Splay Trees and B-Trees

CSE 373

Data Structures

Lecture 9

Readings

- Reading
  - › Sections 4.5-4.7

Self adjusting Trees

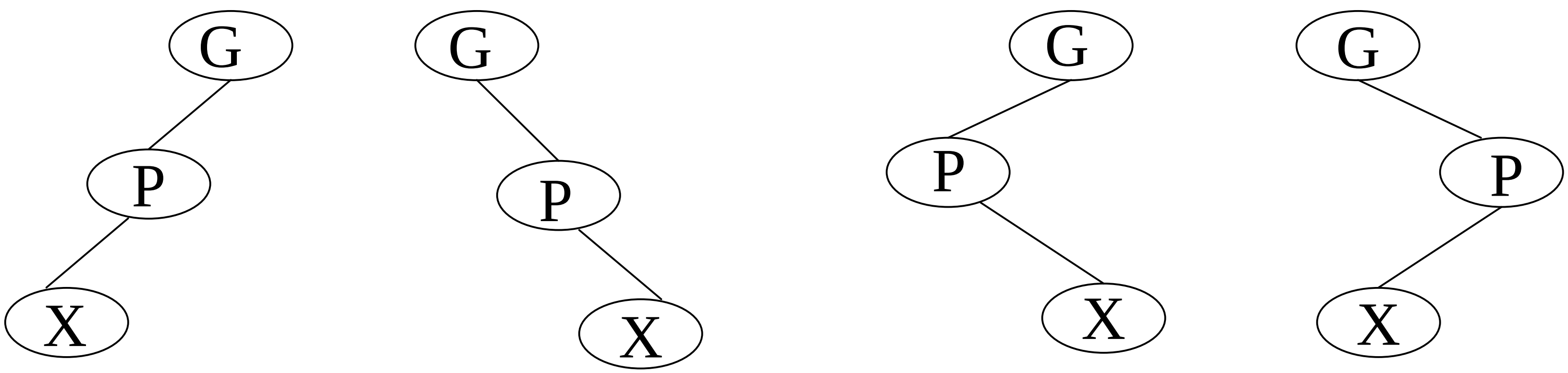
- Ordinary binary search trees have no balance conditions
  - › what you get from insertion order is it
- Balanced trees like AVL trees enforce a balance condition when nodes change
  - › tree is always balanced after an insert or delete
- Self-adjusting trees get reorganized over time as nodes are accessed
  - › Tree adjusts after insert, delete, or find

Splay Trees

- Splay trees are tree structures that:
  - › Are not perfectly balanced all the time
  - › Data most recently accessed is near the root. (principle of locality; 80-20 “rule”)
- The procedure:
  - › After node X is accessed, perform “splaying” operations to bring X to the root of the tree.
  - › Do this in a way that leaves the tree more balanced as a whole

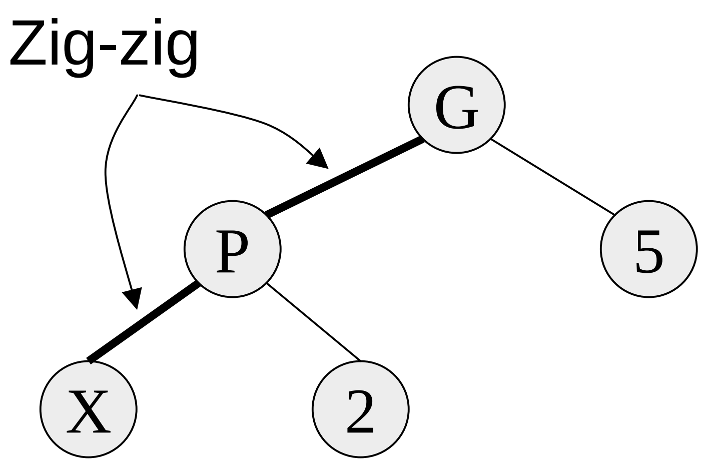
Splay Tree Terminology

- Let X be a non-root node with ≥ 2 ancestors.
  - P is its parent node.
  - G is its grandparent node.

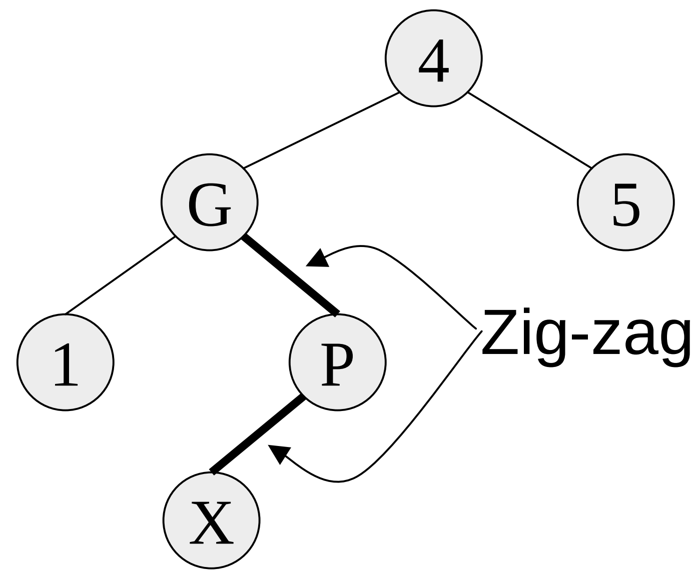


Zig-Zig and Zig-Zag

Parent and grandparent
in same direction.

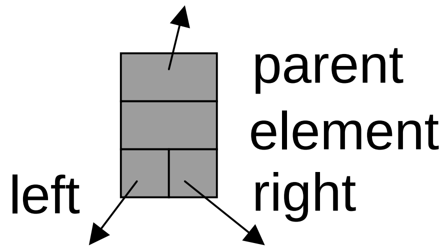


Parent and grandparent
in different directions.



