CSE370 HW2 Solutions (Winter 2010)

1. **CLD2e, 2.14**
We are asked to use the truth table for a half adder and the circuit diagram to show that the circuit computes the truth table of the full adder. The purpose of this exercise is to show how we can compose truth tables to verify a circuit.

First, I will name the internal wires of the circuit as follows: A and B are the input to the first half adder (HA1) and we will call the outputs C1 and S1 for the carry-out and sum. The C_{in} and S1 become the A2 and B2 input to the second half adder (HA2). HA2 outputs S2, which is S_{out}, and C2. The C_{out} then comes from the OR gate combining C1 and C2.

Now we can build the truth table showing the result of this circuit:

C_{in}	A	B	C1	S1	A2 (C_{in})	B2 (S1)	C2	S2	C_{out} (C1 + C2)	S_{out} (S2)
0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	0	1	0	1	0	1
0	1	1	1	0	0	0	0	0	1	0
1	0	0	0	0	1	0	0	1	0	1
1	0	1	0	1	1	1	1	0	1	0
1	1	0	0	1	1	1	1	0	1	0
1	1	1	1	0	1	0	0	1	1	1

By taking the first and last section of this truth table we see that the full diagram is computing the function as illustrated by the full adder's truth table.

Note: Some people had trouble with this. Basically, when asked to verify that a circuit computes a function by using a truth table, for each input you should show how the internal values of the circuit change and lead to the output.

2. **CLD2e, 2.17 part d**
Simplify f(X, Y, Z) = (X + Y) (X' + Y + Z) (X' + Y + Z')

= (X + Y) ((X' + Y) + Z) ((X' + Y) + Z')	Associative [7] (x2)
= (X + Y) (X' + Y)	Simplification Theorem [9D]
= (Y + X) (Y + X')	Commutative Law [6] (x2)
= Y	Simplification Theorem [9D]

We have reduced the equation from 8 literals to 1, this is a reduction of 7 literals.

Note: Some people tried to apply the Duality theorem in the first step as follows:
(X+Y)(X'+Y+Z)(X'+Y+Z') = XY + X'YZ + X'YZ'   by Duality???
This is not how duality is applied. Basically, the theorem is saying that you can take the dual of an equation and apply rules to it as if you were applying them to the original (since every theorem has a dual). To properly use the duality theorem your first step would have to be something like:

$f(X, Y, Z) = (X + Y)(X' + Y + Z)(X' + Y + Z')$
$f(X, Y, Z) = \{XY + X'YZ + X'YZ'\}^D$ by Duality
…
$f = \{Y\}^D$
$f = Y$ by Duality, again

Note: Also, some people had trouble applying deMorgan's in this problem:
$(X + Y)(X' + Y + Z)(X' + Y + Z') \neq X'Y' + XY'Z + XY'Z$
It is equal to:
$(X'Y' + XY'Z + XY'Z)'$

3. **CLD2e, 2.20**
Consider $f(A, B, C, D) = \sum m(1, 2, 3, 5, 8, 13)$

This has the following truth table:

	A	B	C	D	F
0	0	0	0	0	0
1	0	0	0	1	1
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	0
7	0	1	1	1	0
8	1	0	0	0	1
9	1	0	0	1	0
10	1	0	1	0	0
11	1	0	1	1	0
12	1	1	0	0	0
13	1	1	0	1	1
14	1	1	1	0	0
15	1	1	1	1	0

a. **Part a: write in canonical minterm form**
We just write a term for each 1 in the final column
$f = A'B'C'D + A'B'CD' + A'B'CD + A'BC'D + AB'C'D' + ABC'D$

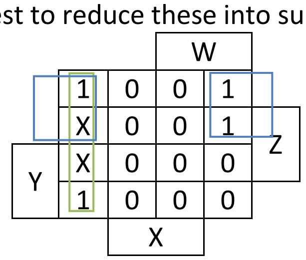
b. **Part b: write in canonical maxterm form**
We write a term for each zero in the final column
$f = (A+B+C+D)(A+B'+C+D)(A+B'+C'+D)(A+B'+C'+D')(A'+B+C+D')(A'+B+C'+D)$
$(A'+B+C'+D')(A'+B'+C+D)(A'+B'+C'+D)(A'+B'+C'+D')$

c. **Part c: write the complement in "little m" and canonical minterm form**
Here we just have all the terms we didn't have in part a
$f = \sum m(0, 4, 6, 7, 9, 10, 11, 12, 14, 15)$
$f = A'B'C'D'+A'BC'D'+ A'BCD' + A'BCD + AB'C'D + AB'CD' +$
$AB'CD + ABC'D' + ABCD' + ABCD$

d. **Part d: write the complement in "big M" and canonical maxterm form**
Here we just have the terms we didn't have in part b
f = ∏ M(1, 2, 3, 5, 8, 13)
f = (A+B+C+D')(A+B+C'+D)(A+B+C'+D')(A+B'+C+D')(A'+B+C+D)(A'+B'+C+D')

Note: Some people forgot to write it out in canonical minterm/maxterm form for part c/d.
Note: Some people also forgot that for the maxterms we write each literal inverted to how we would write it in the minterm: m0 = A'B'C'D' → M0 = (A+B+C+D)

