Lecture 5

- Logistics
  - HW1 was due before lecture
  - HW2 posted today, due in one week
  - Lab2 ongoing
  - Question on how to post lab grades/check list
  - Final exam scheduled: 6/9/ 8:30am here EEB 105
- Last lecture
  - Logic gates and truth tables
  - Implementing logic functions
- Today's lecture
  - Canonical forms
  - NAND and NOR

de Morgan's theorem

- Replace
  - • with +, + with •, 0 with 1, and 1 with 0
  - All variables with their complements
- Example 1: Z = A'B' + A'C'

  Z' = (A'B' + A'C')'

  = (A+B) • (A+C)
- Example 2: Z = A'B'C + A'BC + AB'C + ABC'

  Z' = (A'B'C + A'BC + AB'C + ABC')'

  = (A+B+C') • (A+B'+C') • (A'+B+C') • (A'+B'+C)

Canonical forms

- Canonical forms
  - Standard forms for Boolean expressions
  - Generally not the simplest forms
    - Can be minimized
  - Derived from truth table
- Two canonical forms
  - Sum-of-products (minterms)
  - Product-of-sum (maxterms)

Sum-of-products canonical form (SOP)

- Also called disjunctive normal form (DNF)
  - Commonly called a **minterm expansion**

minterm

$$F = \overset{001}{A'B'C} + \overset{011}{A'BC} + \overset{101}{AB'C} + \overset{110}{ABC'} + \overset{111}{ABC}$$

A	B	C	F	F'
0	0	0	0	1
0	0	1	1	0
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

$$F' = A'B'C' + A'BC' + AB'C'$$

Minterms

- Variables appears exactly once in each minterm
  - In true or inverted form (but not both)

A	B	C	minterms	
0	0	0	A'B'C'	m0
0	0	1	A'B'C	m1
0	1	0	A'BC'	m2
0	1	1	A'BC	m3
1	0	0	AB'C'	m4
1	0	1	AB'C	m5
1	1	0	ABC'	m6
1	1	1	ABC	m7

short-hand notation

F in canonical form:

$$
\begin{aligned}
F(A,B,C) &= \Sigma m(1,3,5,6,7) \\
&= m1 + m3 + m5 + m6 + m7 \\
&= A'B'C + A'BC + AB'C + ABC' + ABC
\end{aligned}
$$

canonical form $\rightarrow$ minimal form

$$
\begin{aligned}
F(A,B,C) &= A'B'C + A'BC + AB'C + ABC + ABC' \\
&= (A'B' + A'B + AB' + AB)C + ABC' \\
&= ((A' + A)(B' + B))C + ABC' \\
&= ABC' + C \\
&= AB + C
\end{aligned}
$$

Product-of-sums canonical form (POS)

- Also called conjunctive normal form (CNF)
  - Commonly called a **maxterm expansion**

A	B	C	F	F'
0	0	0	0	1
0	0	1	1	0
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

maxterm

000 010 100

F = (A + B + C) (A + B' + C) (A' + B + C)

F' = (A+B+C')(A+B'+C')(A'+B+C')(A'+B'+C)(A'+B'+C')

Maxterms

- Variables appears exactly once in each maxterm
  - In true or inverted form (but not both)

A	B	C	maxterms	
0	0	0	A+B+C	M0
0	0	1	A+B+C'	M1
0	1	0	A+B'+C	M2
0	1	1	A+B'+C'	M3
1	0	0	A'+B+C	M4
1	0	1	A'+B+C'	M5
1	1	0	A'+B'+C	M6
1	1	1	A'+B'+C'	M7

short-hand notation

F in canonical form:

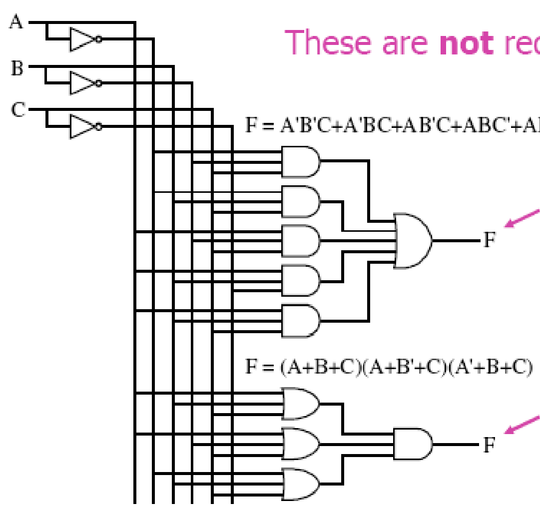
$$\begin{aligned} F(A,B,C) &= \Pi M(0,2,4) \\ &= M0 \bullet M2 \bullet M4 \\ &= (A+B+C)(A+B'+C)(A'+B+C) \end{aligned}$$

canonical form $\rightarrow$ minimal form

$$\begin{aligned} F(A,B,C) &= (A+B+C)(A+B'+C)(A'+B+C) \\ &= (A+B+C)(A+B'+C)\bullet \\ &\quad\ (A+B+C)(A'+B+C) \\ &= (A + C)(B + C) \end{aligned}$$