Splay Tree Operations

1. Helpful if nodes contain a parent pointer.

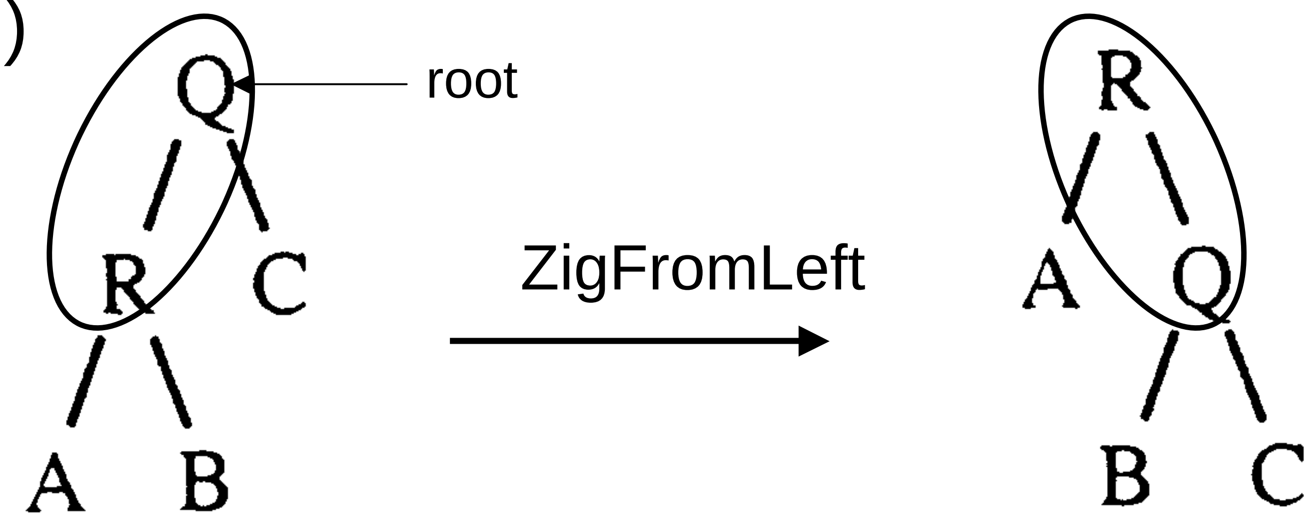


2. When X is accessed, apply one of six rotation routines.

- Single Rotations (X has a P (the root) but no G)
  ZigFromLeft, ZigFromRight
- Double Rotations (X has both a P and a G)
  ZigZigFromLeft, ZigZigFromRight
  ZigZagFromLeft, ZigZagFromRight

Zig at depth 1 (root)

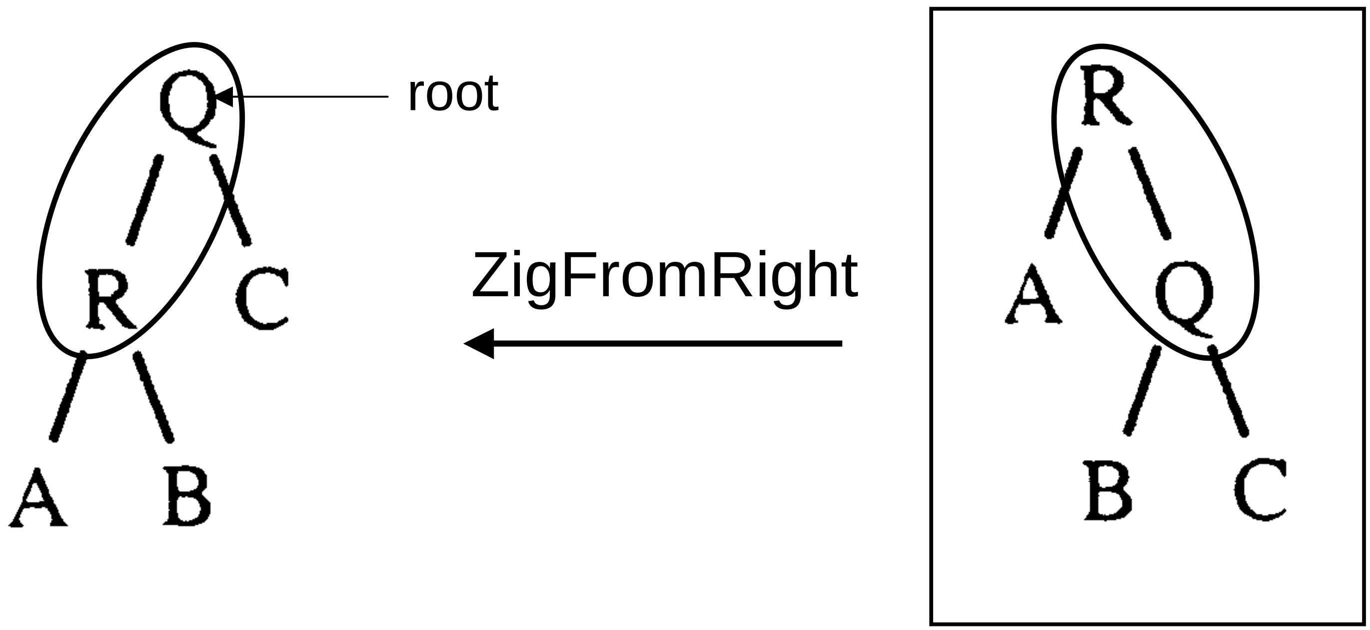
- “Zig” is just a single rotation, as in an AVL tree
- Let R be the node that was accessed (e.g. using Find)



