## Logistics

- Always read the Calendar at http://www.cs.washington.edu/370
- HW1 is posted on the web in the calendar --- due 1/14 11:30am
- Email list: please sign up on the web via the link from the course homepage
- TA Office Hours:
  - Josh Snyder Mondays 3:30-4:20, CSE 220
  - Aaron Miller Tuesdays 12:30-1:20, CSE 220
  - Sara Rolfe Tuesdays 2:30-3:20, CSE 220
- My Office Hours:
  - Mondays 12:20-1:00, CSE 668 (grab me after class)
  - TBA



## Lecture 2: Number Systems

- Last lecture
  - Class introduction and overview
- Today
  - Binary numbers
  - Base conversion
  - Number systems
    - Twos-complement
  - A/D and D/A conversion

There are 10 kinds of people in the world. Those who understand binary… and those who don't.



## The "WHY" slide

- Binary numbers
  - All computers work with 0's and 1's so it is like learning alphabets before learning English
- Base conversion
  - For convenience, people use other bases (like decimal, hexdecimal) and we need to know how to convert from one to another.
- Number systems
  - There are more than one way to express a number in binary. So 1010 could be 10, -2, -5 or -6 and need to know which one.
- A/D and D/A conversion
  - Real world signals come in continuous/analog format and it is good to know generally how they become 0's and 1's (and vice versa).



## Digital

- Digital = discrete
  - Binary codes (example: BCD)
  - Decimal digits 0-9
- Binary codes
  - Represent symbols using binary digits (bits)
- Digital computers:
  - I/O is digital
    - ASCII, decimal, etc.
  - Internal representation is binary
    - Process information in bits

| Decimal Symbols | BCD Code |
|---|---|
| 0 | 0000 |
| 1 | 0001 |
| 2 | 0010 |
| 3 | 0011 |
| 4 | 0100 |
| 5 | 0101 |
| 6 | 0110 |
| 7 | 0111 |
| 8 | 1000 |
| 9 | 1001 |



## The basics: Binary numbers

- Bases we will use
  - Binary: Base 2
  - Octal: Base 8
  - Decimal: Base 10
  - Hexadecimal: Base 16 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F
- Positional number system
  - $101_2 = 1\times2^2 + 0\times2^1 + 1\times2^0$
  - $63_8 = 6\times8^1 + 3\times8^0$
  - $A1_{16} = 10\times16^1 + 1\times16^0$
- Addition and subtraction

$$\begin{array}{r} 1011 \\ +\ 1010 \\ \hline 10101 \end{array} \qquad \begin{array}{r} 1011 \\ -\ 0110 \\ \hline 0101 \end{array}$$



## Binary → hex/decimal/octal conversion

- Conversion from binary to octal/hex
  - Binary: 10011110001
  - Octal: 10 | 011 | 110 | 001=$2361_8$
  - Hex: 100 | 1111 | 0001=$4F1_{16}$
- Conversion from binary to decimal
  - $101_2 = 1\times2^2 + 0\times2^1 + 1\times2^0 = 5_{10}$
  - $63.4_8 = 6\times8^1 + 3\times8^0 + 4\times8^{-1} = 51.5_{10}$
  - $A1_{16} = 10\times16^1 + 1\times16^0 = 161_{10}$

## Decimal→ binary/octal/hex conversion

**Binary**

| | Quotient | Remainder |
|---|---|---|
| 56÷2= | 28 | 0 |
| 28÷2= | 14 | 0 |
| 14÷2= | 7 | 0 |
| 7÷2= | 3 | 1 |
| 3÷2= | 1 | 1 |
| 1÷2= | 0 | 1 |

**Octal**

| | Quotient | Remainder |
|---|---|---|
| 56÷8= | 7 | 0 |
| 7÷8= | 0 | 7 |

$56_{10}=111000_2$
$56_{10}=70_8$

- Why does this work?
  - $N=56_{10}=111000_2$
  - Q=N/2=56/2=111000/2=11100 remainder 0
- Each successive divide liberates an LSB (least significant bit)



## Number systems

- How do we write negative binary numbers?
- Historically: 3 approaches
  - Sign-and-magnitude
  - Ones-complement
  - Twos-complement
- For all 3, the most-significant bit (MSB) is the sign digit
  - 0 ≡ positive
  - 1 ≡ negative
- twos-complement is the important one
  - Simplifies arithmetic
  - Used almost universally



## Sign-and-magnitude

- The most-significant bit (MSB) is the sign digit
  - 0 ≡ positive
  - 1 ≡ negative
- The remaining bits are the number's magnitude
- Problem 1: Two representations for zero
  - 0 = 0000 and also –0 = 1000
- Problem 2: Arithmetic is cumbersome

| Add | | Subtract | | | Compare and subtract | | |
|---|---|---|---|---|---|---|---|
| 4 | 0100 | 4 | 0100 | 0100 | – 4 | 1100 | 1100 |
| + 3 | + 0011 | – 3 | + 1011 | – 0011 | + 3 | + 0011 | – 0011 |
| = 7 | = 0111 | = 1 | ≠ 1111 | = 0001 | – 1 | ≠ 1111 | = 1001 |



## Ones-complement

- Negative number: Bitwise complement positive number
  - 0111 ≡ $7_{10}$
  - 1000 ≡ $–7_{10}$
- Solves the arithmetic problem

| Add | | Invert, add, add carry | | Invert and add | |
|---|---|---|---|---|---|
| 4 | 0100 | 4 | 0100 | – 4 | 1011 |
| + 3 | + 0011 | – 3 | + 1100 | + 3 | + 0011 |
| = 7 | = 0111 | = 1 | 1 0000 | – 1 | 1110 |
| | | add carry: | +1 | | |
| | | | = 0001 | | |

