Graph Searching

CSE 373

Data Structures

Lecture 20

Graph Searching

- Find Properties of Graphs
  - Connected components
  - Bipartite structure
  - Biconnected components
- Applications
  - Alternating paths for matching
  - Garbage collection – used in Java run time system
  - Finding dead code
  - Finding the web graph – used by Google and others

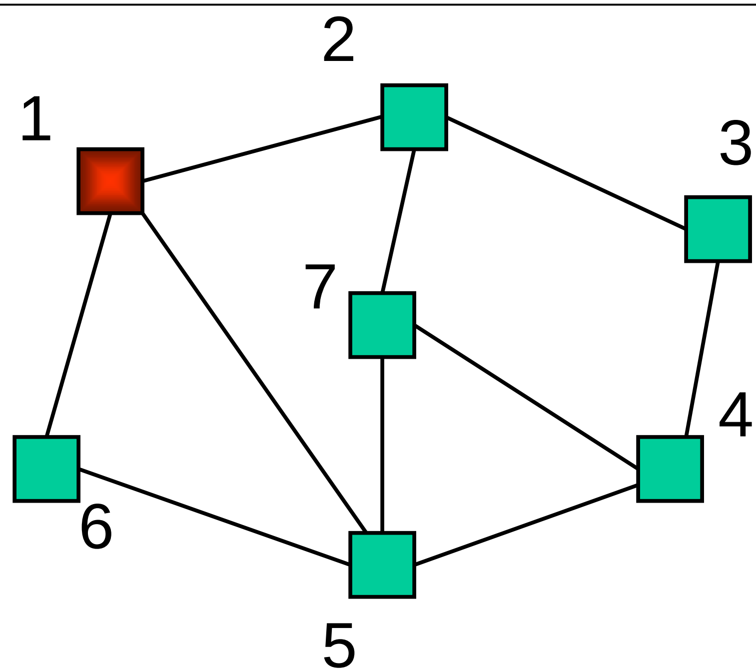
Depth First Search Algorithm

- Recursive marking algorithm
- Initially every vertex is unmarked

```
DFS(i: vertex)
  mark i;
  for each j adjacent to i do
    if j is unmarked then DFS(j)
end{DFS}
```

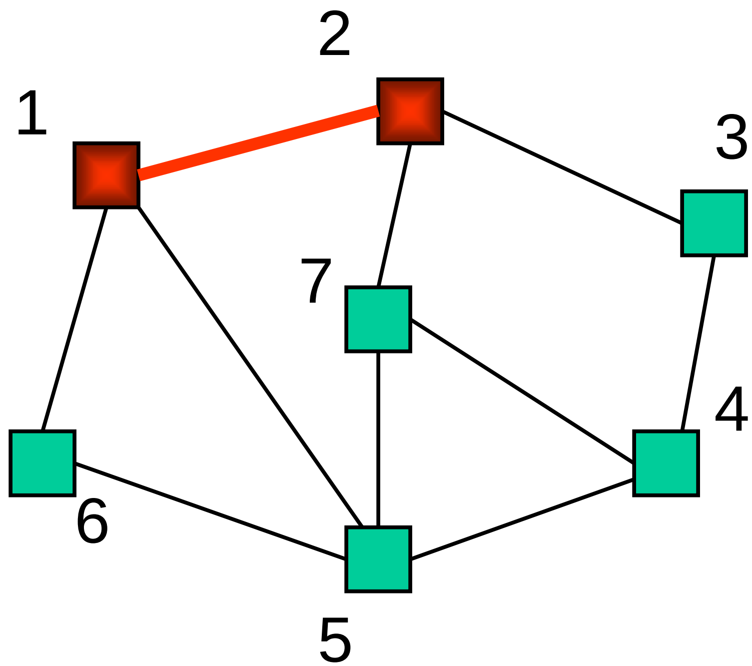
Marks all vertices reachable from i

Example of Depth First Search



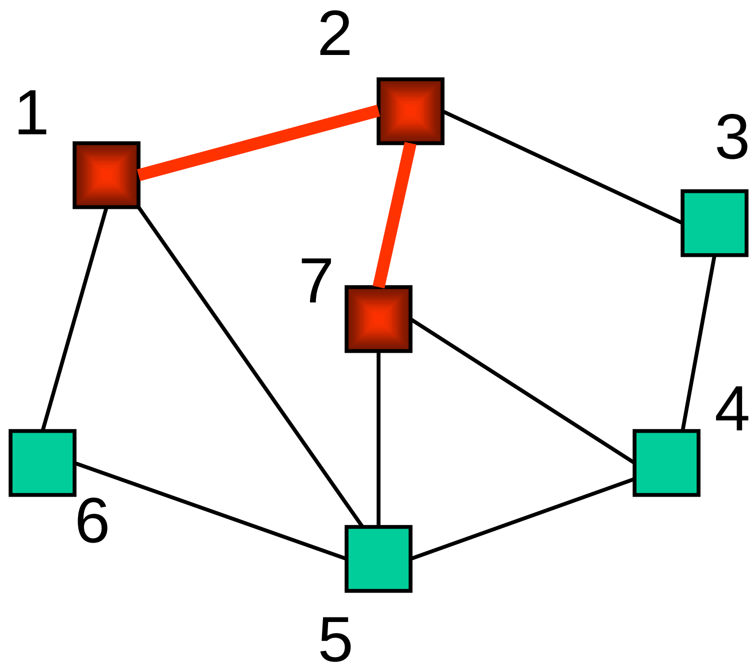
DFS(1)

Example Step 2



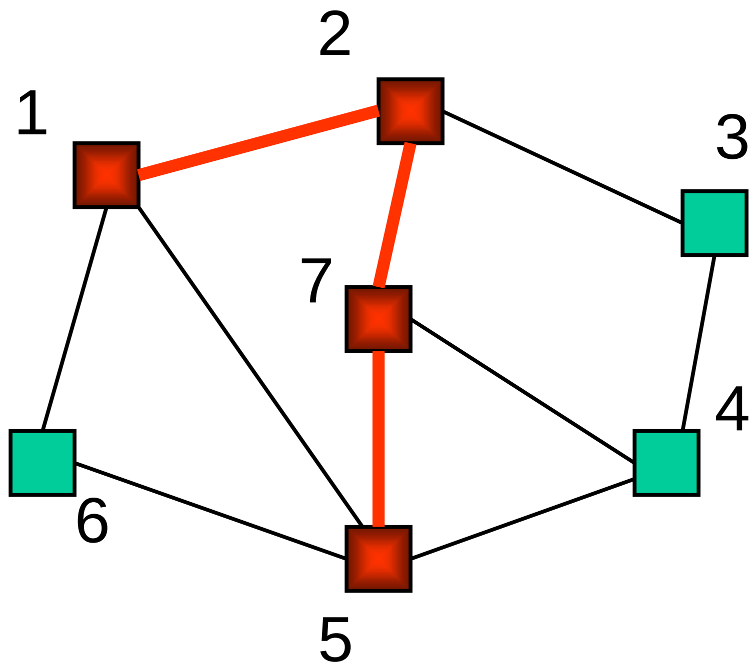
DFS(1)
DFS(2)