- ZigFromLeft moves R to the top $\rightarrow$faster access next time

Zig at depth 1

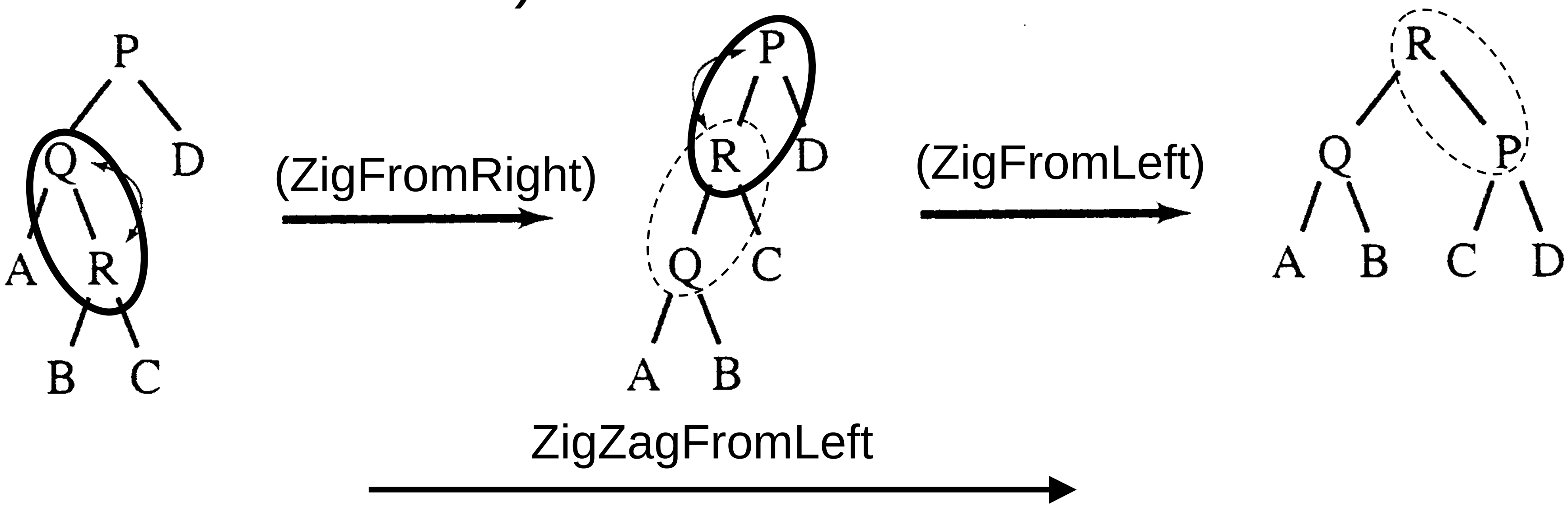
- Suppose Q is now accessed using Find



- ZigFromRight moves Q back to the top

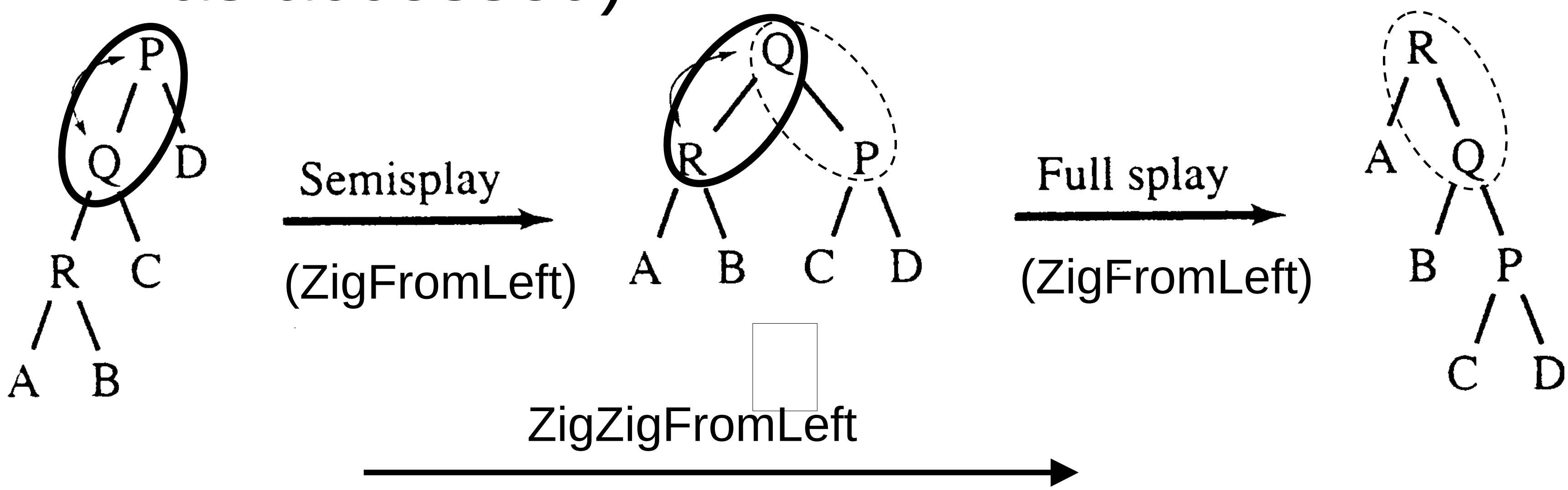
Zig-Zag operation

- “Zig-Zag” consists of two rotations of the opposite direction (assume R is the node that was accessed)

