Lecture 6

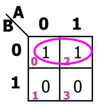
- Karnaugh maps (K-maps)
- K-maps with “don’t cares”

Karnaugh map (K-map)

- Flat representation of Boolean cubes
  - Easy to use for 2– 4 dimensions
  - Harder for 5 – 6 dimensions
  - Virtually impossible for >6 dimensions
    - Use CAD tools
- Help visualize adjacencies
  - On-set elements that have one variable changing are adjacent

Karnaugh map (K-map)

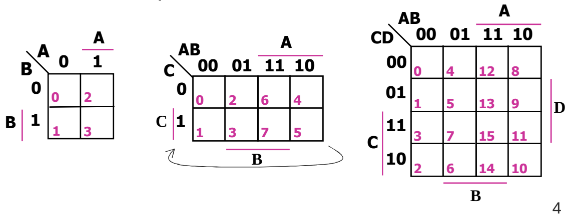
	A	B	F
0	0	0	1
1	0	1	0
2	1	0	1
3	1	1	0



F = B'

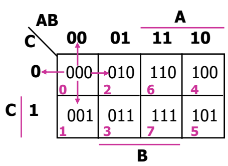
2, 3, and 4 dimensional K-maps

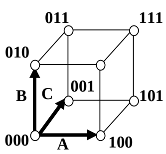
- Uses Gray code: Only one bit changes between cells
  - Example: 00 → 01 → 11 → 10



Adjacencies

- Wrap–around at edges
  - First column to last column
  - Top row to bottom row





K-map minimization

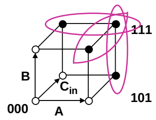
- Find the least number of subcubes, each as large as possible, that cover the ON-set.
- Make sure subcubes contain 1, 2, 4, or 8 items (remember the Boolean cube)

Example 1

A	B	Cin	Cout
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Cin \ AB	00	01	11	10
0	0	0	1	0
1	0	1	1	1

Cout = AB + BCin + ACin

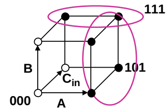


Example 2

A	B	C	D
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

C \ AB	00	01	11	10
0	0	0	1	1
1	0	1	1	1

D = A + BC



Example 3

F(A,B,C) = Σm(0,4,5,7)
= B'C'+AC

C \ AB	00	01	11	10
0	1	0	0	1
1	0	0	1	1

Finding the complement

C \ AB	00	01	11	10
0	1	0	0	1
1	0	0	1	1

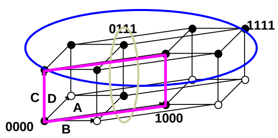
F(A,B,C) = Σm(0,4,5,7)
= B'C'+AC

F'(A,B,C) = Σm(1,2,3,6)
= A'C + BC'

Example: 4 variables

- Minimize F(A,B,C,D) = Σm(0,2,3,5,6,7,8,10,11,14,15)
- Answer: F = C+A'BD+B'D'

CD \ AB	00	01	11	10
00	1	0	0	1
01	0	1	0	0
11	1	1	1	1
10	1	1	1	1



Exercise

F(A,B,C) = Σm(0,3,6,7)

F(A,B,C) =

F'(A,B,C) =

F(A,B,C,D) = Σm(0,3,7,8,11,15)

F(A,B,C,D) =

F'(A,B,C,D) =

C \ AB	00	01	11	10
0				
1				

CD \ AB	00	01	11	10
00				
01				
11				
10				

6-dimensions

K-maps become 3D for 5 & 6 variables

AB = 00

EF \ CD	00	01	11	10
00	0	0	0	0
01	0	0	1	1
11	0	0	1	1
10	0	0	0	0

AB = 01

EF \ CD	00	01	11	10
00	1	0	0	0
01	0	0	1	1
11	1	0	1	1
10	1	0	0	0

AB = 11

EF \ CD	00	01	11	10
00	0	0	0	0
01	0	0	1	1
11	1	0	1	1
10	1	0	0	0

AB = 10

EF \ CD	00	01	11	10
00	0	0	0	0
01	0	0	1	1
11	0	0	1	1
10	0	0	0	0

OUTPUT =
A'BC'D'F' +
CF + BC'D'E

Incompletely specified functions

- Functions of n inputs have 2^n possible configurations
  - Some combinations may be unused
  - Call unused combinations "don't cares"
  - Exploit don't cares during logic minimization
  - Don't care ≠ no output
    - There will always be an output signal

Don't cares

- Example: A BCD increment-by-1
  - Function F computes the next number in a BCD sequence
    - If the input is 0010_2, the output is 0011_2
  - BCD encodes decimal digits 0–9 as 0000_2–1001_2
    - Don't care about binary numbers 1010_2–1111_2

BCD increment-by-one

INPUTS				OUTPUTS			
A	B	C	D	W	X	Y	Z
0	0	0	0	0	0	0	1
0	0	0	1	0	0	1	0
0	0	1	0	0	0	1	1
0	0	1	1	0	1	0	0
0	1	0	0	0	1	0	1
0	1	0	1	0	1	1	0
0	1	1	0	0	1	1	1
0	1	1	1	1	0	0	0
1	0	0	0	1	0	0	1
1	0	0	1	0	0	0	0
1	0	1	0	X	X	X	X
1	0	1	1	X	X	X	X
1	1	0	0	X	X	X	X
1	1	0	1	X	X	X	X
1	1	1	0	X	X	X	X
1	1	1	1	X	X	X	X

off-set for W: m0–m6, m9

on-set for W: m7 and m8

Don't care set for W:
We **don't care**
about the output values

Don't care notation

- Minterm expansion
  W = m7+m8+d10+d11+d12+d13+d14+d15
  = Σm(7,8) + d(10,11,12,13,14,15)
- Maxterm expansion
  W = M0•M1•M2•M3•M4•M5•M6•M9•D10•D11•D12•D13•D14•D15
  = ΠM(0,1,2,3,4,5,6,9) • D(10,11,12,13,14,15)
- In K-maps, can treat 'don't cares' as 0s or 1s depending on which is more advantageous

Example

- F(A,B,C,D) = Σm(1,3,5,7,9) + d(6,12,13)
  - F = A'D + B'C'D *without using* don't cares
  - F = A'D + C'D *using* don't cares

CD \ AB	00	01	11	10
00	0	0	X	0
01	1	1	X	1
11	1	1	0	0
10	0	X	0	0

A B C D

Assign X == "1"
⇒ allows a 2-cube rather than a 1-cube