Example Step 3



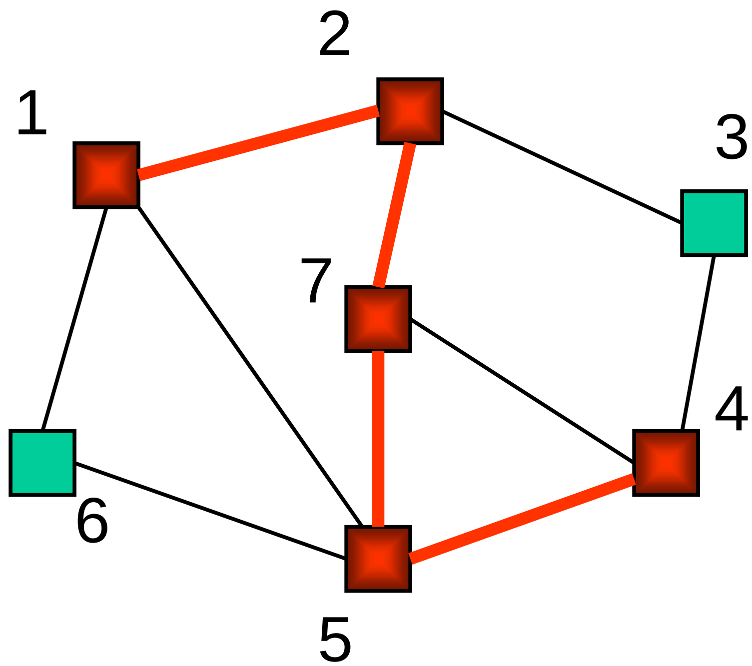
DFS(1)
DFS(2)
DFS(7)

Example Step 4



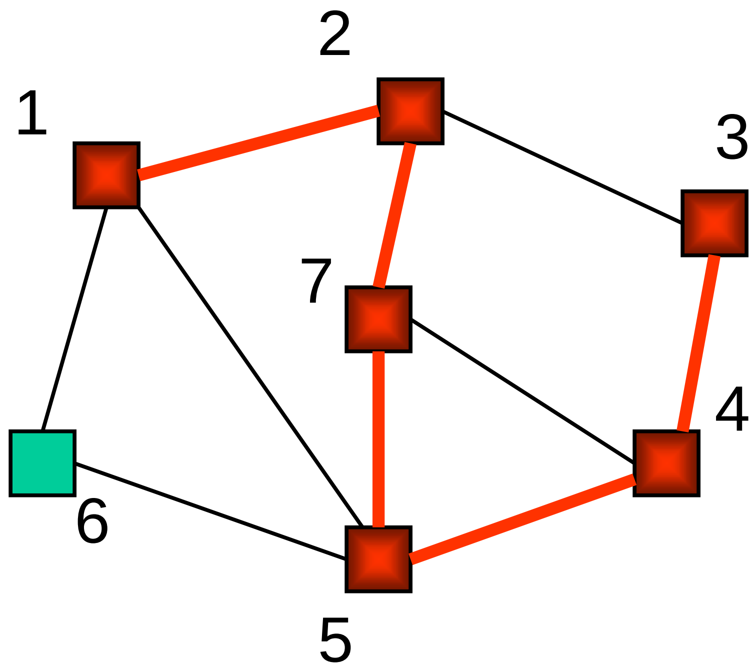
DFS(1)
DFS(2)
DFS(7)
DFS(5)

Example Step 5



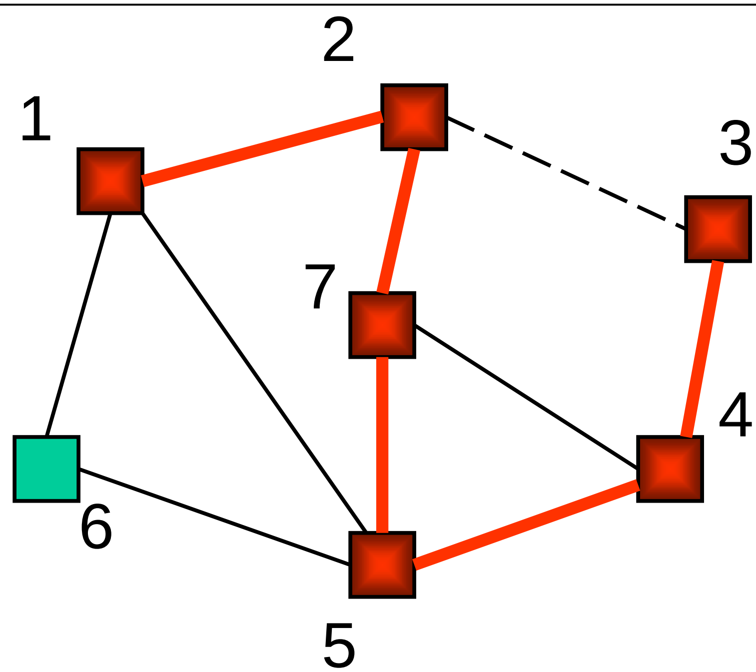
DFS(1)
DFS(2)
DFS(7)
DFS(5)
DFS(4)

Example Step 6



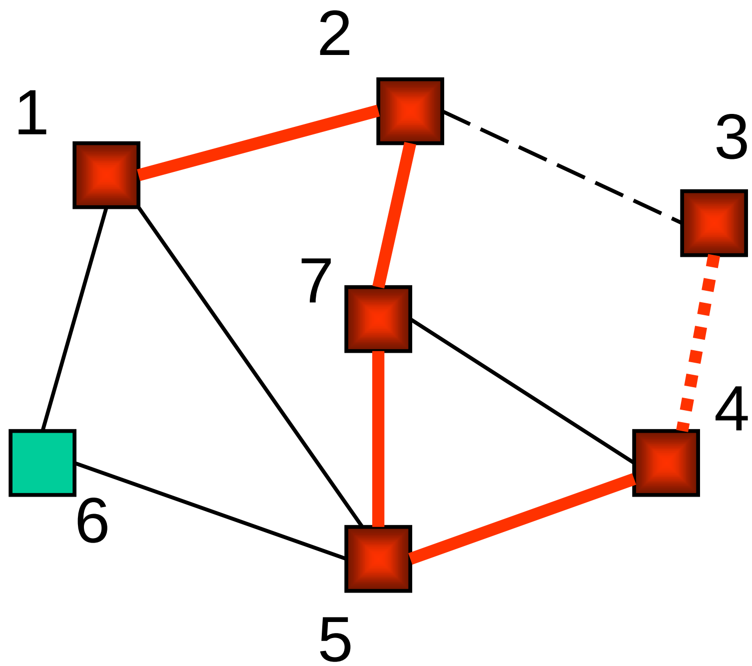
DFS(1)
DFS(2)
DFS(7)
DFS(5)
DFS(4)
DFS(3)

Example Step 7



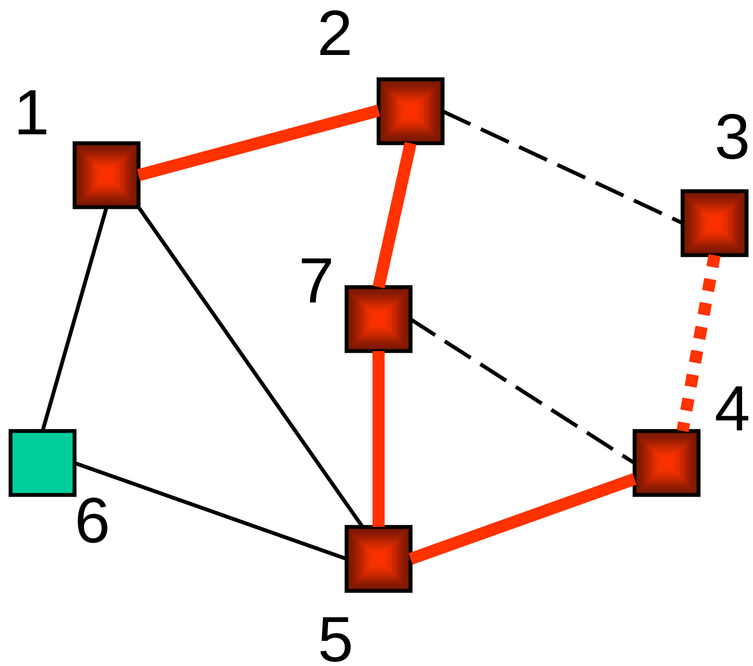
DFS(1)
DFS(2)
DFS(7)
DFS(5)
DFS(4)
DFS(3)

