Splay Trees

CSE 373

Data Structures

Unit 8

Reading: Sections 4.5-4.6

Self adjusting Trees

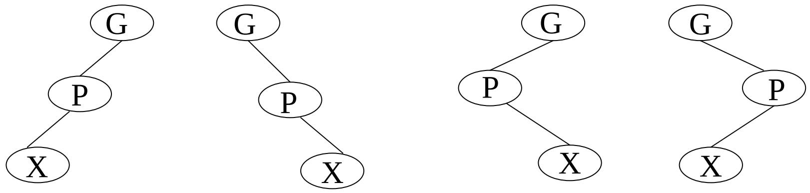
- Ordinary binary search trees have no balance conditions
  - › what you get from insertion order is it
- Balanced trees like AVL trees enforce a balance condition when nodes change
  - › tree is always balanced after an insert or delete
- Self-adjusting trees get reorganized over time as nodes are accessed
  - › Tree adjusts after insert, delete, or find

Splay Trees

- Splay trees are tree structures that:
  - › Are not perfectly balanced all the time
  - › Data most recently accessed is near the root. (principle of locality; 80-20 "rule")
- The procedure:
  - › After node X is accessed, perform "splaying" operations to bring X to the root of the tree.
  - › Do this in a way that leaves the tree more balanced as a whole

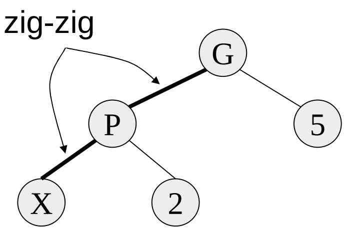
Splay Tree Terminology

- Let X be a non-root node with ≥ 2 ancestors.
  - P is its parent node.
  - G is its grandparent node.

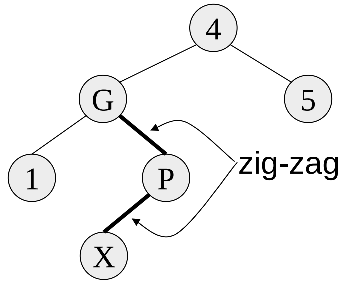


Zig-Zig and Zig-Zag

Parent and grandparent
in same direction.

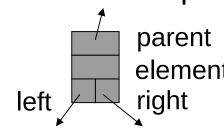


Parent and grandparent
in different directions.



Splay Tree Operations

1. Helpful if nodes contain a parent pointer.

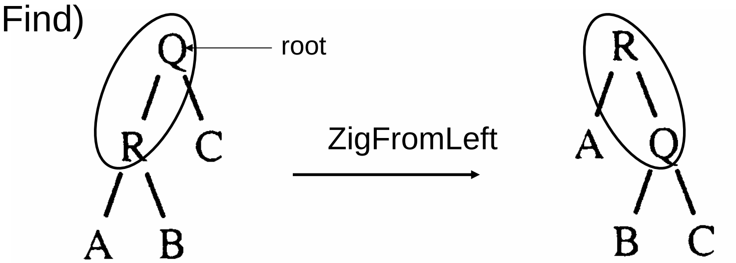


2. When X is accessed, apply one of six rotation routines.

- Single Rotations (X has a P (the root) but no G)
  ZigFromLeft, ZigFromRight
- Double Rotations (X has both a P and a G)
  ZigZigFromLeft, ZigZigFromRight
  ZigZagFromLeft, ZigZagFromRight

Zig at depth 1 (root)

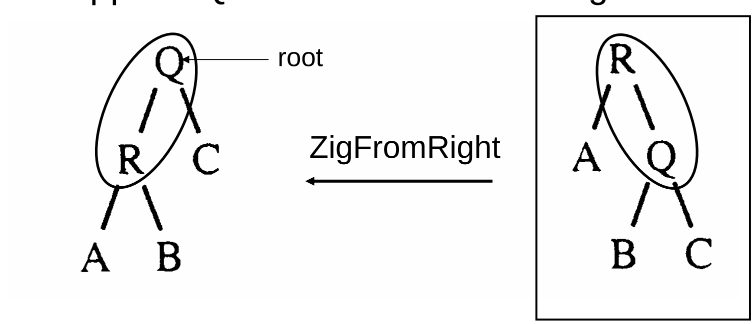
- “Zig” is just a single rotation, as in an AVL tree
- Let R be the node that was accessed (e.g. using Find)