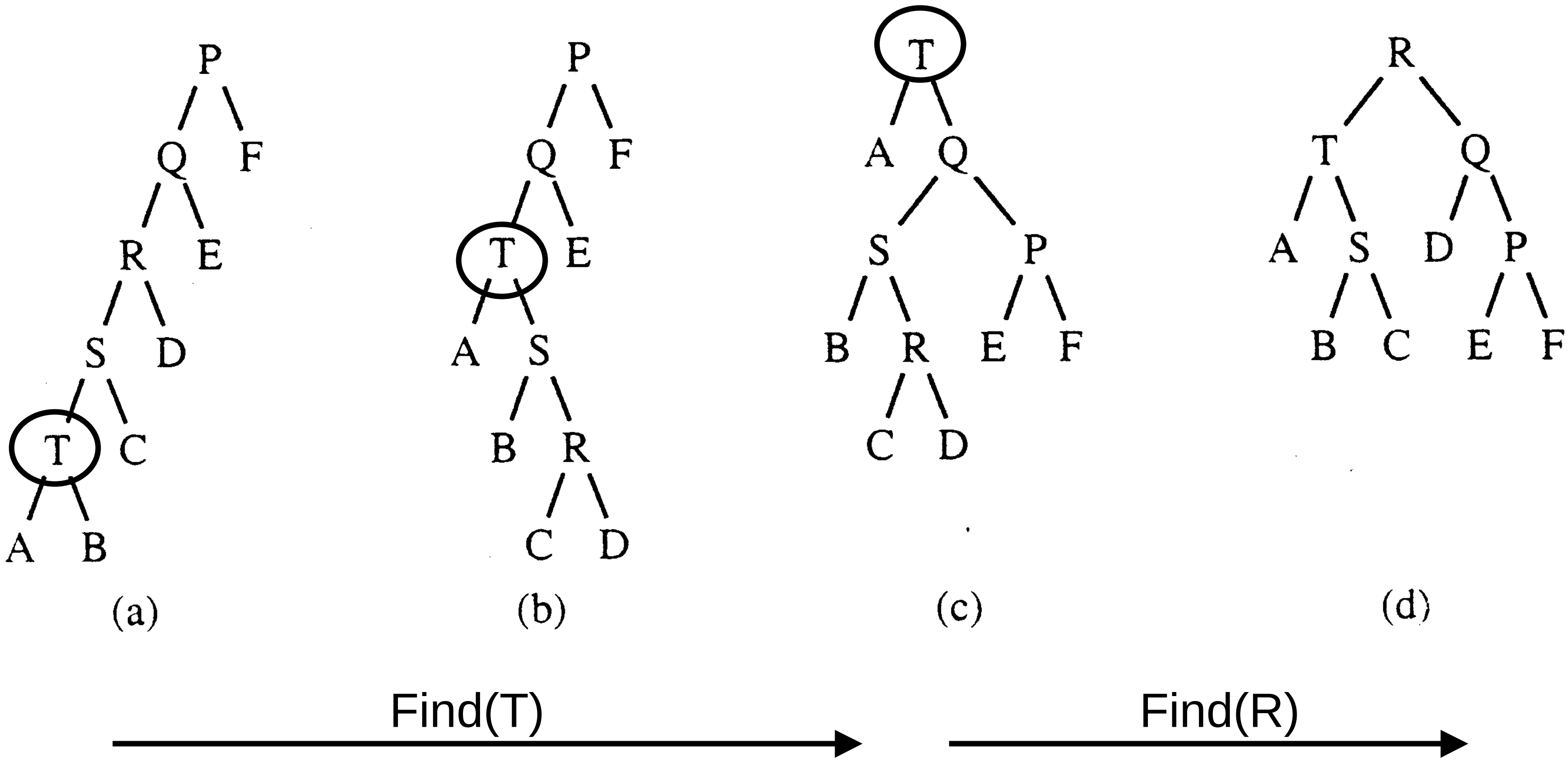


Zig-Zig operation

- “Zig-Zig” consists of two single rotations of the same direction (R is the node that was accessed)



Decreasing depth - "autobalance"