Example Step 8



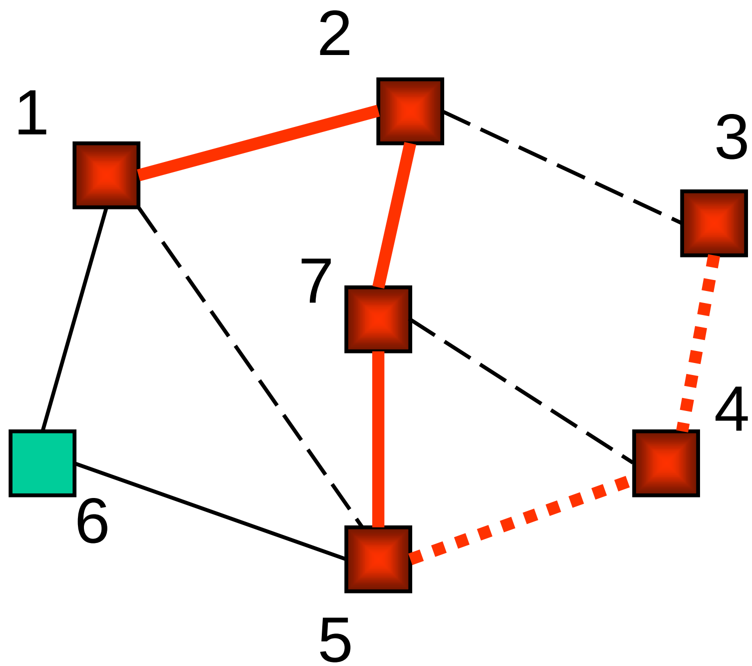
DFS(1)
DFS(2)
DFS(7)
DFS(5)
DFS(4)

Example Step 9



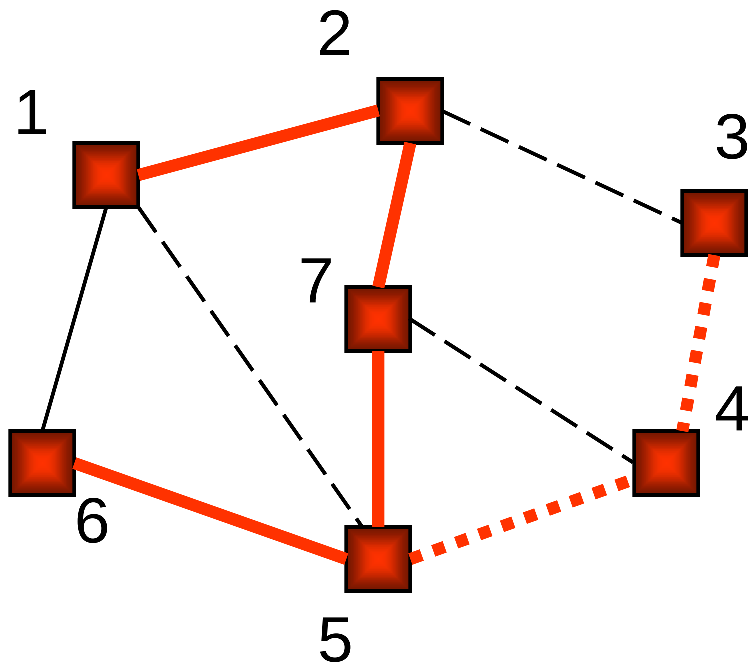
DFS(1)
DFS(2)
DFS(7)
DFS(5)
DFS(4)

Example Step 10



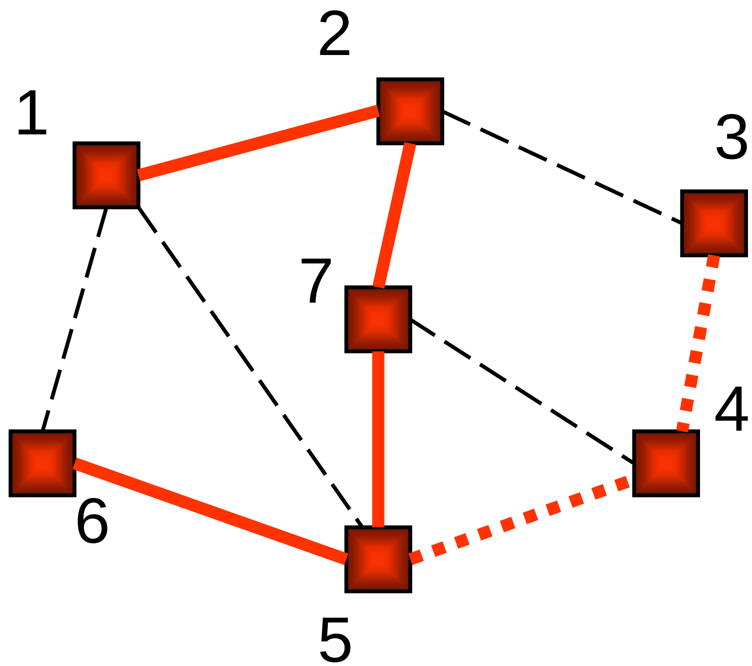
DFS(1)
DFS(2)
DFS(7)
DFS(5)

Example Step 11



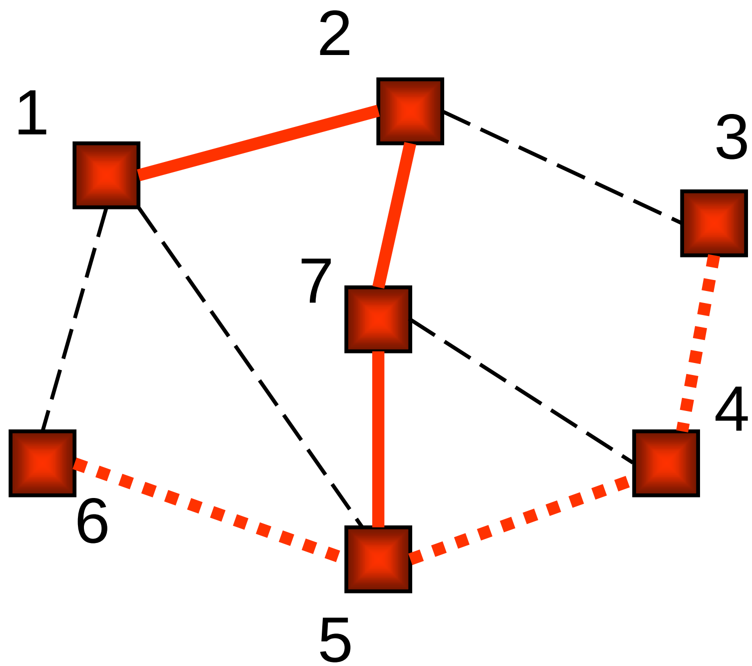
DFS(1)
DFS(2)
DFS(7)
DFS(5)
DFS(6)