Canonical implementations of F = AB + C

These are **not** reduced forms for F



canonical sum-of-products

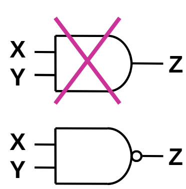
canonical product-of-sums

Conversion between canonical forms

- Minterm to maxterm
  - Use maxterms that aren’t in minterm expansion
  - $F(A,B,C) = \sum m(1,3,5,6,7) = \prod M(0,2,4)$
- Maxterm to minterm
  - Use minterms that aren’t in maxterm expansion
  - $F(A,B,C) = \prod M(0,2,4) = \sum m(1,3,5,6,7)$
- Minterm of F to minterm of F'
  - Use minterms that don’t appear
  - $F(A,B,C) = \sum m(1,3,5,6,7)$ $F'(A,B,C) = \sum m(0,2,4)$
- Maxterm of F to maxterm of F'
  - Use maxterms that don’t appear
  - $F(A,B,C) = \prod M(0,2,4)$ $F'(A,B,C) = \prod M(1,3,5,6,7)$

NAND/NOR more common/efficient

- CMOS logic gates are more common and efficient in the inverted forms
  - NAND, NOR, NOT
  - Even though Canonical forms discussed so far used AND/OR, NAND/NOR preferred for real hardware implementation



X	Y	Z
0	0	1
0	1	1
1	0	1
1	1	0

NAND and NOR (truth table)

(X + Y)' = X' • Y'
NOR is equivalent to AND
with inputs complemented

X	Y	X'	Y'	(X + Y)'	X' • Y'
0	0	1	1	1	1
0	1	1	0	0	0
1	0	0	1	0	0
1	1	0	0	0	0

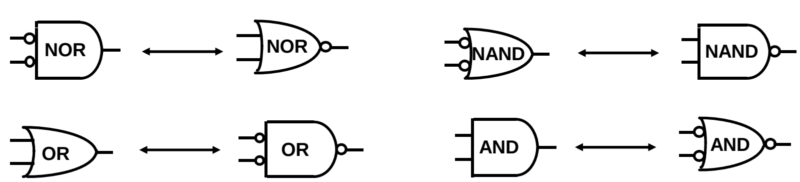
(X • Y)' = X' + Y'
NAND is equivalent to OR
with inputs complemented

X	Y	X'	Y'	(X • Y)'	X' + Y'
0	0	1	1	1	1
0	1	1	0	1	1
1	0	0	1	1	1
1	1	0	0	0	0

NAND and NOR (logic gates)

- de Morgan's
  - Standard form: A'B' = (A + B)' A' + B' = (AB)'
  - Inverted: A + B = (A'B')' (AB) = (A' + B')'
  - AND with complemented inputs ≡ NOR
  - OR with complemented inputs ≡ NAND
  - OR ≡ NAND with complemented inputs
  - AND ≡ NOR with complemented inputs

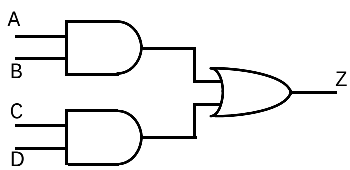
pushing the bubble

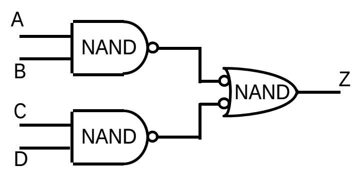


Converting to use NAND/NOR

- Introduce inversions ("bubbles")
  - Introduce bubbles in pairs
    - Conserve inversions
    - Do not alter logic function
- Example
  - AND/OR to NAND/NAND

Z = AB + CD
= (A'+B')'+(C'+D')'
= [(A'+B')(C'+D')]'
= [(AB)'(CD)']'

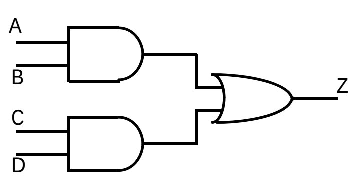


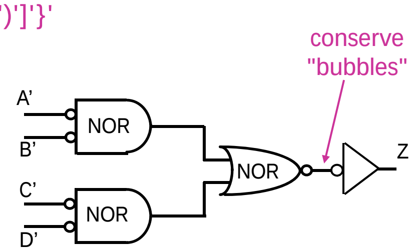


Converting to use NAND/NOR (con't)

- Example: AND/OR network to NOR/NOR

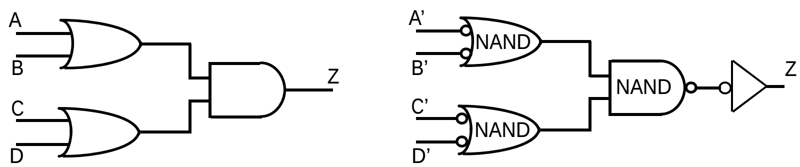
Z = AB+CD
= (A'+B')'+(C'+D')'
= [(A'+B')'+(C'+D')']''
= {[(A'+B')'+(C'+D')']'}'





Converting to use NAND/NOR (con't)

- Example: OR/AND to NAND/NAND



Converting between forms (con't)

- Example: OR/AND to NOR/NOR