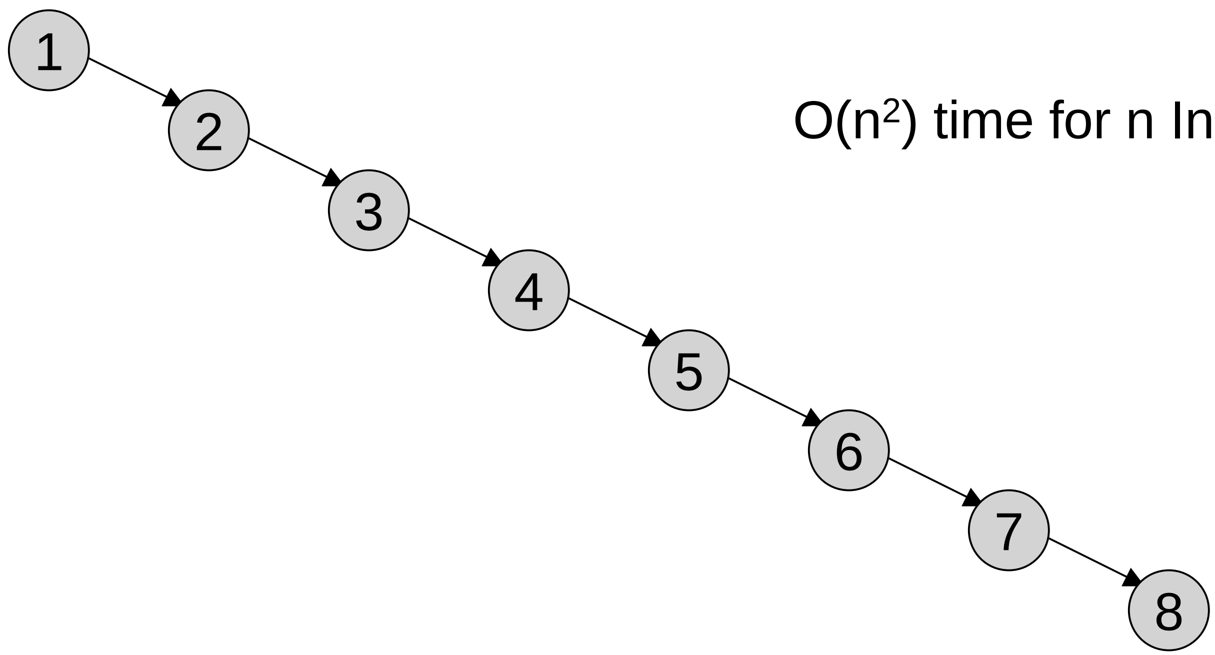


Splay Tree Insert and Delete

- Insert x
  - › Insert x as normal then splay x to root.
- Delete x
  - › Splay x to root and remove it. (note: the node does not have to be a leaf or single child node like in BST delete.) Two trees remain, right subtree and left subtree.
  - › Splay the max in the left subtree to the root
  - › Attach the right subtree to the new root of the left subtree.