Example Step 12



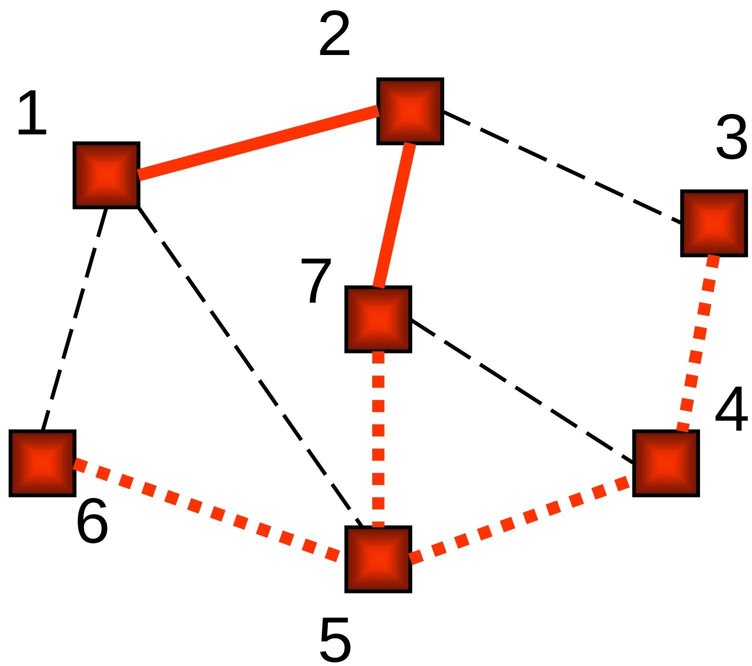
DFS(1)
DFS(2)
DFS(7)
DFS(5)
DFS(6)

Example Step 13



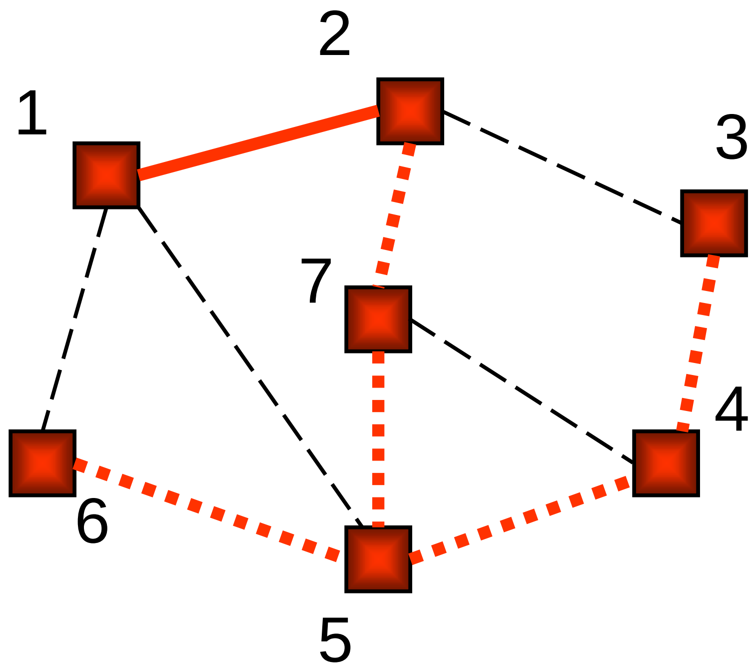
DFS(1)
DFS(2)
DFS(7)
DFS(5)

Example Step 14



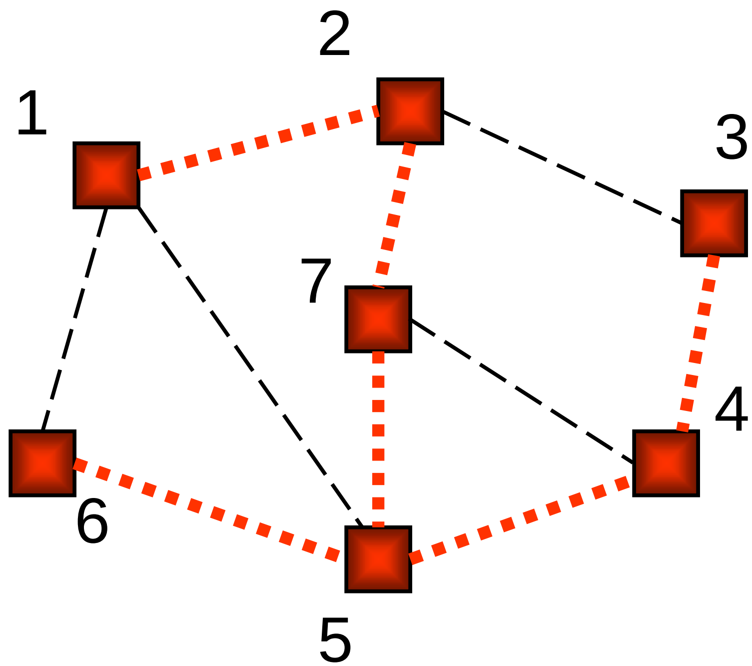
DFS(1)
DFS(2)
DFS(7)

Example Step 15



DFS(1)
DFS(2)

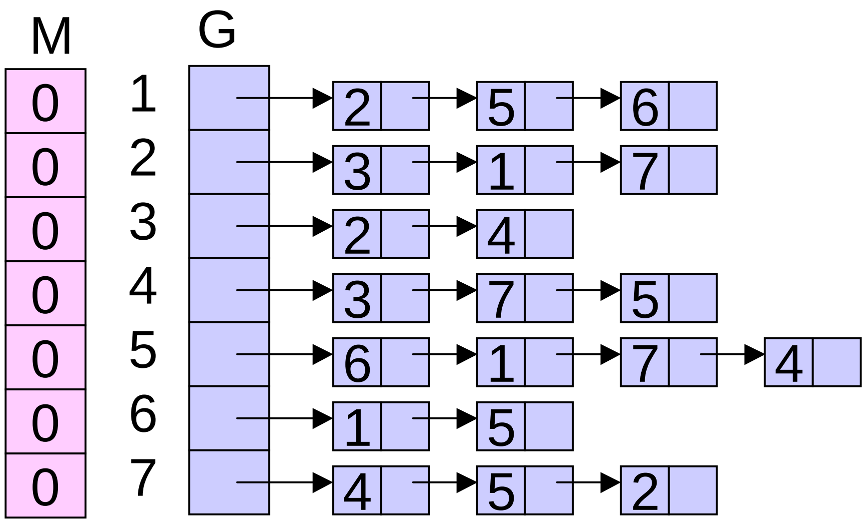
Example Step 16

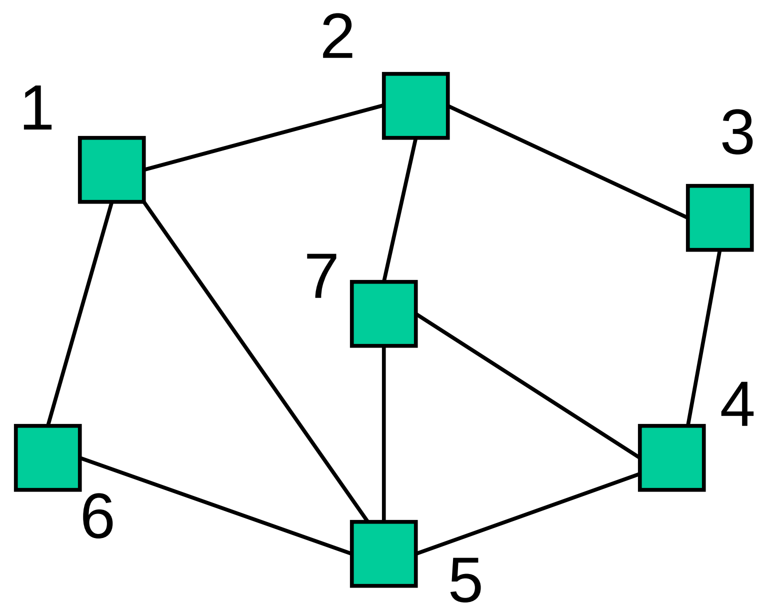


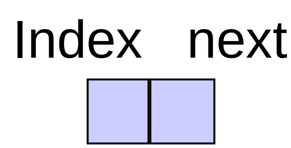
DFS(1)

