CSE 373 Lecture 13: Hashing

- Today's Topics:
  - Collision Resolution
    - Separate Chaining
    - Open Addressing
      - Linear/Quadratic Probing
      - Double Hashing
    - Rehashing
    - Extendible Hashing
- Covered in Chapter 5 in the text

Review of Hashing

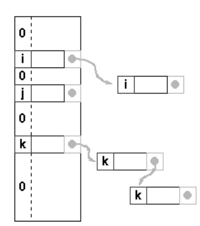
- Idea: Store data record in array slot A[i] where i = Hash(*key*)
- If keys are integers, we can use the hash function:
  - Hash(*key*) = *key* mod *TableSize*
  - *TableSize* is size of the array (preferably a prime number)
- If keys are strings (in the form `char *key`), get integers by treating characters as digits in base 27 (using "a" = 1, "b" = 2, "c" = 3, "d" = 4 etc.)
  - Hash(*key*) = StringInt(*key*) mod *TableSize*
  - StringInt("abc") = $1*27^2 + 2*27^1 + 3 = 786$
  - StringInt("bca") = $2*27^2 + 3*27^1 + 1 = 1540$
  - StringInt("cab") = $3*27^2 + 1*27^1 + 2 = 2216$

Collisions and their Resolution

- A collision occurs when two different keys hash to the same value
  - E.g. For *TableSize* = 17, keys 18 and 35 hash to the same value
  - 18 mod 17 = 1 and 35 mod 17 = 1
- Cannot store both data records in the same slot in array!
- Two different methods for collision resolution:
  - **Separate Chaining:** Use data structure (such as a linked list) to store multiple items that hash to the same slot
  - **Open addressing (or probing):** search for other slots using a second function and store item in first empty slot that is found

Collision Resolution by Separate Chaining

- Each hash table cell holds pointer to linked list of records with same hash value (i, j, k in figure)
- Collision: Insert item into linked list
- To Find an item: compute hash value, then do Find on linked list
- Can use List ADT for Find/Insert/Delete in linked list
- Can also use BSTs: O(log N) time instead of O(N). But lists are usually small – not worth the overhead of BSTs



Separate Chaining: In-Class Example

- Insert 10 random keys between 0 and 100 into a hash table with *TableSize* = 10

Load Factor of a Hash Table

- Let N = number of items to be stored
- **Load factor λ = N/*TableSize***
- What is λ for our example?
- Suppose *TableSize* = 2 and number of items N = 10
  - $\lambda = 5$
- Suppose *TableSize* = 10 and number of items N = 2
  - $\lambda = 0.2$
- Average length of chained list = λ
- Average time for accessing an item = O(1) + O(λ)
  - Want λ to be close to 1 (i.e. *TableSize* ≈ N)
  - But chaining continues to work for $\lambda > 1$

Collision Resolution by Open Addressing

- Linked lists can take up a lot of space…
- Open addressing (or probing): When collision occurs, try alternative cells in the array until an empty cell is found
- Given an item X, try cells $h_0(X)$, $h_1(X)$, $h_2(X)$, …, $h_i(X)$
- $h_i(X)$ = (Hash(X) + F(i)) mod *TableSize*
  - Define F(0) = 0
- F is the collision resolution function. Three possibilities:
  - Linear: F(i) = i
  - Quadratic: F(i) = i^2
  - Double Hashing: F(i) = i $\cdot$ $Hash_2(X)$

Open Addressing I: Linear Probing

- Main Idea: When collision occurs, scan down the array one cell at a time looking for an empty cell
  - $h_i(X)$ = (Hash(X) + i) mod *TableSize* (i = 0, 1, 2, …)
  - Compute hash value and increment it until a free cell is found
- Example: Insert {18, 19, 20, 29, 30, 31} into empty hash table with *TableSize* = 10

Load Factor Analysis of Linear Probing

- Recall: **Load factor λ = N/*TableSize***
- Fraction of empty cells = $1 - \lambda$
- Number of such cells we expect to probe = $1/(1-\lambda)$
- Can show that expected number of probes for:
  - Successful searches = $O(1+1/(1-\lambda))$
  - Insertions and unsuccessful searches = $O(1+1/(1-\lambda)^2)$
- Keep $\lambda \leq 0.5$ to keep number of probes small (between 1 and 5). (E.g. What happens when $\lambda = 0.99$)

Drawbacks of Linear Probing

- Works until array is full, but as number of items N approaches *TableSize* ($\lambda \approx 1$), access time approaches O(N)
- Very prone to cluster formation (as in our example)
  - If a key hashes into a cluster, finding a free cell involves going through the entire cluster
  - Inserting this key at the end of cluster *causes the cluster to grow* → future Inserts will be even more time consuming!
  - This type of clustering is called *Primary Clustering*
- Can have cases where table is empty except for a few clusters
  - Does not satisfy good hash function criterion of distributing keys uniformly

Open Addressing II: Quadratic Probing

- Main Idea: Spread out the search for an empty slot – Increment by i^2 instead of i
- $h_i(X) = (Hash(X) + i^2) \bmod TableSize$ (i = 0, 1, 2, …)
  - No primary clustering but secondary clustering possible
- Example 1: Insert {18, 19, 20, 29, 30, 31} into empty hash table with *TableSize* = 10
- Example 2: Insert {1, 2, 5, 10, 17} with *TableSize* = 16
  - Note: 25 mod 16 = 9, 36 mod 16 = 4, 49 mod 16 = 1, etc.
- Theorem: If *TableSize* is prime and $\lambda < 0.5$, quadratic probing will always find an empty slot

Open Addressing III: Double Hashing

- Idea: Spread out the search for an empty slot by using a second hash function
  - No primary or secondary clustering
- $h_i(X) = (Hash(X) + i \cdot Hash_2(X)) \bmod TableSize$ for i = 0, 1, 2, …
- E.g. $Hash_2(X) = R - (X \bmod R)$
  - R is a prime smaller than *TableSize*
- Try this example: Insert {18, 19, 20, 29, 30, 31} into empty hash table with *TableSize* = 10 and R = 7
- No clustering but slower than quadratic probing due to $Hash_2$

Rehashing

- Need to use *lazy deletion* if we use probing (why?)
  - Need to mark array slots as deleted after Delete
- If table gets too full ($\lambda \approx 1$) or if many deletions have occurred, running time gets too long and Inserts may fail
- Solution: *Rehashing* – Build a bigger hash table (of size 2**TableSize*) when λ exceeds a particular value
  - Cannot just copy data from old table → bigger table has a new hash function
  - Go through old hash table, ignoring items marked deleted
  - Recompute hash value for each non-deleted key and put the item in new position in new table
- Running time is O(N) but happens very infrequently

Extendible Hashing

- A method of hashing used when large amounts of data are stored on disks → can find data in 2 disk accesses
- Could use B-trees but deciding which of many children contains the data takes time
- Extendible Hashing: Store data according to bit patterns
  - Root contains pointers to sorted data bit patterns stored in leaves
  - Leaves contain ≤ M data bit patterns with d_L identical leading bits
  - Root is known as the directory; M is the size of a disk block
  - Requires bits to be nearly random, so hash keys to long integers
- E.g.: Leaves store bit patterns with 2 identical leading bits
  - See text (page 169)

Wednesday's Class will be a *Lab Session* for help

with the programming assignment (no lecture)

Where: Communications Bldg. B-027 and B-022

When: 11:30am-12:30pm

Charles and Jiwon will be in the lab to answer questions

To Do:

Finish reading Chapter 5

Programming Assignment #1 (due April 27)