- Remaining problem: Two representations for zero
  - 0 = 0000 and also –0 = 1111



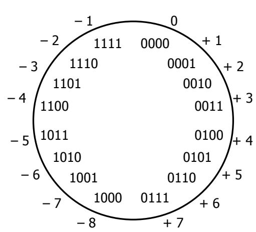
## Why ones-complement works

- The ones-complement of an 8-bit positive y is $11111111_2$ – y
- What is $11111111_2$ ?
  - 1 less than 1 $00000000_2$ ≡ $2^8$ ≡ $256_{10}$
  - So in ones-complement –y is represented by ($2^8$ -1) – y
- Adding representations of x and –y where x, y are positive we get ($2^8$ -1) + x – y
  - If x < y then x - y < 0 there is no carry and get –ve number
    - Just add the representations if no carry
  - If x > y then x - y > 0 there is a carry and get +ve number
    - Need to add 1 and ignore the $2^8$, i.e. "add the carry"
  - If x = y then answer should be 0, get $2^8$-1 =$11111111_2$



## Twos-complement

- Negative number: Bitwise complement plus one
  - 0111 ≡ $7_{10}$
  - 1001 ≡ $–7_{10}$
- Number wheel




- Only one zero!
- MSB is the sign digit
  - 0 ≡ positive
  - 1 ≡ negative

## Twos-complement (con't)

- Complementing a complement ➜ the original number
- Arithmetic is easy
  - Subtraction = negation and addition
    - Easy to implement in hardware

| Add | | Invert and add | | Invert and add | |
|---|---|---|---|---|---|
| 4<br>+ 3 | 0100<br>+ 0011 | 4<br>– 3 | 0100<br>+ 1101 | – 4<br>+ 3 | 1100<br>+ 0011 |
| = 7 | = 0111 | = 1<br>drop carry | 1 0001<br>= 0001 | – 1 | 1111 |



## Why twos-complement works better

- Recall: The ones-complement of a b-bit positive y is $(2^b-1) - y$
- Adding 1 to get the twos-complement represents –y by $2^b - y$
  - So -y and $2^b - y$ are equal mod $2^b$ (leave the same remainder when divided by $2^b$)
  - Ignoring carries is equivalent to doing arithmetic mod $2^b$
- Adding representations of x and –y yields $2^b + x - y$
  - If there is a carry then that means $x \geq y$ and dropping the carry yields x-y
  - If there is no carry then $x < y$ and then we can think of it as $2^b - (y-x)$



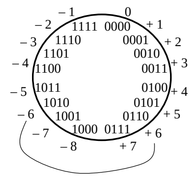
## Miscellaneous

- Twos-complement of non-integers
  - $1.6875_{10} = 01.1011_2$
  - $-1.6875_{10} = 10.0101_2$
- Sign extension
  - Write +6 and –6 as twos complement
    - 0110 and 1010
  - Sign extend to 8-bit bytes
    - 00000110 and 11111010
- Can't infer a representation from a number
  - 11001 is 25 (unsigned)
  - 11001 is –9 (sign magnitude)
  - 11001 is –6 (ones complement)
  - 11001 is –7 (twos complement)

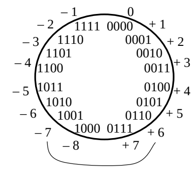


## Twos-complement overflow

- Answers only correct mod $2^b$
  - Summing two positive numbers can give negative result
  - Summing two negative numbers can give a positive result




$6 + 4 \Rightarrow -6$




$-7 - 3 \Rightarrow +6$

- Make sure to have enough bits to handle overflow



## BCD (Binary-Coded Decimal) and Gray codes

| Decimal Symbols | BCD Code |
|---|---|
| 0 | 0000 |
| 1 | 0001 |
| 2 | 0010 |
| 3 | 0011 |
| 4 | 0100 |
| 5 | 0101 |
| 6 | 0110 |
| 7 | 0111 |
| 8 | 1000 |
| 9 | 1001 |

| Decimal Symbols | Gray Code |
|---|---|
| 0 | 0000 |
| 1 | 0001 |
| 2 | 0011 |
| 3 | 0010 |
| 4 | 0110 |
| 5 | 0111 |
| 6 | 0101 |
| 7 | 0100 |
| 8 | 1100 |
| 9 | 1101 |

Only one bit changes per step

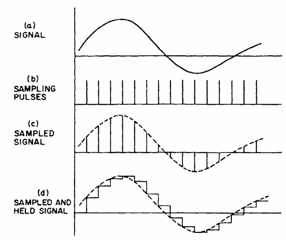


## The physical world is analog

- Digital systems need to
  - Measure analog quantities
    - Speech waveforms, etc
  - Control analog systems
    - Drive motors, etc
- How do we connect the analog and digital domains?
  - Analog-to-digital converter (A/D)
    - Example: CD recording
  - Digital-to-analog converter (D/A)
    - Example: CD playback

## Sampling

- **Quantization**
  - Conversion from analog to discrete values
- Quantizing a signal
  - We sample it



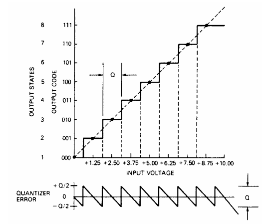

Signal Sampling

Datel Data Acquisition and Conversion Handbook



## Conversion

- **Encoding**
  - Assigning a digital word to each discrete value
- Encoding a quantized signal
  - Encode the samples
  - Typically Gray or binary codes




Transfer Function of Ideal 3-Bit Quantizer

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