Adjacency List Implementation

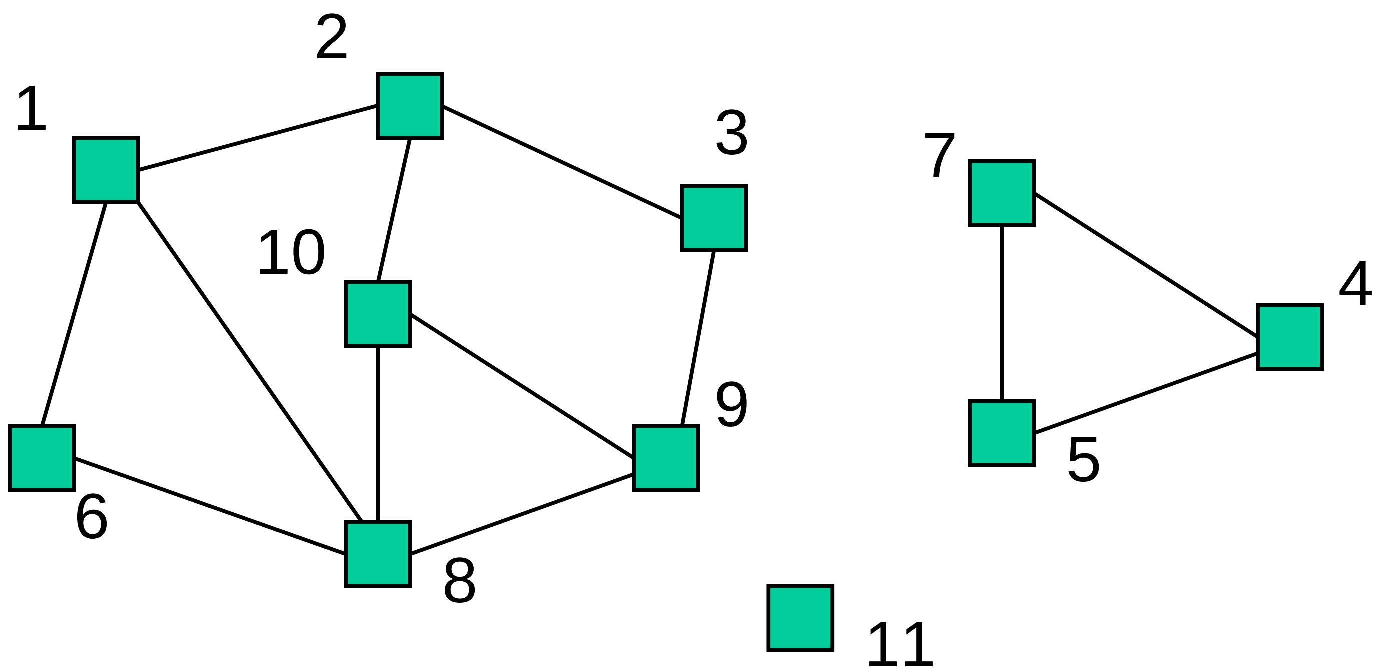
- Adjacency lists





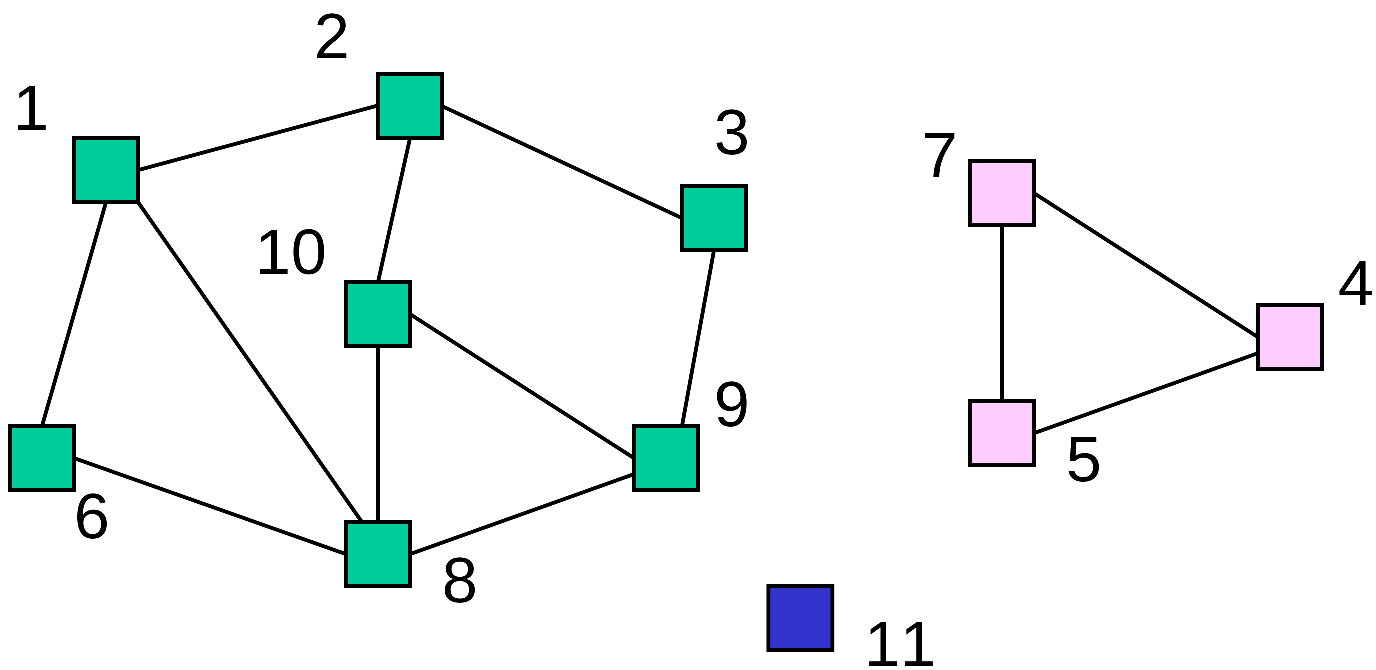


Connected Components



3 connected components

Connected Components

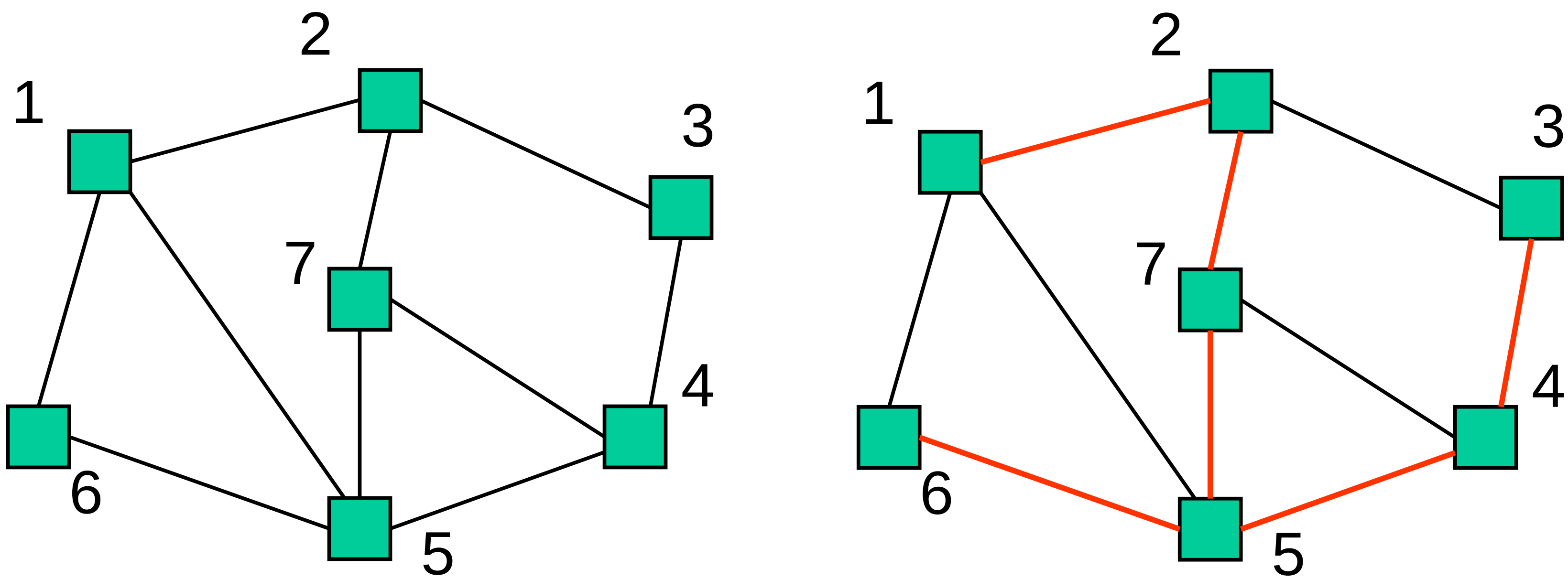


3 connected components are labeled

Depth-first Search for Labeling Connected components

```
Main {
i : integer
for i = 1 to n do M[i] := 0;
label := 1;
for i = 1 to n do
  if M[i] = 0 then DFS(G,M,i,label);
  label := label + 1;
}
DFS(G[]: node ptr array, M[]: int array, i,label: int) {
  v : node pointer;
  M[i] := label;
  v := G[i];
  while v ≠ null do
    if M[v.index] = 0 then DFS(G,M,v.index,label);
    v := v.next;
}
```

Spanning Tree



Spanning tree – no cycles and connects all vertices

Exercise

- Design a depth-first algorithm to output a spanning tree of a connected graph.

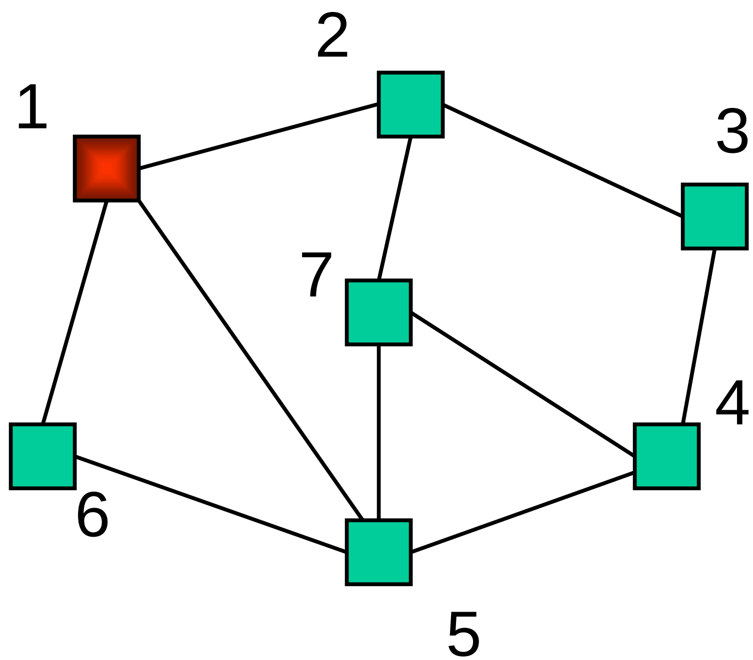
```
Main {
  i : integer
  for i = 1 to n M[i] := 0;
  T := EmptySet;
  STree(1);
}
STree(G[]: node ptr array, M[]: int array, i : int) : {
???
}
```

Performance DFS

- n vertices and m edges
- Storage complexity O(n + m)
- Time complexity O(n + m)
- Linear Time!

