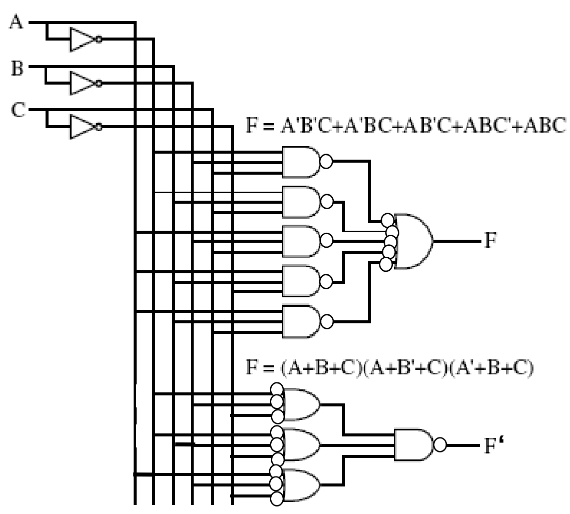
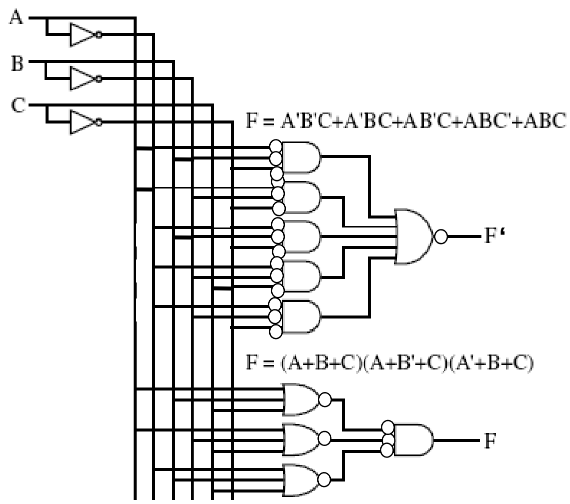
Lecture 6

- Logistics
  - HW2 out, due 4/16 Wednesday
  - Lab 2 ongoing
- Last lecture
  - Canonical forms
  - NAND and NOR
- Today's lecture
  - One more pushing bubble example
  - Logic simplification
    - Boolean cubes
    - Karnaugh maps

Example of bubble pushing: NAND/NAND



Example of bubble pushing: NOR/NOR



Goal: Minimize two-level logic expression

- Algebraic simplification
  - not an systematic procedure
  - hard to know when we reached the minimum
- Computer-aided design tools
  - require very long computation times (NP hard)
  - heuristic methods employed – "educated guesses"
- Visualization methods are useful
  - our brain is good at figuring things out over computers
  - many real-world problems are solvable by hand

Key tool: The Uniting Theorem

- The uniting theorem → A(B'+B) = A
- The approach:
  - Find some variables don't change (the A's above) and others do (the B's above)
  - Eliminate the changing variables (the B's)

A	B	F
0	0	1
0	1	1
1	0	0
1	1	0

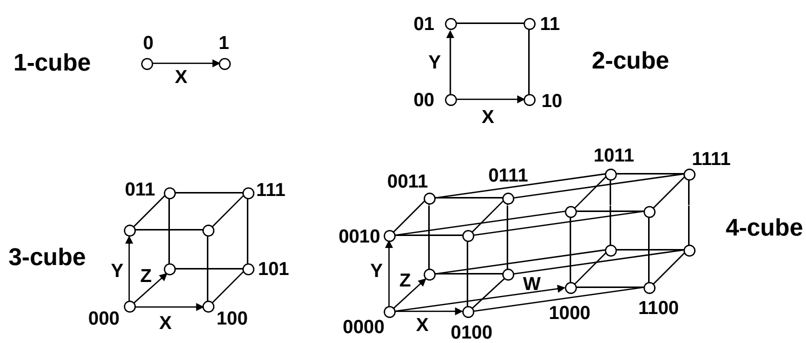
A has the same value in both "on-set" rows
⇒ keep A