Example Insert

- Inserting in order 1,2,3,…,8
- Without self-adjustment

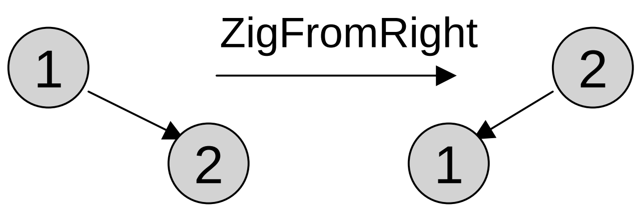


O(n^2) time for n Insert

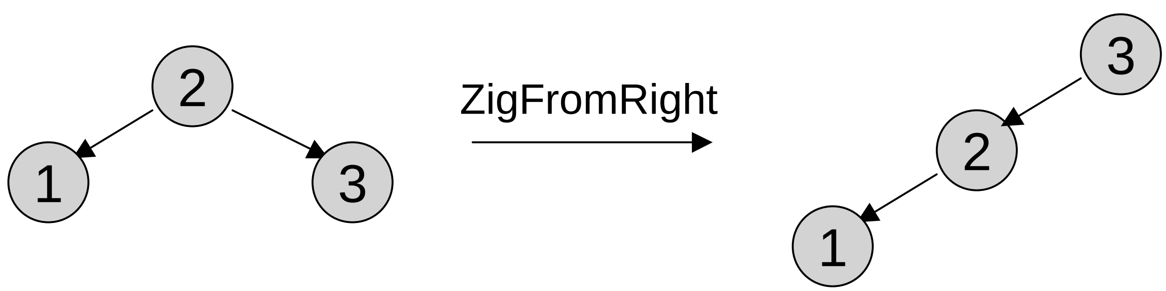
With Self-Adjustment

1

2

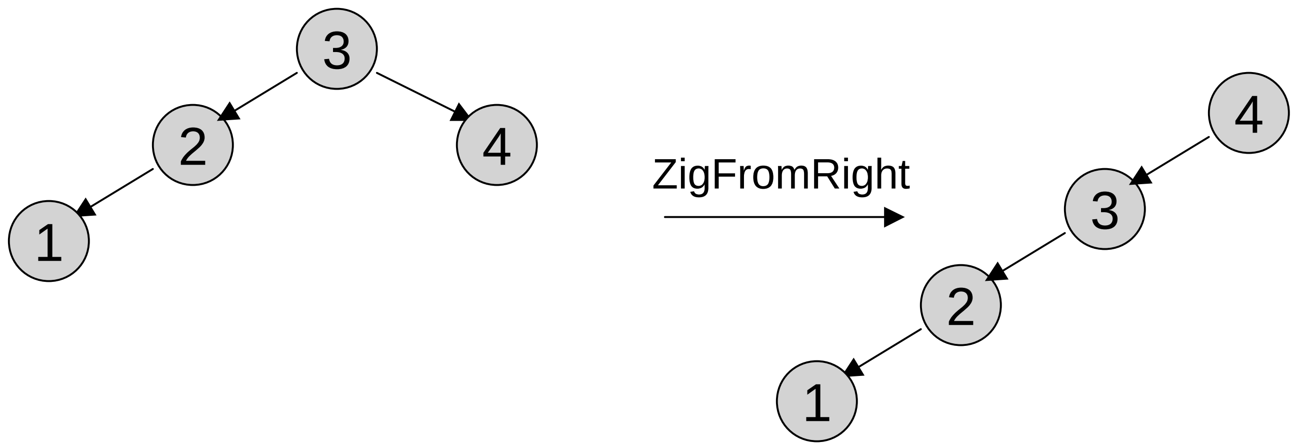


3



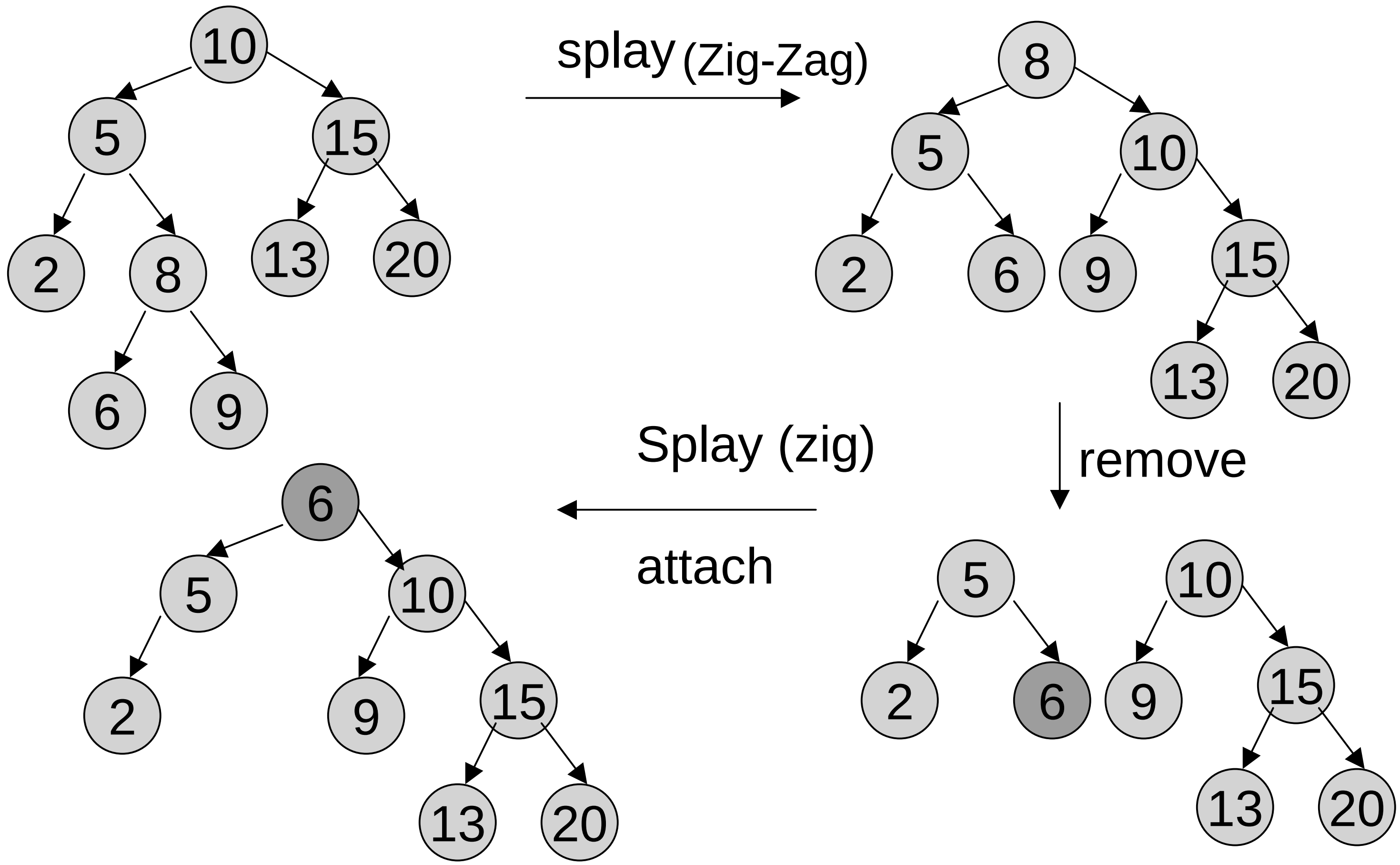
With Self-Adjustment

4



Each Insert takes O(1) time therefore O(n) time for n Insert!!

Example Deletion

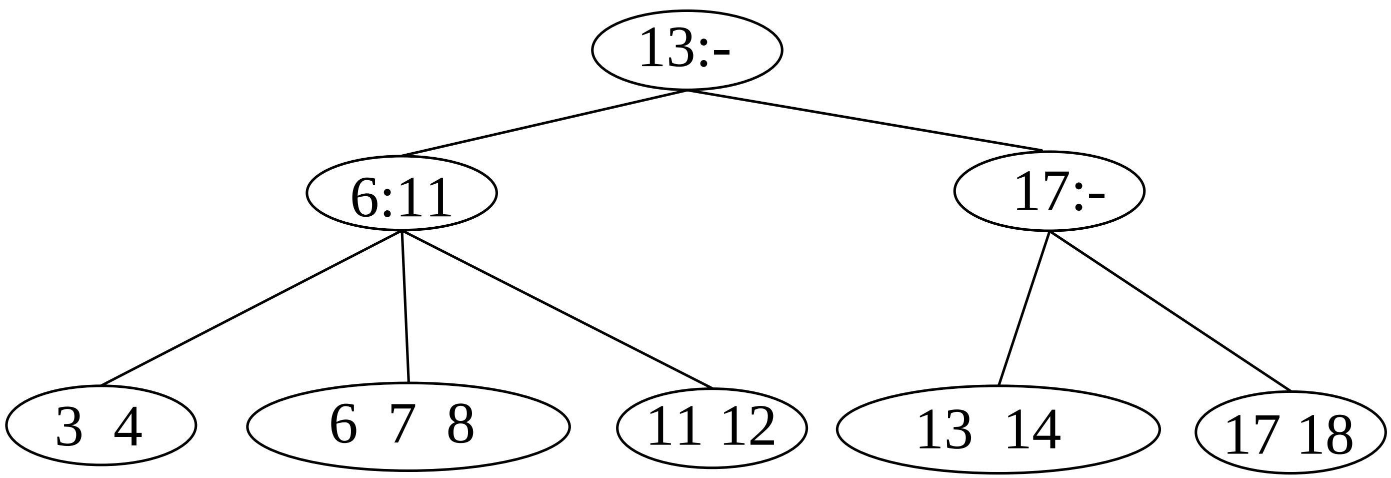


Analysis of Splay Trees

- Splay trees tend to be balanced
  - M operations takes time O(M log N) for M $\geq$ N operations on N items. (proof is difficult)
  - Amortized O(log n) time.
- Splay trees have good “locality” properties
  - Recently accessed items are near the root of the tree.
  - Items near an accessed one are pulled toward the root.