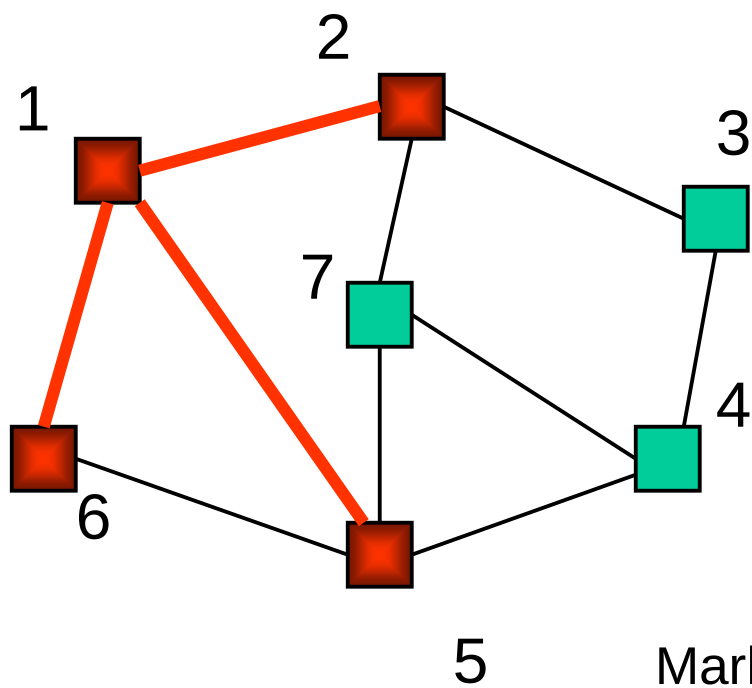
Breadth First Search 1

- Uses a queue to order search



Queue = 1

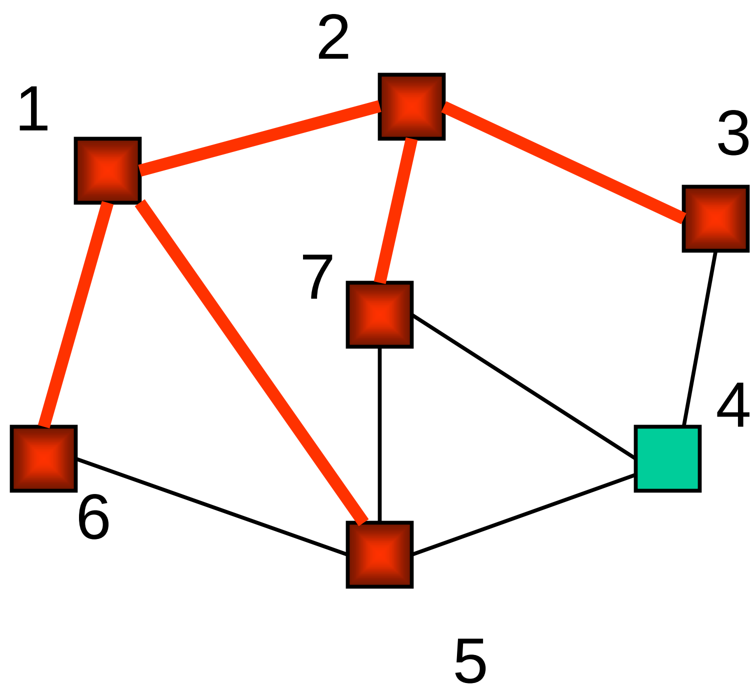
Breadth First Search 2



Queue = 2,6,5

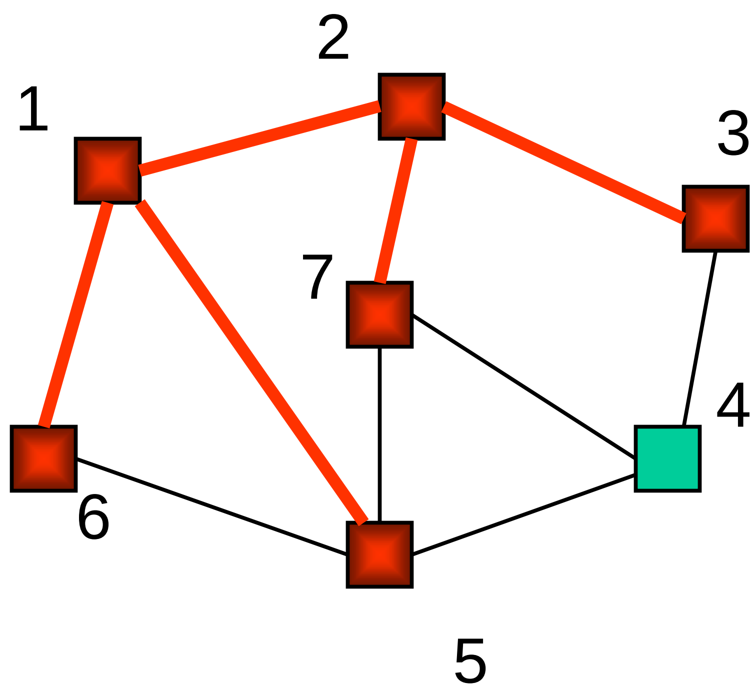
Mark while on queue
to avoid putting in
queue more than once