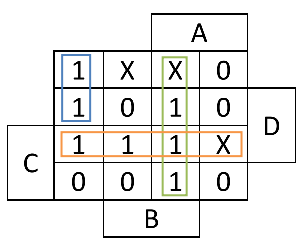
4. **CLD2e, 2.26 parts a and d**
   a. **Part a: f(W,X,Y,Z) = ∑ m(0, 2, 8, 9) + ∑ d(1, 3)**
   It is easiest to reduce these into sum-of-products form by using a K-map



   f = W'X' + X'Y'

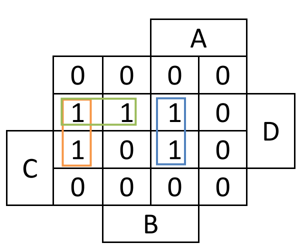
   Note: A lot of people missed that you can wrap around to make the second term smaller at the top.

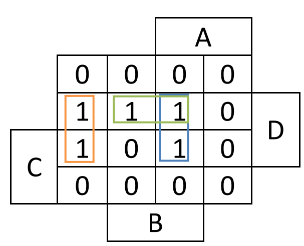
   b. **Part d: f(A,B,C,D) = ∏ M(2, 5, 6, 8, 9, 10) * ∏ D(4, 11, 12)**
   It is easiest to reduce these into sum-of-products form by using a K-map. Remember when using "big M" notation, those terms represent the zeros in the map



   f = AB + CD + A'B'C'

5. **CLD2e, 2.32**
Here is an example of two K-maps that have multiple ways to cover all the 1's





For the first k-map we have the formula: $f = A'B'D + ABD + A'C'D$
For the second map we have the formula: $f = A'B'D + ABD + BC'D$

Note: On this problem a lot of people used “don’t cares” in their solution. You cannot use don’t cares here because your two resulting equations will *not* be the same. We saw this in lecture. Instead you need to show an instance like above, where we have a choice of how to cover one of the 1’s.

Note: Also a lot of people had solutions with two equations, but neither was in minimized form…in fact the k-map had only a single minimized equation. In the example above, both equations are completely minimized and logically equivalent.

6. **CLD2e, 2.43 parts a, b, and d**
   a. **Part a**
   The truth table for this function is as follows:

	A	B	C	D	f
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	1
7	0	1	1	1	0
8	1	0	0	0	0
9	1	0	0	1	1
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	1
13	1	1	0	1	0
14	1	1	1	0	0
15	1	1	1	1	0

   b. **Part b**
   To write in “little m” notation, we just use the rows that contain a 1 in column f to find the minterms:

   $f(A, B, C, D) = \sum m(3, 5, 6, 9, 10, 12)$

   c. **Part d**
   From the minterms we can easily construct the K-maps by putting 1’s in the minterms and a 0 in all the other positions.

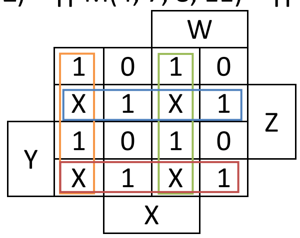
<table>
<tr><td></td><td></td><td></td><td colspan="2">A</td><td></td></tr>
<tr><td></td><td>0</td><td>0</td><td>1</td><td>0</td><td></td></tr>
<tr><td></td><td>0</td><td>1</td><td>0</td><td>1</td><td rowspan="2">D</td></tr>
<tr><td rowspan="2">C</td><td>1</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>1</td><td>0</td><td>1</td><td></td></tr>
<tr><td></td><td></td><td colspan="2">B</td><td></td><td></td></tr>
</table>

Unfortunately, there are no 1’s next to each other so there are no opportunities for reducing. This gives the following formula:
f(A, B, C, D) = A’B’CD + A’BC’D + A’BCD’ + AB’C’D + AB’CD’ + ABC’D’

7. **CLD2e, 3.3/3.4, parts a and b**

   a. **Part a**

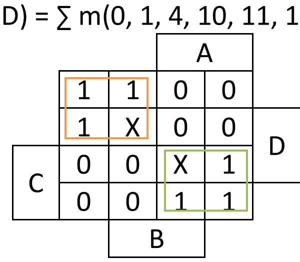
   f(W, X, Y, Z) = ∏ M(4, 7, 8, 11) * ∏ D(1, 2, 13, 14)



   f(W,X,Y,Z) = WX + W’X’ + YZ’ + Y’Z
   Prime implicants are: WX, W’X’, YZ’, Y’Z
   Essential implicants: All
   Redundant implicants: None
   Don’t cares set to 1: 4

   b. **Part b**

   f(A, B, C, D) = ∑ m(0, 1, 4, 10, 11, 14) + ∑ d(5, 15)



   f(A, B, C, D) = AC + A’C’
   Prime implicants: AC, A’C’
   Essential implicants: All
   Redundant implicants: None
   Don’t cares set to 1: 2

Note: A couple people had confusion with how to determine the prime/essential/redundant implicants. Prime implicants are those implicants that are not fully within another larger implicant. Essential implicants are any implicant that covers a 1 and is the only implicant to cover that 1. All others are redundant.