	18	23	15	16	17	14
1	2	3	7	8	9	10	12	18	15	23	16	17	14
1	2	3	7	8	9	10	12	15	18	23	16	17	14
1	2	3	7	8	9	10	12	15	18	16	23	17	14
1	2	3	7	8	9	10	12	15	16	18	23	17	14

Example

1	2	3	7	8	9	10	12	15	16	18	17	23	14
1	2	3	7	8	9	10	12	15	16	17	18	23	14
1	2	3	7	8	9	10	12	15	16	17	18	14	23
1	2	3	7	8	9	10	12	15	16	17	14	18	23
1	2	3	7	8	9	10	12	15	16	14	17	18	23
1	2	3	7	8	9	10	12	15	14	16	17	18	23
1	2	3	7	8	9	10	12	14	15	16	17	18	23

Insertion Sort Characteristics

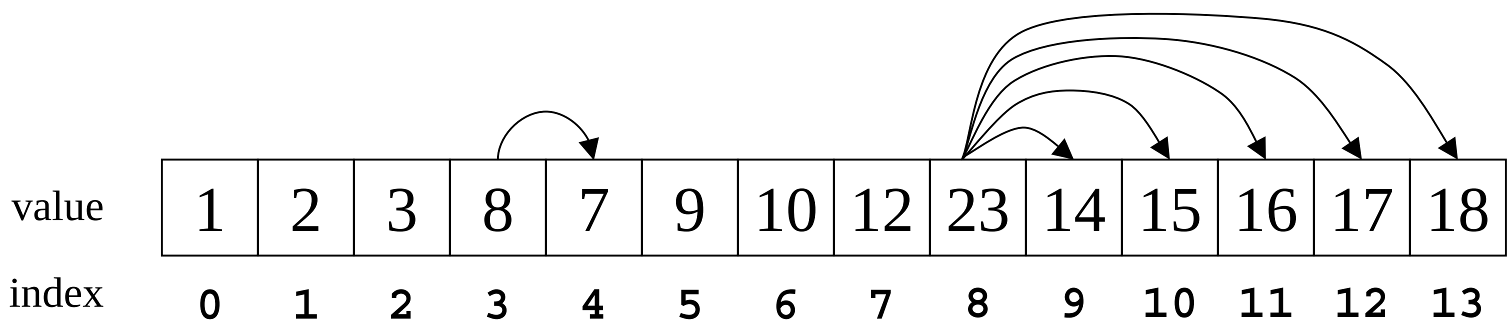
- In place and Stable
- Running time
  - › Worst case is $O(N^2)$
    - reverse order input
    - must copy every element every time
- Good sorting algorithm for almost sorted data
  - › Each item is close to where it belongs in sorted order.

Inversions

- An inversion is a pair of elements in wrong order
  - › i < j but A[i] > A[j]
- By definition, a sorted array has no inversions
- So you can think of sorting as the process of removing inversions in the order of the elements