Breadth First Search 3



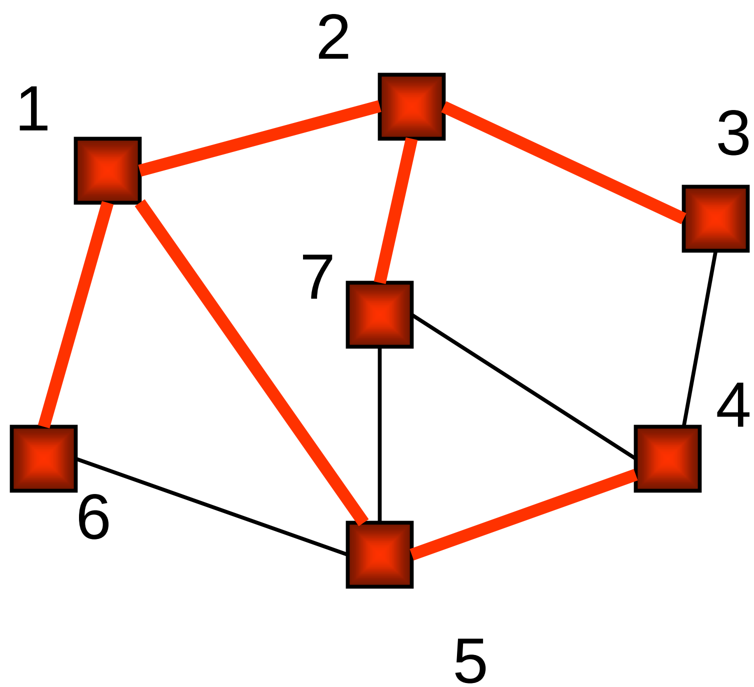
Queue = 6,5,7,3

Breadth First Search 4



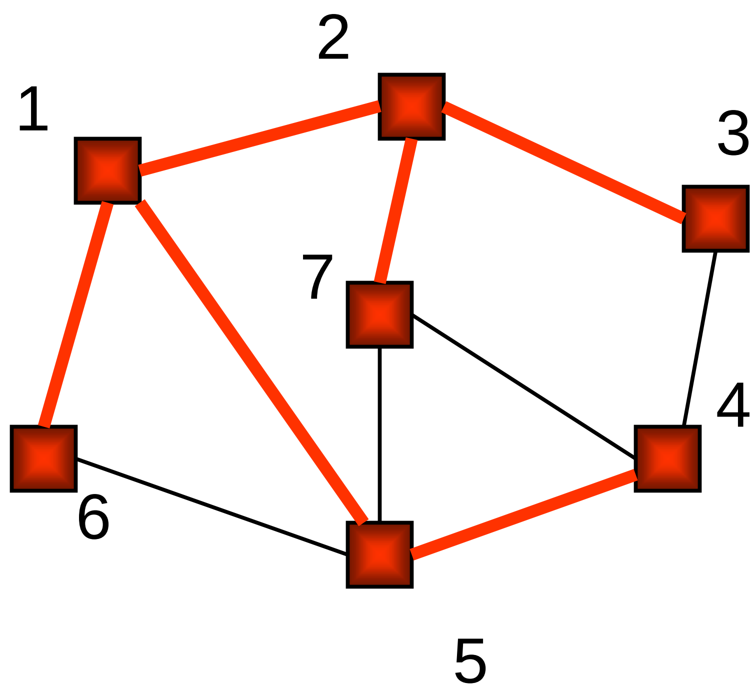
Queue = 5,7,3

Breadth First Search 5



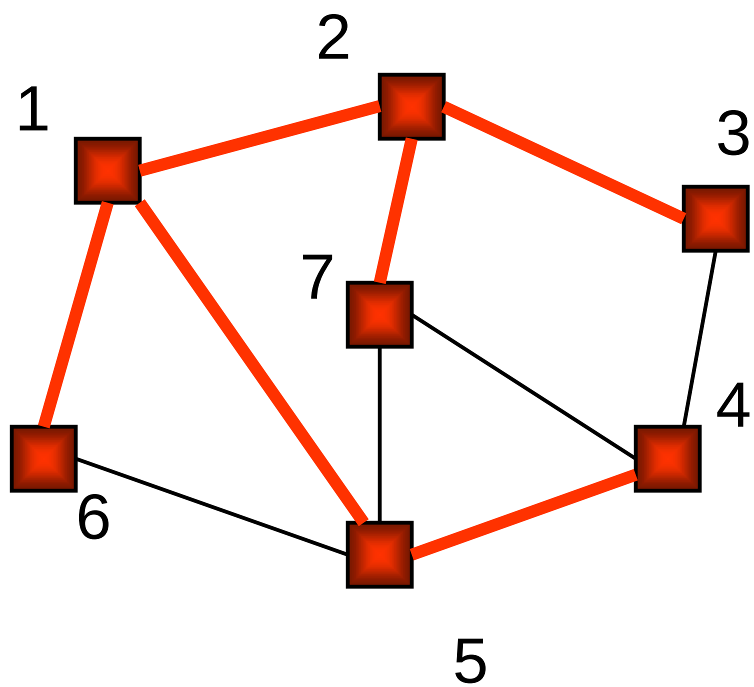
Queue = 7,3,4

Breadth First Search 6



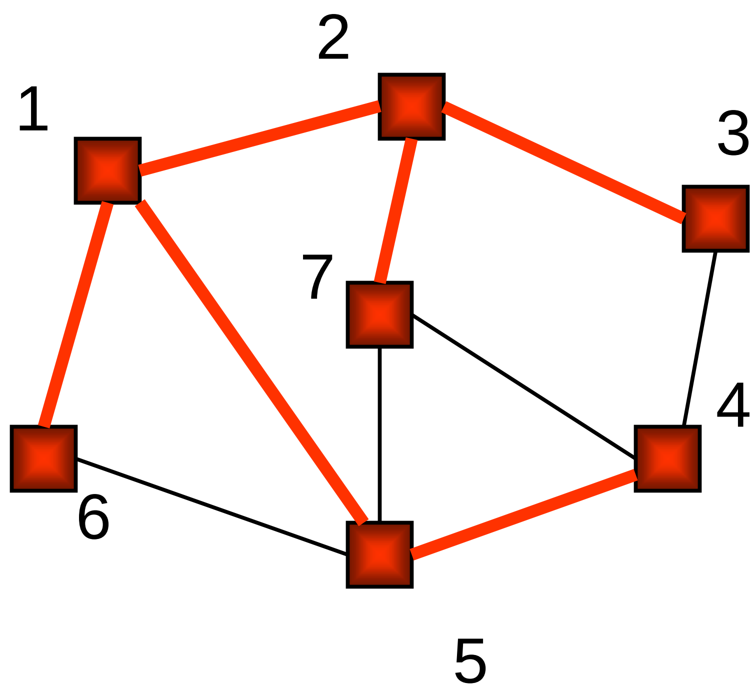
Queue = 3,4

Breadth First Search 7



Queue = 4

Breadth First Search 8



Queue =

Breadth-First Search

```
BFS
Initialize Q to be empty;
Enqueue(Q,1) and mark 1;
while Q is not empty do
    i := Dequeue(Q);
    for each j adjacent to i do
        if j is not marked then
            Enqueue(Q,j) and mark j;
end{BFS}
```

Depth-First vs Breadth-First

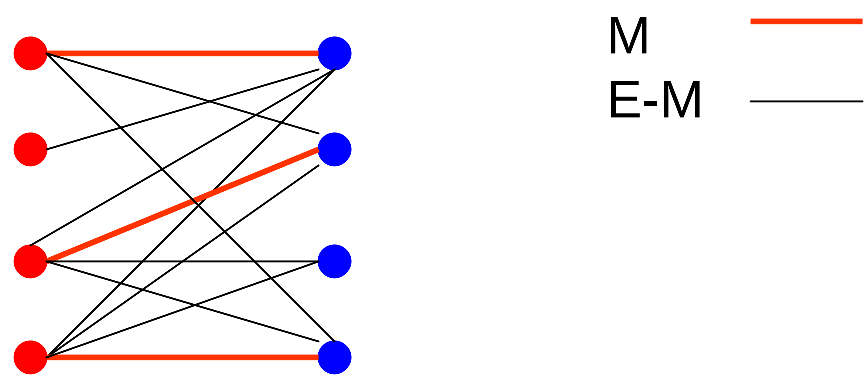
- Depth-First
  - › Stack or recursion
  - › Many applications
- Breadth-First
  - › Queue (recursion no help)
  - › Can be used to find shortest paths from the start vertex
  - › Can be used to find short alternating paths for matching

Bipartite Matching Algorithm

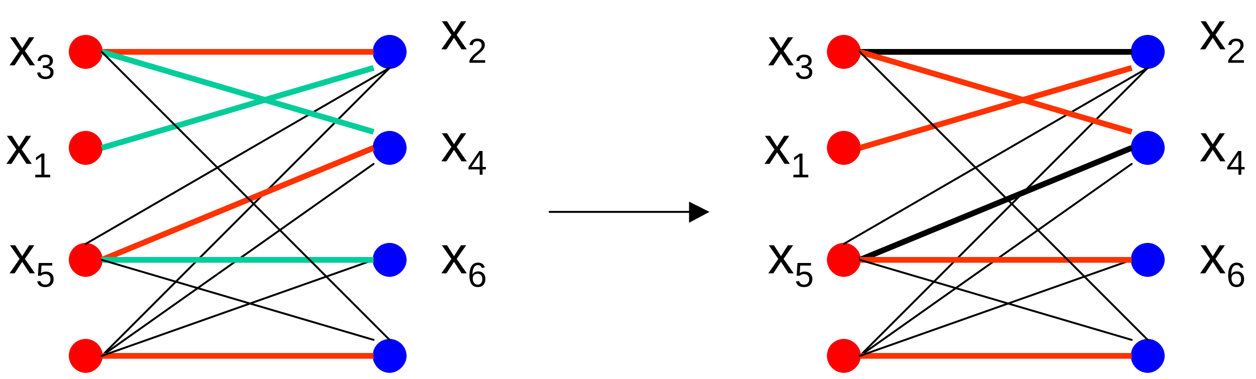
set M to be the empty set initially
repeat
  find an alternating path $x_1,x_2,\ldots,x_{2n}$
  // (x_i,x_{i+1}) in E – M if i is odd and (x_i,x_{i+1}) in M if i is even
    neither x_1 nor x_{2n} are matched//
  delete (x_i,x_{i+1}) from M if i is even
  add (x_i,x_{i+1}) to M if i is odd
until no alternating path can be found

if M has every vertex in U then M is a matching
