DS.T.1

Trees

Chapter 4 Overview

- **Tree Concepts**
- **Traversals**
- **Binary Trees**
- **Binary Search Trees**
- **AVL Trees**
- **Splay Trees**
- **B-Trees**

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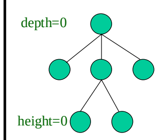
Terminology

Trees are hierarchic structures.



- root
- leaves
- parent
- children
- ancestors
- descendants
- path
- path length
- depth / level
- height
- subtrees

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Recursive Definition:
A tree is a set of nodes that is
a. empty or
b. has one node called the root from which zero or more trees descend.

General (n-ary) Arithmetic Expression Tree

(A + B + ((C * D * E) / F) + G) - H

How can we implement general trees with whose nodes can have variable numbers of children?

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Common Traveral Orders for General Trees

- Preorder
- Postorder

```
void print_preorder ( TreeNode T)
{
TreeNode P;
if  ( T == NULL )      return;
else  {
        print T-> Element;
        P = T -> FirstChild;
        while (P != NULL)
          {
          print_preorder ( P );
          P = P -> NextSibling;
          }
        }
}
```

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A **binary tree** is a tree in which each node has two subtrees--left and right.

Either or both may be empty.

(A + B + ((C * D * E) / F) + G) - H

What operations are required for a binary tree?
That depends …

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- Binary Arithmetic Expression Trees

  - construct from infix expression
  - add or delete nodes
  - traverse in preorder to produce prefix expression
  - traverse in postorder to evaluate
  - traverse in inorder to output infix expression

- Binary Decision Trees

- Binary Search Trees

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Recursive Preorder Traversal

```
void RPT (TreeNode T)
{
if (T != NULL) {
  "process" T -> Element;


}
```

Preorder Traversal with a Stack

```
void SPT (TreeNode T, Stack S)
{
if (T == NULL) return; else push(T,S);
while (!isempty(S)) {
  T = pop(S);
  "process" T -> Element;
  if (T -> Right != NULL) push(T -> Right, S);
  if (T -> Left  != NULL) push(T -> Left, S);
  }
}
```

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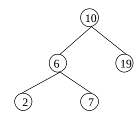
Binary Search Trees

Search trees are look-up tables that are used to find a given key value and return associated data.

Example: look up SSN, return name and address.

A binary tree satisfies the ordering property if the key value in any given node is

> all key values in the node's left subtree
≤ all key values in the node's right subtree



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Operations

- Find the node with a given key
- FindMin / FindMax key in the tree
- Insert a new key (and associated data)
- Delete a key (and associated data)

Find, FindMin, FindMax, Insert are easy.

Delete is a little bit tricky.

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Deletion of a Node from a Binary Search Tree

1. Find the node with the given key value.

2. Delete that node from the tree.

Problem: When you delete a node, what do you replace it by?

- If it has no children, by NULL.
- If it has one child, by that child.
- If it has two children, by the node with the smallest key in its right subtree.

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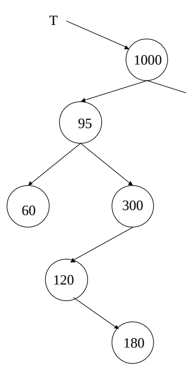
Delete node with key 95.



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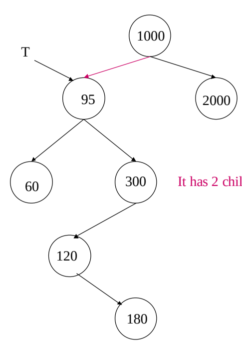
Find the node.



It has 2 children.

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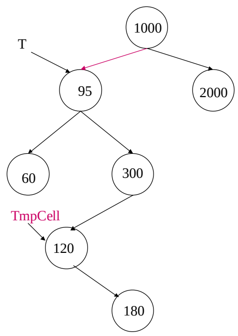
Find smallest in its right subtree.



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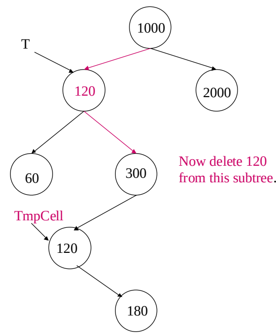
Replace T's key value with that of TmpCell.



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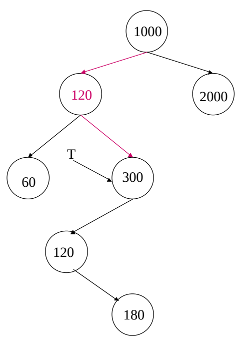
Delete 120 from this subtree T.



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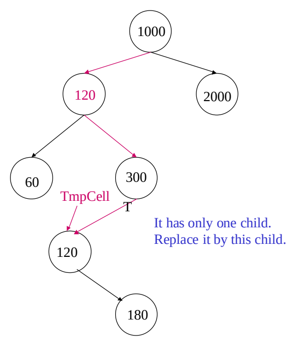
Find the 120 again.



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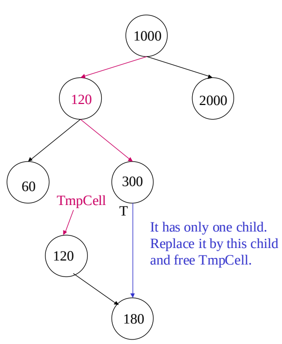

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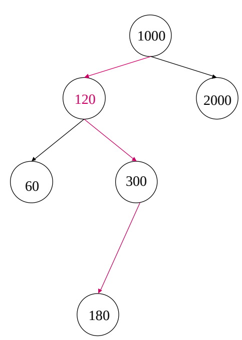

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What do you think of this delete procedure?

Is it readable?

Is it efficient?

How would YOU do it?