CSE 373 – Data Structures and Algorithms
Autumn 2003
Dry assignment #5. Due date: 12/05/03

1. *(26 points)*

1.a A *directed* graph is given by the following adjacency matrix, M, in which M(i,j)=1 if and only if there is an edge from i to j. Find a topological sort of the graph.

	A	B	C	D	E	F
A	0	1	0	0	0	0
B	0	0	1	0	0	0
C	0	0	0	1	0	0
D	0	0	0	0	0	0
E	0	1	1	0	0	0
F	1	0	1	0	0	0

1.b A weighted undirected graph G=(V,E) is given by the following adjacency matrix, M, in which M=(j,i)=M(i,j)=c(i,j). Use Dijkstra's algorithm to find the length of the shortest paths from S to any other vertex in G. For each vertex v, describe $\lambda(v)$ in any stage of the algorithm. Remark: M(i,j)= $\propto$ means that $(i,j) \notin E$.

	S	A	B	C	D	E
S	$\propto$	3	1	$\propto$	8	$\propto$
A	3	$\propto$	$\propto$	2	6	$\propto$
B	1	$\propto$	$\propto$	5	$\propto$	2
C	$\propto$	2	5	$\propto$	$\propto$	4
D	8	6	$\propto$	$\propto$	$\propto$	4
E	$\propto$	$\propto$	2	4	4	$\propto$

2. (*26 points*) Let $G=(V, E)$ be an undirected graph with weights on the edges.
Let $s,t \in V$, $e \in E$. Give efficient algorithms for each of the following problems:

a. Does *e* belong to *all* shortest paths connecting *s* and *t*?
b. Does *e* belong to *some* shortest path connecting *s* and *t*?

Explain why your algorithms are correct and analyze their time complexity.

The more efficient your algorithms are, the more points you are going to receive.

3. (*24 points*) Consider each of the statements below independently of each other. If it is true, justify it; if it is false, give a counter example.

3.1. Suppose that E' is a subset of the edges of a graph G_1 such that there is a minimum spanning tree of G_1 that contains all edges in E'. Let *e* be the minimum weight edge in the graph such that *e* is not contained in E'. Then $E' \cup \{e\}$ is contained in some minimum spanning tree of G_1.

3.2. Let (u,v) be the minimum weight edge in a graph G_2. Then (u,v) belongs to some minimum spanning tree of G_2.

3.3. In a graph $G_3=(V, E)$, if the shortest distance between vertices *A* and *B* is a single edge that connects them, this edge is guaranteed to be in some minimum spanning tree of G_3.

4. (*24 points*) How can we use the Bellman-Ford algorithm to detect in $O(|E|*|V|)$ steps if a given directed graph contains a negative cycle? Explain.