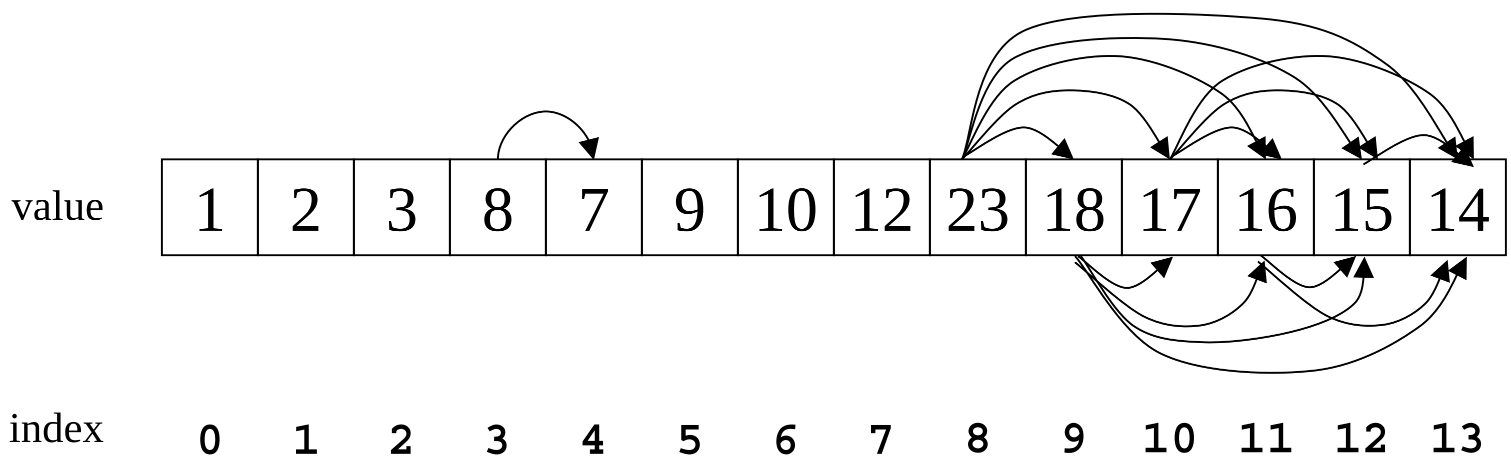
Inversions

- A single value out of place can cause several inversions



Reverse order

- All values out of place (reverse order) causes numerous inversions



Inversions

- Our simple sorting algorithms so far swap adjacent elements (explicitly or implicitly) and remove just 1 inversion at a time
  - › Their running time is proportional to number of inversions in array
- Given N distinct keys, the maximum possible number of inversions is

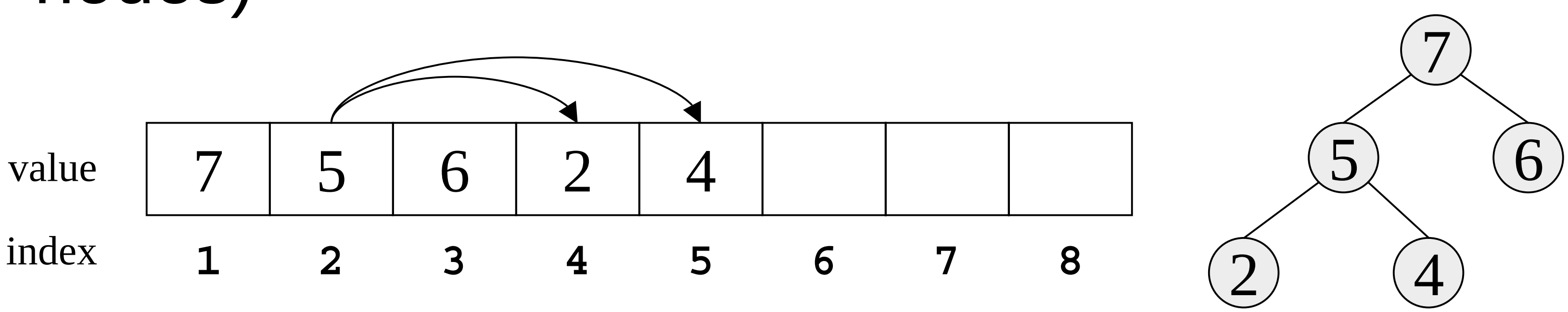
$$(n-1)+(n-2)+...+1=\sum_{i=1}^{n-1} i=\frac{(n-1)n}{2}$$

Inversions and Adjacent Swap Sorts

- "Average" list will contain half the max number of inversions = $\frac{(n-1)n}{4}$
  - › So the average running time of Insertion sort is $\Theta(N^2)$ (i.e, $O(N^2)$ is a tight bound)
- Any sorting algorithm that only swaps adjacent elements requires $\Omega(N^2)$ time because each swap removes only one inversion (lower bound)