- ZigFromLeft moves R to the top $\rightarrow$faster access next time

Zig at depth 1

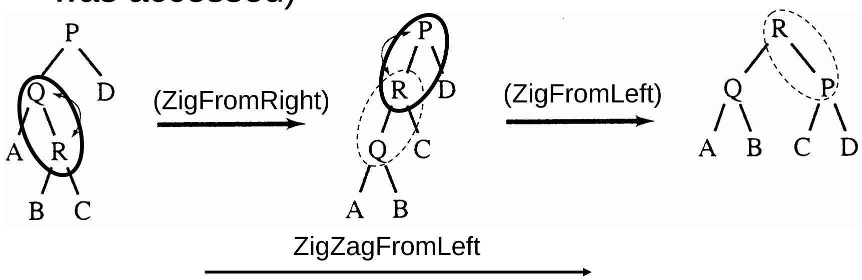
- Suppose Q is now accessed using Find



- ZigFromRight moves Q back to the top

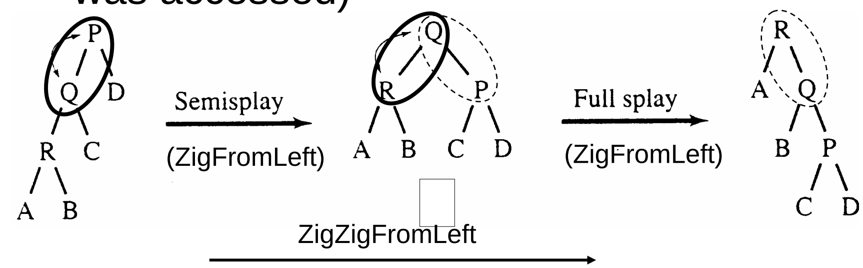
Zig-Zag operation

- “Zig-Zag” consists of two rotations of the opposite direction (assume R is the node that was accessed)

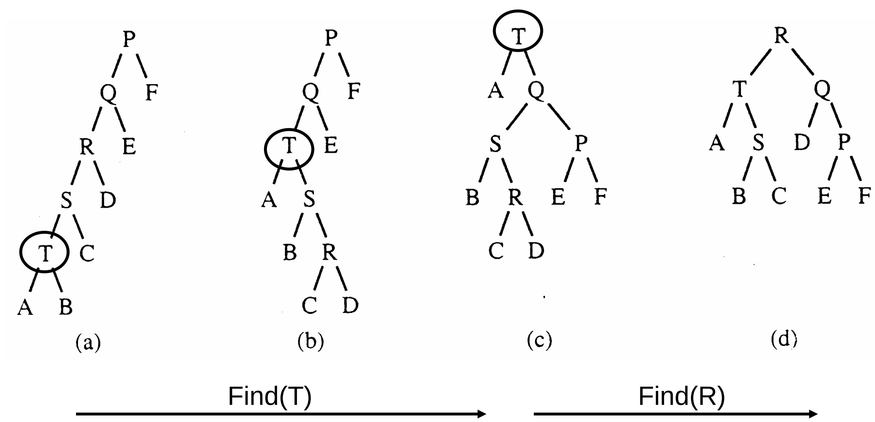


Zig-Zig operation

- “Zig-Zig” consists of two single rotations of the same direction (R is the node that was accessed)



Decreasing depth - "autobalance"

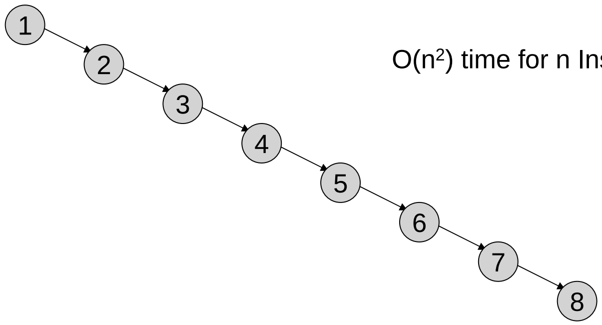


Splay Tree Insert and Delete

- Insert x
  - Insert x as normal then splay x to root.
- Delete x
  - Splay x to root and remove it. (note: the node does not have to be a leaf or single child node like in BST delete.) Two trees remain, right subtree and left subtree.
  - Splay the max in the left subtree to the root
  - Attach the right subtree to the new root of the left subtree.