Beyond Binary Search Trees: Multi-Way Trees

- Example: B-tree of order 3 has 2 or 3 children per node



- Search for 8

B-Trees

B-Trees are multi-way search trees commonly used in database systems or other applications where data is stored externally on disks and keeping the tree shallow is important.

A B-Tree of order M has the following properties:

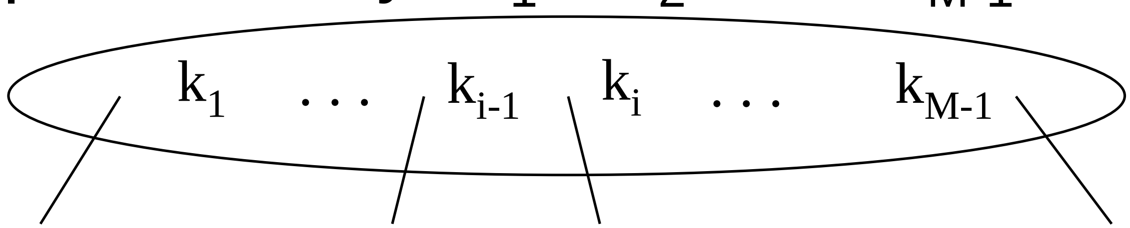
1. The root is either a leaf or has between 2 and M children.
2. All nonleaf nodes (except the root) have between $\lceil M/2 \rceil$ and M children.
3. All leaves are at the same depth.

All data records are stored at the leaves.
Internal nodes have "keys" guiding to the leaves.
Leaves store between $\lceil M/2 \rceil$ and M data records.

B-Tree Details

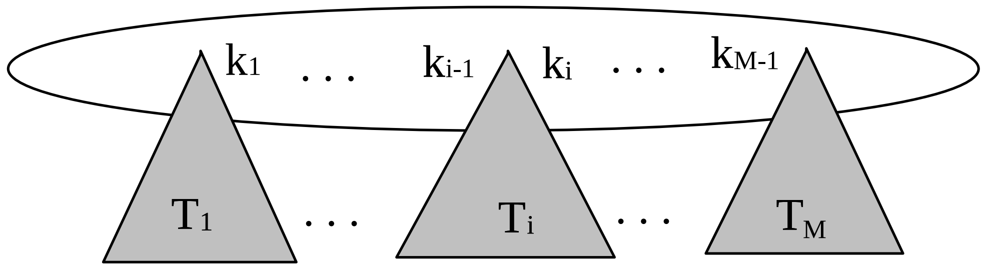
Each (non-leaf) internal node of a B-tree has:

- Between $\lceil M/2 \rceil$ and M children.
- up to M-1 keys $k_1 < k_2 < ... < k_{M-1}$



Keys are ordered so that:
$k_1 < k_2 < ... < k_{M-1}$

Properties of B-Trees



Children of each internal node are "between" the items in that node.
Suppose subtree T_i is the *i*th child of the node:

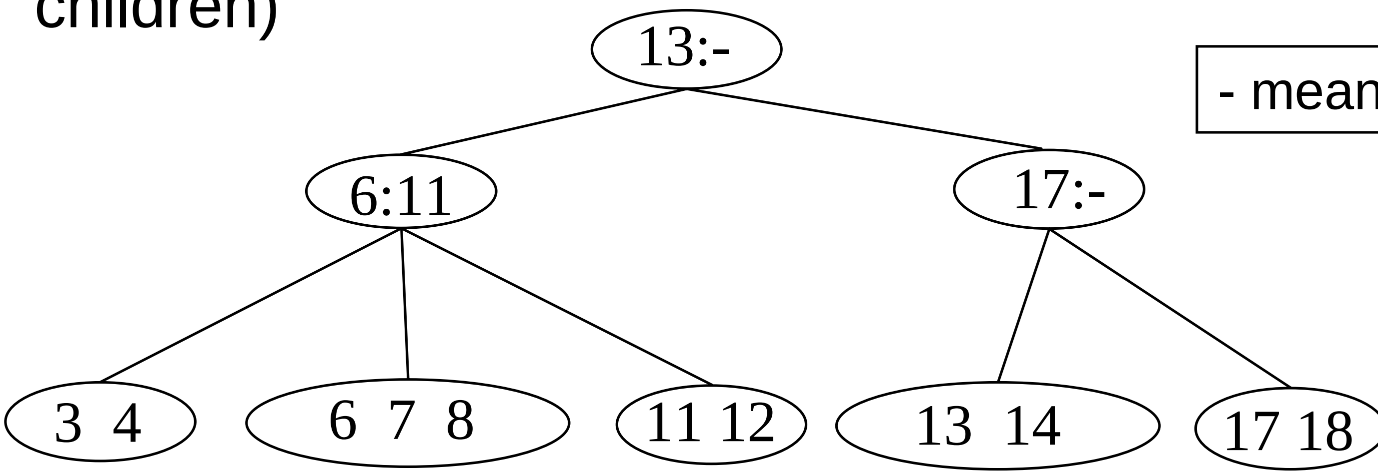
all keys in T_i must be between keys k_{i-1} and k_i

i.e. $k_{i-1} \leq T_i < k_i$

k_{i-1} is the smallest key in T_i
All keys in first subtree $T_1 < k_1$
All keys in last subtree $T_M \geq k_{M-1}$

Example: Searching in B-trees

- B-tree of order 3: also known as 2-3 tree (2 to 3 children)