Heap Sort

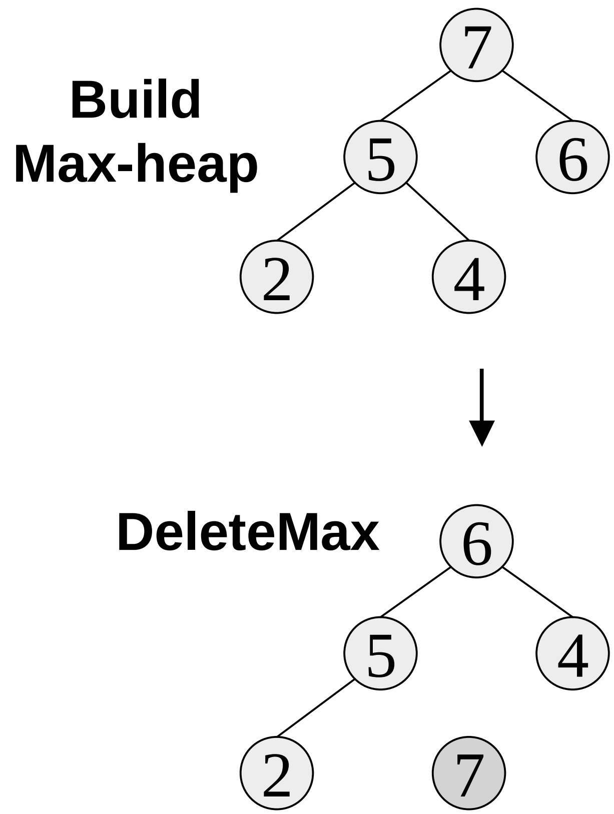
- We use a Max-Heap
- Root node = A[1]
- Children of A[i] = A[2i], A[2i+1]
- Keep track of current size N (number of nodes)



N = 5

Using Binary Heaps for Sorting

- Build a max-heap
- Do N DeleteMax operations and store each Max element as it comes out of the heap
- Data comes out in largest to smallest order
- Where can we put the elements as they are removed from the heap?

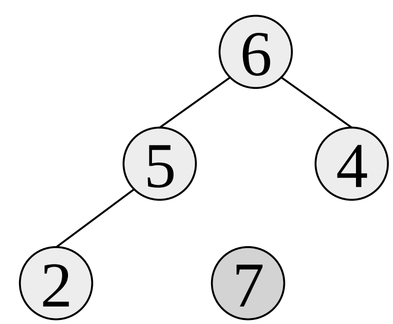


1 Removal = 1 Addition

- Every time we do a DeleteMax, the heap gets smaller by one node, and we have one more node to store
  - › Store the data at the end of the heap array
  - › Not "in the heap" but it is in the heap array

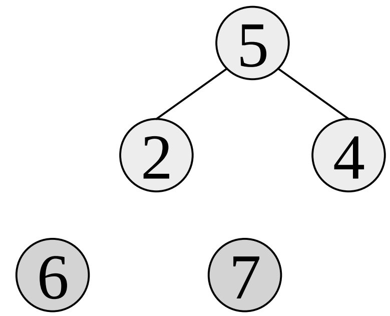
value	6	5	4	2	7			
index	1	2	3	4	5	6	7	8

N = 4



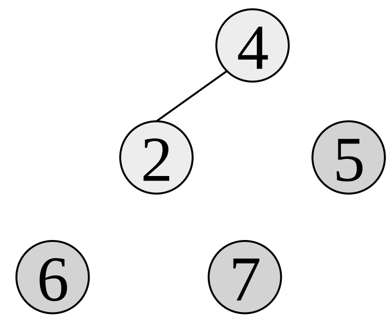
Repeated DeleteMax

5	2	4	6	7			
1	2	3	4	5	6	7	8



N = 3

4	2	5	6	7			
1	2	3	4	5	6	7	8

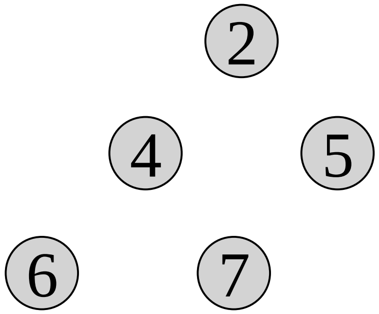


N = 2

Heap Sort is In-place

- After all the DeleteMaxs, the heap is gone but the array is full and is in sorted order

value	2	4	5	6	7			
index	1	2	3	4	5	6	7	8



N = 0

Heapsort: Analysis

- Running time
  - › time to build max-heap is O(N)
  - › time for N DeleteMax operations is N O(log N)
  - › total time is **O(N log N)**
- Can also show that running time is Ω(N log N) for some inputs,
  - › so *worst case* is Θ**(N log N)**
  - › *Average case* running time is also O(N log N)
- Heapsort is in-place but not stable (why?)