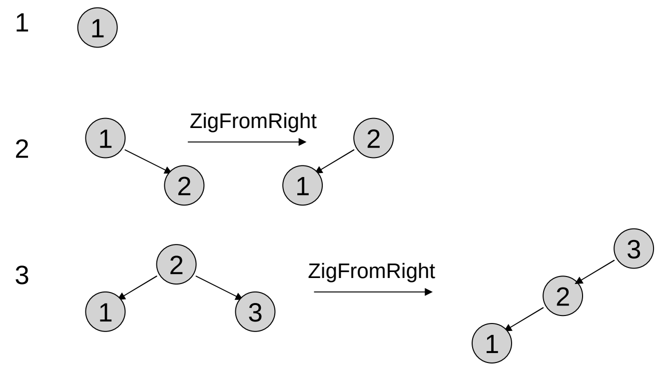
Example Insert

- Inserting in order 1,2,3,…,8
- Without self-adjustment

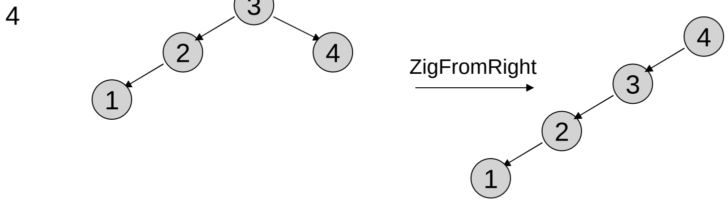


$O(n^2)$ time for n Insert

With Self-Adjustment

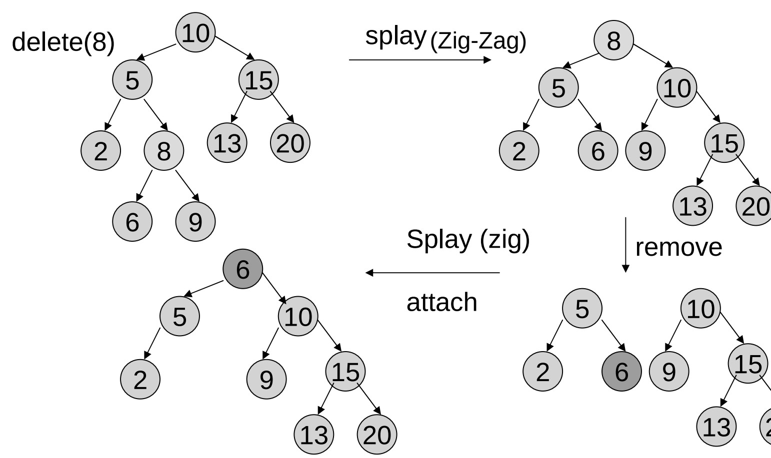


With Self-Adjustment



Each Insert takes O(1) time therefore O(n) time for n Insert!!

Example Deletion



Analysis of Splay Trees

- Splay trees tend to be balanced
  - › M operations takes time O(M log N) for M $\geq$ N operations on N items. (proof is difficult)
  - › Amortized O(log n) time.
- Splay trees have good “locality” properties
  - › Recently accessed items are near the root of the tree.
  - › Items near an accessed one are pulled toward the root.

Summary of Search Trees

- Problem with Binary Search Trees: Must keep tree balanced to allow fast access to stored items

- AVL trees: Insert/Delete operations keep tree balanced
- Splay trees: Repeated Find operations produce balanced trees
- Multi-way search trees (e.g. B-Trees):
  - › More than two children per node allows shallow trees; all leaves are at the same depth.
  - › Keeping tree balanced at all times.
  - › Excellent for indexes in database systems.