# Trees - Intro

CSE 373 - Data Structures
April 15, 2002

# Readings and References

- Reading
  - › Chapter 4.1-4.3, *Data Structures and Algorithm Analysis in C*, Weiss
- Other References



# Why Do We Need Trees?

- Lists, Stacks, and Queues are linear relationships
- Information often contains hierarchical relationships
  - › File directories or folders on your computer
  - › Moves in a game
  - › Employee hierarchies in organizations
- Can build a tree to support fast searching



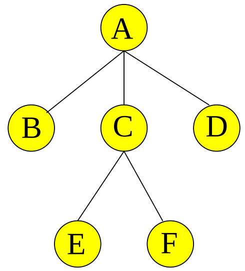
# Tree Jargon

- root
- nodes and edges
- leaves

- parent, children, siblings
- ancestors, descendants

- subtrees

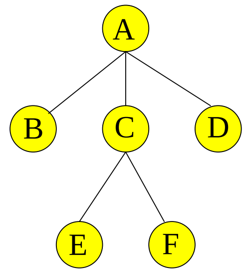
- path, path length
- height, depth



# More Tree Jargon

- **Length** of a path = number of edges
- **Depth** of a node N = length of path from root to N
- **Height** of node N = length of longest path from N to a leaf
- **Depth of tree** = depth of deepest node
- **Height of tree** = height of root

depth=0, height = 2




depth = 2, height=0



# Definition and Tree Trivia

- A tree is a set of nodes
  - that is an empty set of nodes, or
  - has one node called the root from which zero or more trees (subtrees) descend
- A tree with N nodes always has N-1 edges
- Two nodes in a tree have at most one path between them



# Paths

- Can a non-zero path from node N reach node N again?
    - No. Trees can never have cycles (loops)
- Does depth of nodes in a non-zero path increase or decrease?
  - › Depth always increases in a non-zero path



# Implementation of Trees

- One possible pointer-based Implementation
  - › tree nodes with value and a pointer to each child
  - › but how many pointers should we allocate space for?
- A more flexible pointer-based implementation
  - › 1st Child / Next Sibling List Representation
  - › Each node has 2 pointers: one to its first child and one to next sibling
  - › Can handle arbitrary number of children

# Application: Arithmetic Expression Trees

Example Arithmetic Expression:

A + (B * (C / D) )

How would you express this as a tree?

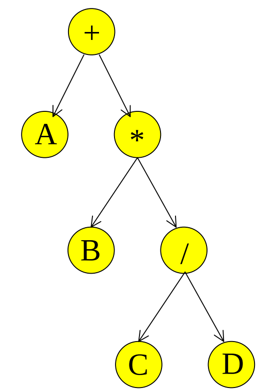


# Application: Arithmetic Expression Trees

Example Arithmetic Expression:

A + (B * (C / D) )

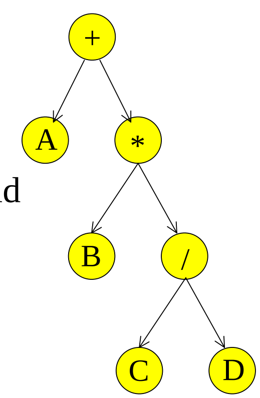
Tree for the above expression:




- Used in most compilers
- No parenthesis need – use tree structure
- Can speed up calculations e.g. replace / node with C/D if C and D are known
- Calculate by traversing tree (how?)



# Traversing Trees




- Preorder: Node, then Children
  + A * B / C D
- Inorder: Left child, Node, Right child
  A + B * C / D
- Postorder: Children, then Node
  A B C D / * +



# Binary Trees

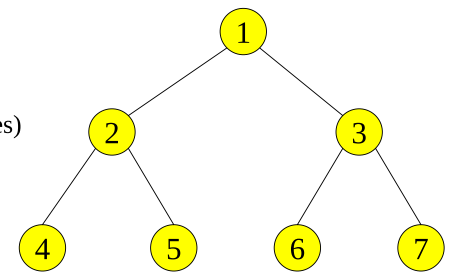
- Every node has at most two children
  - › Most popular tree in computer science
  - › Easy to implement, fast in operation
- Given N nodes, what is the minimum depth of a binary tree?
  - › At depth d, you can have $N = 2^d$ to $2^{d+1}-1$ nodes
  - › minimum depth d is: $\log N \leq d \leq \log(N+1)-1$ or $\Theta(\log N)$