if M does not have some vertex then there is complete matching, but M is a maximum size matching

Partial Matching

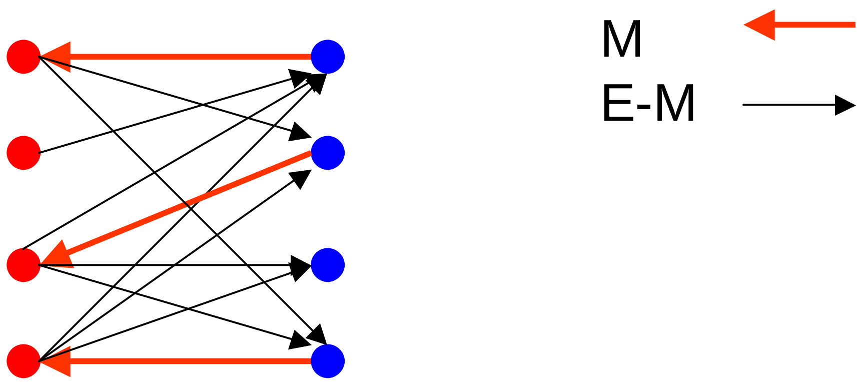


Alternating Path



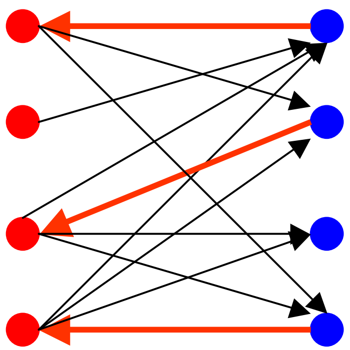
Finding an Alternating Path 1

- Direct the edges
  - › U to V if edge in E – M
  - › V to U if edge in M



Finding an Alternating Path 2

- For each u in U which is not matched do a breadth-first search until an unmatched v in V is found.
  - › If no unmatched v is found then no alternating path from u
  - › Each visited node is marked and not visited again



Can also use depth-first search, but longer longer alternating paths would be found.

Running Time of Maximum Matching

- Parameters
  - Number of edges m
  - Number of vertices n
- Iterations of find alternating path – n
  - Each iteration increases the matching size by 1
- Time to find an alternating path
  - Direct the edges O(m) (assuming $m \geq n$)
  - Breadth-first search O(m) (assuming $m \geq n$)
- Total time O(nm) (Comment: $nm \leq n^3$)
- Can be solved in $O(n^{2.5})$ by clever tricks.

Exercise Solution

- Design a depth-first algorithm to output a spanning tree of a connected graph.

```
STree(G[]: node ptr array, M[]: int array i : integer) : {
  v : node pointer;
  M[i] := 1;
  v := G[i];
  while v ≠ null do
    if M[v.index] = 0 then
      Insert({i,v.index}, T); STree(v.index);
    v := v.next
}
```