B has a different value in the two rows
⇒ eliminate B

F = A'B'+A'B = A'(B+B') = A'

Boolean cubes

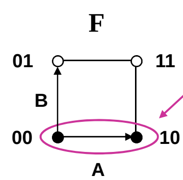
- Visualization tool for the uniting theorem
  - n input variables = n-dimensional "cube"



Mapping truth tables onto Boolean cubes

- ON set = solid nodes
- OFF set = empty nodes

A	B	F
0	0	1
0	1	0
1	0	1
1	1	0



Look for on-set adjacent to each other

Sub-cube (a line) comprises two nodes

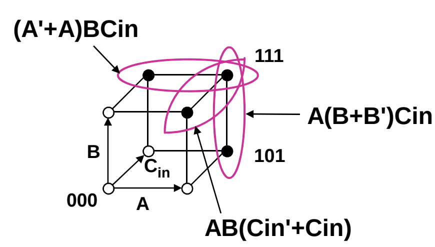
A varies within the sub-cube; B does not

This sub-cube represents B'

Example using Boolean cube

- Binary full-adder carry-out logic
  - On-set is covered by the OR of three 2-D subcubes

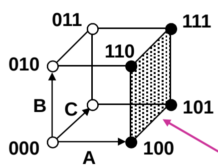
A	B	Cin	Cout
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1



Cout = BCin+AB+ACin

M-dimensional cubes in n-dimensional space

- In a 3-cube (three variables):
  - A 0-cube (a single node) yields a term in 3 literals
  - A 1-cube (a line of two nodes) yields a term in 2 literals
  - A 2-cube (a plane of four nodes) yields a term in 1 literal
  - A 3-cube (a cube of eight nodes) yields a constant term "1"



$F(A,B,C) = \sum m(4,5,6,7)$

On-set forms a square (a 2-D cube)

A is asserted (true) and unchanging
B and C vary

This sub-cube represents the literal A

Karnaugh maps (K-map)

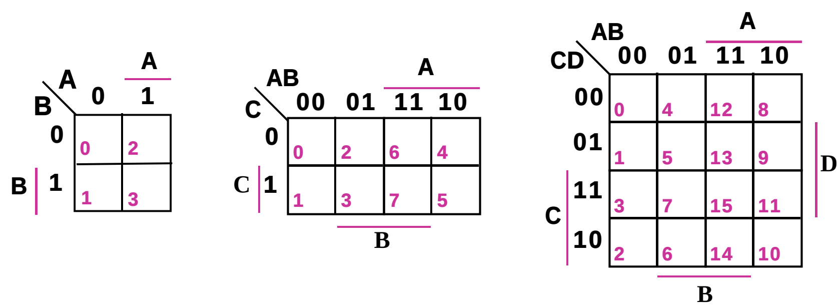
- Flat representation of Boolean cubes
  - Easy to use for 2– 4 dimensions
  - Hard for 4 – 6 dimensions
  - Virtually impossible for 6+ dimensions
    - Use CAD tools
- Help visualize adjacencies
  - On-set elements that have one variable changing are adjacent

	A	B	F
0	0	0	1
1	0	1	0
2	1	0	1
3	1	1	0

B \ A	0	1
0	(0) 1	(2) 1
1	(1) 0	(3) 0

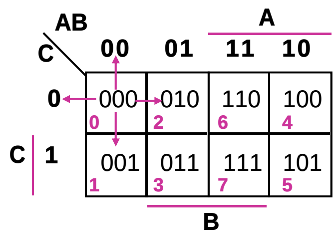
2, 3, and 4 dimensional K-maps

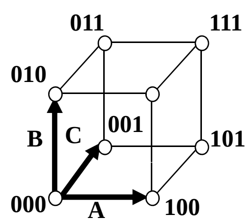
- Uses Gray code: Only one bit changes between cells
  - Example: 00 → 01 → 11 → 10