# Minimum depth vs node count

- At depth d, you can have $N = 2^d$ to $2^{d+1}-1$ nodes
- minimum depth d is $\log N \leq d \leq \log(N+1)-1$ or $\Theta(\log N)$

d=2

$N=2^2$ to $2^3-1$ (ie, 4 to 7 nodes)



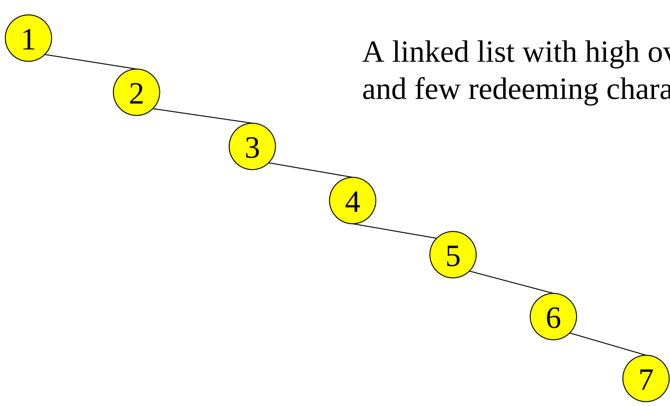



# Maximum depth vs node count

- What is the maximum depth of a binary tree?
  - › Degenerate case: Tree is a linked list!
  - › Maximum depth = N-1
- Goal: Would like to keep depth at around log N to get better performance than linked list for operations like Find



# A degenerate tree

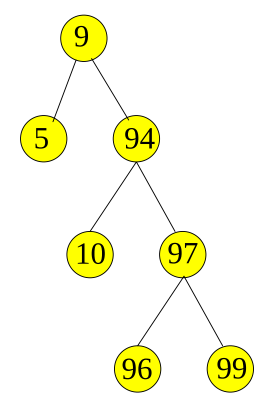



A linked list with high overhead and few redeeming characteristics



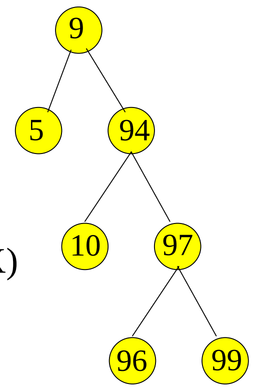
# Binary Search Trees

- Binary search trees are binary trees in which
  - › all values in the node's left subtree are less than node value
  - › all values in the node's right subtree are greater than node value
- Operations:
  - › Find, FindMin, FindMax, Insert, Delete



# Operations on Binary Search Trees

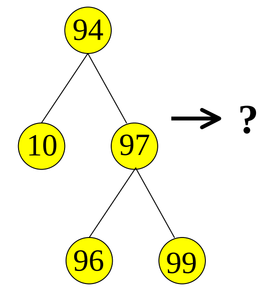
- How would you implement these?
  - › Recursive definition of binary search trees allows recursive routines
- Position FindMin(Tree T)
- Position FindMax(Tree T)
- Position Find(Tree T, ElementType X)
- Tree Insert(Tree T,ElementType X)
- Tree Delete(Tree T, ElementType X)





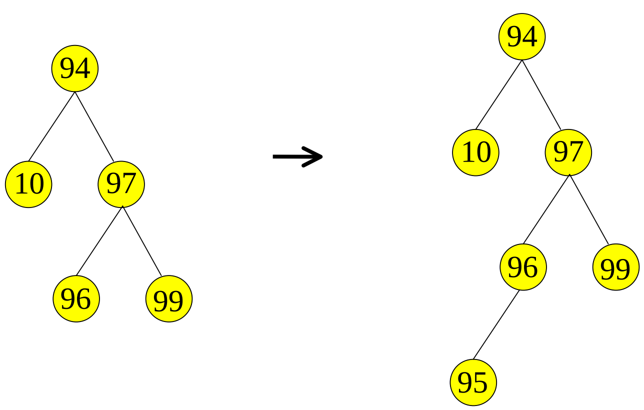

# Insert Operation

- **`Tree Insert(Tree T, ElementType X)`**
  - › Do a “Find” operation for X
  - › If X is found → update duplicates counter
  - › Else, “Find” stops at a NULL pointer
  - › Insert Node with X there
- Example: Insert 95





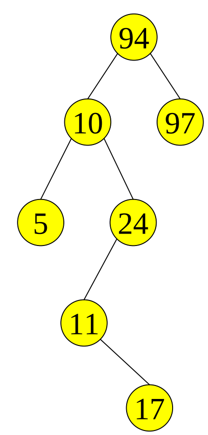

# Insert 95





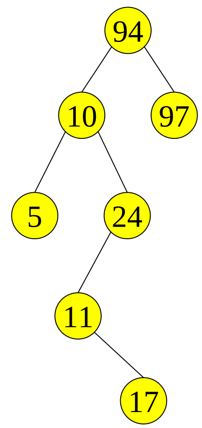

# Delete Operation

- Delete is a bit trickier…Why?
- Suppose you want to delete 10
- Strategy:
  - › Find 10
  - › Delete the node containing 10
- Problem: When you delete a node, what do you replace it by?



