Karnaugh maps (K-maps)
- Flat representation of Boolean cubes
- Easy to use for 2–4 dimensions
- Harder for 5–6 dimensions
- Virtually impossible for >6 dimensions
- Use CAD tools
- Help visualize adjacencies
- On-set elements that have one variable changing are adjacent

Uses Gray code: Only one bit changes between cells
- Example: \(00 \rightarrow 01 \rightarrow 11 \rightarrow 10\)

Find the least number of subcubes, each as large as possible, that cover the ON-set.
- Make sure subcubes contain 1, 2, 4, or 8 items (remember the Boolean cube)
Example 1

\[ A B C \mid \text{Cout} \begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{array} \]

Cout = AB + BCin + ACin

Example 2

\[ A B C \mid D \begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{array} \]

D = A + BC

Example 3

\[ F(A,B,C) = \Sigma m(0,4,5,7) = B'C' + AC \]

Exercise

\[ F(A,B,C,D) = \Sigma m(0,3,7,8,11,15) \]

F(A,B,C,D) =

\[ F'(A,B,C,D) = \Sigma m(1,2,3,6) = A'C + B'C' \]
6-dimensions

K-maps become 3D for 5 & 6 variables

Incompletely specified functions

- Functions of n inputs have $2^n$ possible configurations
  - Some combinations may be unused
  - Call unused combinations “don’t cares”
  - Exploit don’t cares during logic minimization
  - Don’t care ≠ no output
    - There will always be an output signal

Don't cares

- Example: A BCD increment-by-1
  - Function $F$ computes the next number in a BCD sequence
    - If the input is $0010_2$, the output is $0011_2$
  - BCD encodes decimal digits 0–9 as $0000_2$–$1001_2$
  - Don’t care about binary numbers $1010_2$–$1111_2$

Don't care notation

- Minterm expansion
  \[ W = m_7 + m_8 + d_{10} + d_{11} + d_{12} + d_{13} + d_{14} + d_{15} \]
  \[ = \Sigma(m,7,8) + d(10,11,12,13,14,15) \]

- Maxterm expansion
  \[ W = M_0 + M_1 + M_2 + M_3 + M_4 + M_5 + M_6 + M_9 + D_{10} + D_{11} + D_{12} + D_{13} + D_{14} + D_{15} \]
  \[ = \Pi(M,0,1,2,3,4,5,6,9) + D(10,11,12,13,14,15) \]

- In K-maps, can treat ‘don’t cares’ as 0s or 1s depending on which is more advantageous

Example

- $F(A, B, C, D) = \Sigma m(1,3,5,7,9) + d(6,12,13)$
  - $F = A'D + B'CD$ without using don’t cares
  - $F = A'D + CD$ using don’t cares

Assign $X = 1$
  \[ \Rightarrow \] allows a 2-cube rather than a 1-cube