Adjacencies

- Wrap–around at edges
  - First column to last column
  - Top row to bottom row





K-map minimization example: 2 variables

	A	B	F
0	0	0	1
1	0	1	0
2	1	0	1
3	1	1	0

B \ A	0	1
0	$_0$ 1	$_2$ 1
1	$_1$ 0	$_3$ 0

F = B'

K-map minimization example: 3 variables

A	B	Cin	Cout
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

			A	A
Cin \ AB	**00**	**01**	**11**	**10**
0	$_0$ 0	$_2$ 0	$_6$ 1	$_4$ 0
1	$_1$ 0	$_3$ 1	$_7$ 1	$_5$ 1
		B	B	

Cout = AB + BCin + ACin

K-map minimization example: minterms

F(A,B,C) = Σm(0,4,5,7)
= B'C'+AC

			A	
C \ AB	00	01	11	10
0	$_0$ 1	$_2$ 0	$_6$ 0	$_4$ 1
1	$_1$ 0	$_3$ 0	$_7$ 1	$_5$ 1
		B		

K-map minimization example: complement

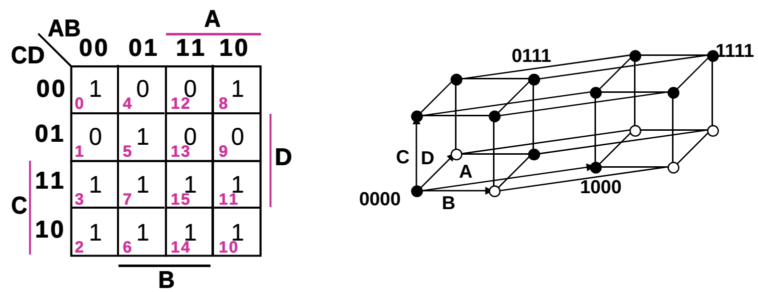
			A	
C \ AB	00	01	11	10
0	$_0$ 1	$_2$ 0	$_6$ 0	$_4$ 1
1	$_1$ 0	$_3$ 0	$_7$ 1	$_5$ 1
		B		

F(A,B,C) = Σm(0,4,5,7)
= B'C'+AC

F'(A,B,C) = Σm(1,2,3,6)
= A'C + BC'

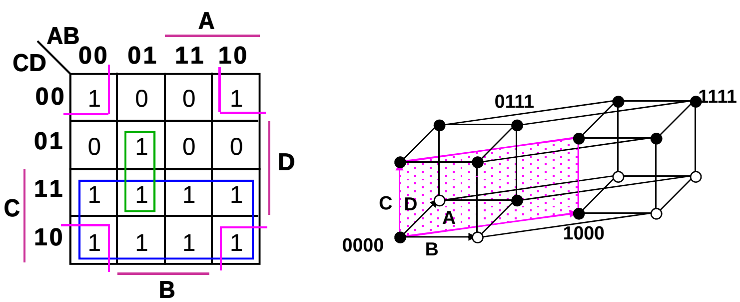
K-map minimization example: 4 variables

- Minimize F(A,B,C,D) = Σm(0,2,3,5,6,7,8,10,11,14,15)
  - Find the least number of subcubes, each as large as possible, that cover the ON-set



K-map minimization example: 4 variables

- Minimize F(A,B,C,D) = Σm(0,2,3,5,6,7,8,10,11,14,15)
- Answer: F = C+A'BD+B'D'



K-map minimization examples: do it yourself

F(A,B,C) = Σm(0,3,6,7)

F(A,B,C) =
F'(A,B,C) =

C \ AB	00	01	11	10
0				
1				

F(A,B,C,D) = Σm(0,3,7,8,11,15)

F(A,B,C,D) =
F'(A,B,C,D) =

CD \ AB	00	01	11	10
00				
01				
11				
10				