# Delete Operation

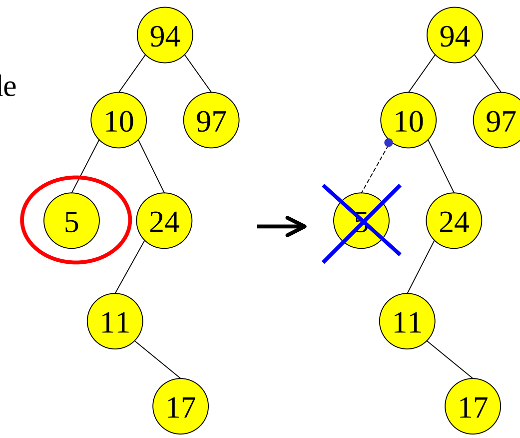
- Problem: When you delete a node, what do you replace it by?
- Solution:
  - › If it has no children, by NULL
  - › If it has 1 child, by that child
  - › If it has 2 children, by the node with the smallest value in its right subtree
- Examples:
  - › Delete 5
  - › Delete 24
  - › Delete 10 (note: recursive deletion)






# Delete “5” - No children

Find 5 node

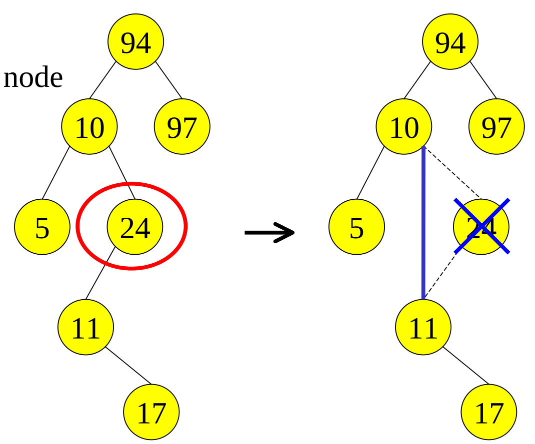



Then Free the 5 node and NULL the pointer to it



# Delete “24” - One child

Find 24 node

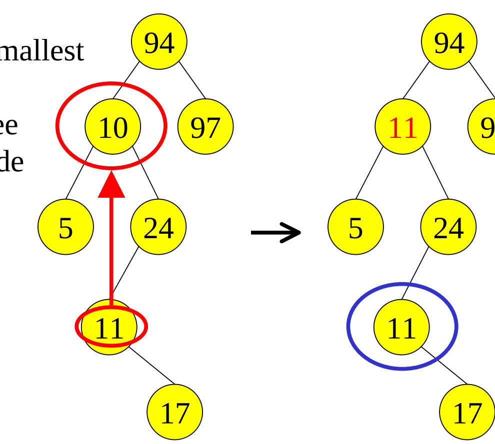



Then Free the 24 node and replace the pointer to it with a pointer to its child



# Delete “10” - two children

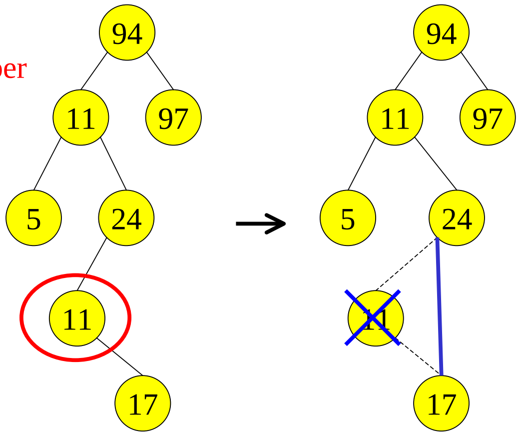
Find 10, Copy the smallest value in right subtree into the node




Then recursively Delete node with smallest value in right subtree
Note: it does not have two children

# Delete "11" - One child

Remember
11 node




Then Free
the 11 node and
replace the
pointer to it with
a pointer to its
child