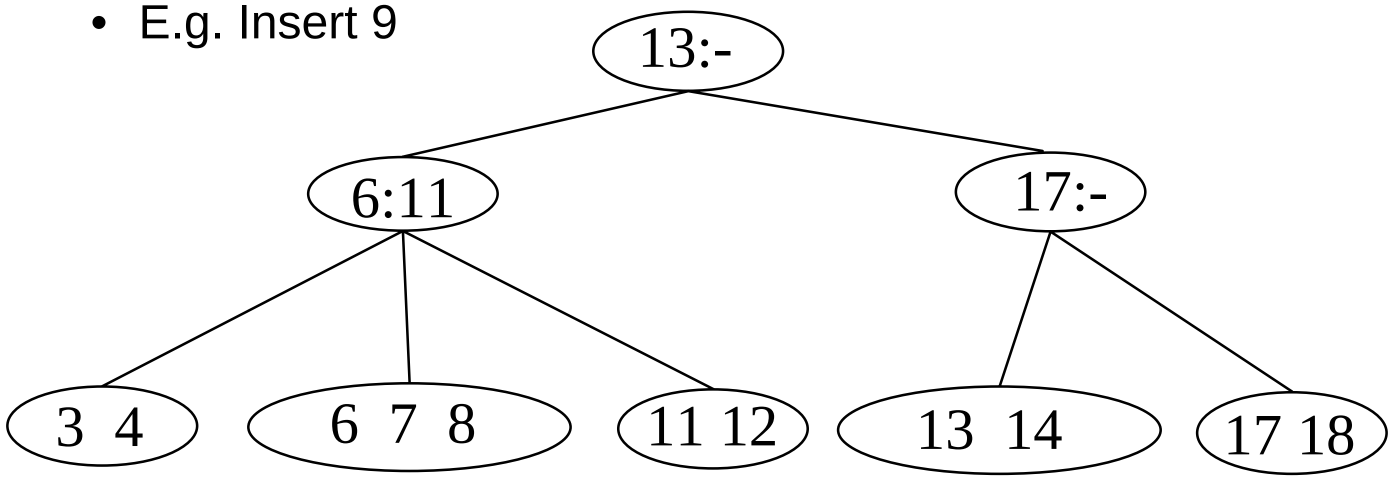


- means empty slot

- Examples: Search for 9, 14, 12
- Note: If leaf nodes are connected as a Linked List, B-tree is called a B+ tree – Allows sorted list to be accessed easily

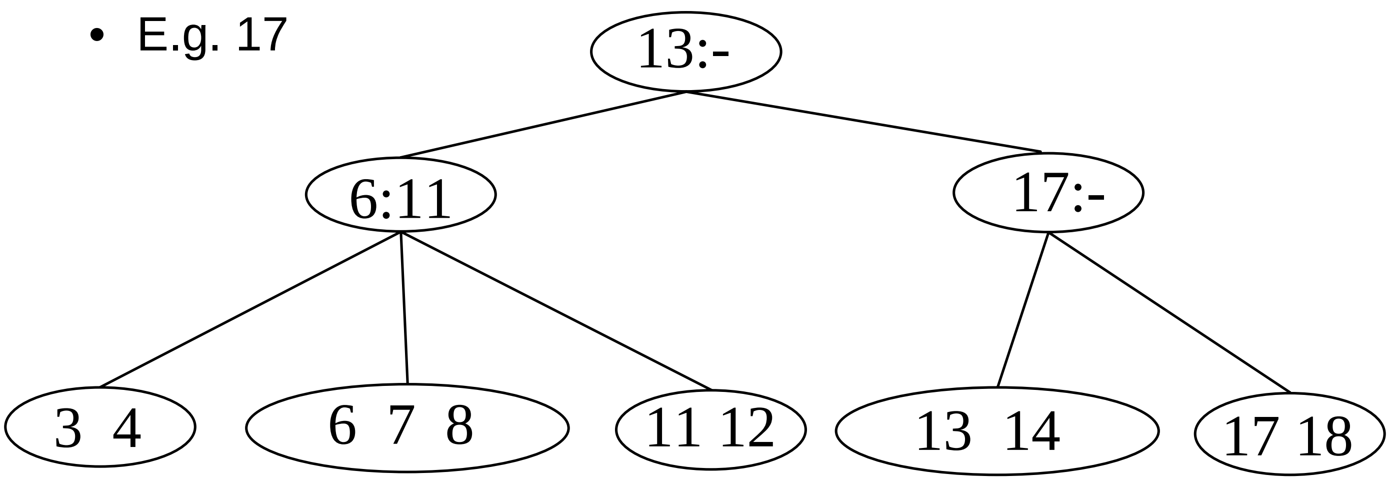
Inserting into B-Trees

- Insert X: Do a Find on X and find appropriate leaf node
  - › If leaf node is not full, fill in empty slot with X
    - E.g. Insert 5
  - › If leaf node is full, split leaf node and adjust parents up to root node
    - E.g. Insert 9



Deleting From B-Trees

- Delete X : Do a find and remove from leaf
  - › Leaf underflows – borrow from a neighbor
    - E.g. 11
  - › Leaf underflows and can't borrow – merge nodes, delete parent
    - E.g. 17



Run Time Analysis of B-Tree Operations

- For a B-Tree of order M
  - › Each internal node has up to M-1 keys to search
  - › Each internal node has between $\lceil M/2 \rceil$ and M children
  - › Depth of B-Tree storing N items is $O(\log_{\lceil M/2 \rceil} N)$
- Find: Run time is:
  - › O(log M) to binary search which branch to take at each node. But M is small compared to N.
  - › Total time to find an item is O(depth*log M) = O(log N)

Summary of Search Trees

- Problem with Binary Search Trees: Must keep tree balanced to allow fast access to stored items
- AVL trees: Insert/Delete operations keep tree balanced
- Splay trees: Repeated Find operations produce balanced trees
- Multi-way search trees (e.g. B-Trees): More than two children
  - › per node allows shallow trees; all leaves are at the same depth
  - › keeping tree balanced at all times