Lecture 2: The Magical Base-2

CSE 370, Autumn 2007
Benjamin Ylvisaker

1

Daily Quiz

- Have you added yourself to the class mailing list?
- Do it by 5:30 this afternoon to get a 4 on today's daily quiz
- Tell classmates who didn't make it to class on time at your own discretion

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Administrivia

- Office hours

Monday	Ramkumar	???	lab
Tuesday	Josh	1:30-2:30	lab
Wednesday	Benjamin	1:30-2:30	210
Thursday	Benjamin	9:30-10:30	210
Friday	Nikhil	11:30-12:30	lab

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Elementary Math Review

- Positional number notation
  - `2,104 = 2×1,000 + 1×100 + 0×10 + 4×1`
    `= 2×10³ + 1×10² + 0×10¹ + 4×10⁰`
- Generalize to arbitrary base b
  - $XYZ = X \times b^2 + Y \times b^1 + Z \times b^0$
    where X, Y and Z are digits with values in the range [0..b-1]

Bases of Interest

- In 370, we are interested in the following bases:
  - Binary [0,1]
  - Octal [0..7]
  - Decimal [0..9]
  - Hexadecimal [0..9,A..F]
    - A=10, B=11, C=12, D=13, E=14, F=15

Conversion to Decimal

- 1001101_2
  $= 1\times2^6 + 0\times2^5 + 0\times2^4 + 1\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0$
  $= 1\times64 + 0\times32 + 0\times16 + 1\times8 + 1\times4 + 0\times2 + 1\times1$
  $= 64 + 8 + 4 + 1$
  $= 77$
- $92A70_{16}$
  $= 9\times16^4 + 2\times16^3 + 10\times16^2 + 7\times16^1 + 0\times16^0$
  $= 9\times65536 + 2\times4096 + 10\times256 + 7\times16 + 0\times1$
  $= 589824 + 8192 + 2560 + 112$
  $= 600688$

Arithmetic is the Same in All Bases

- $$\begin{array}{r} 1001101_2 \\ +\ 101011_2 \\ \hline 1111000_2 \end{array} \quad \begin{array}{r} 32175_8 \\ +\ 1622_8 \\ \hline 34017_8 \end{array} \quad \begin{array}{r} \mathrm{27AA32}_{16} \\ +\ \mathrm{92A70}_{16} \\ \hline \mathrm{30D4A2}_{16} \end{array}$$
- $$\begin{array}{r} 1001101_2 \\ -\ 101011_2 \\ \hline 100010_2 \end{array} \quad \begin{array}{r} 32175_8 \\ -\ 1622_8 \\ \hline 30353_8 \end{array} \quad \begin{array}{r} \mathrm{27AA32}_{16} \\ -\ \mathrm{92A70}_{16} \\ \hline \mathrm{1E7FC2}_{16} \end{array}$$

Multiplication, Too

- $$\begin{array}{r} 1101101_2 \\ \times\ 101011_2 \\ \hline 1101101_2 \\ 1101101_2 \\ 0000000_2 \\ 1101101_2 \\ 0000000_2 \\ +1101101_2 \\ \hline 1001001001111_2 \end{array} \qquad \begin{array}{r} \mathrm{A3}_{16} \\ \times 17_{16} \\ \hline 475_{16} \\ +\mathrm{A3}_{16} \\ \hline \mathrm{EA5}_{16} \end{array}$$

Division, Too

- $$\begin{array}{r} 1001 \\ 101\overline{)110001} \\ -101 \\ \hline 10 \\ -\ 0 \\ \hline 100 \\ -\ \ 0 \\ \hline 1001 \\ -\ 101 \\ \hline 100 \end{array}$$ Remainder: 100

Conversion to Binary by Successive Division

- $154_{10} \div 2_{10} = 77_{10}$ Remainder 0 10011010
  $77_{10} \div 2_{10} = 38_{10}$ Remainder 1
  $38_{10} \div 2_{10} = 19_{10}$ Remainder 0
  $19_{10} \div 2_{10} = 9_{10}$ Remainder 1
  $9_{10} \div 2_{10} = 4_{10}$ Remainder 1
  $4_{10} \div 2_{10} = 2_{10}$ Remainder 0
  $2_{10} \div 2_{10} = 1_{10}$ Remainder 0
  $1_{10} \div 2_{10} = 0_{10}$ Remainder 1

Read the result "up"

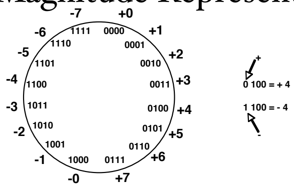
... and Back Again

- $10011010_2 \div 1010_2 = 1111_2$ Remainder 100_2
  $1111_2 \div 1010_2 = 1_2$ Remainder 101_2
  $1_2 \div 1010_2 = 0_2$ Remainder 1_2
- Converting from base B to C
  - Do divisions in base B
  - Divide by C

The Trouble with Negative Numbers

- The symbol "-" for negative can be used in any base, when doing arithmetic by hand
- Computers only have two symbols: 1, 0. No "-"
- Also, computers usually do arithmetic with numbers that are a fixed number of bits "wide" (like, 8, 16, 32, 64)

Sign/Magnitude Representation

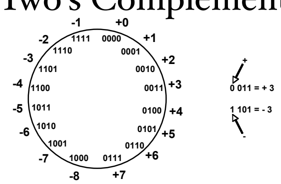


- High-order (left-most) bit is the sign. `0`=positive, `1`=negative
- Remaining bits are the magnitude
- With N bits, represent numbers between $-2^{N-1}+1$ and $2^{N-1}-1$
- Two representations of `0`!

Sign/Magnitude

- Pro: easy to read and write for humans
- Con: harder to do basic arithmetic correctly with a computer
- Result: rarely used

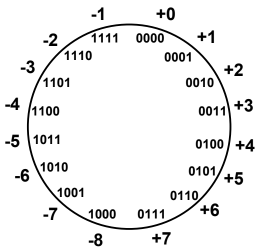
Two's Complement



- High-order (left-most) bit is the sign. `0`=positive, `1`=negative
- Remaining bits are the magnitude (encoded in a funny way)
- With N bits, represent numbers between -2^{N-1} and $2^{N-1}-1$
- Just one representations of `0`

Negation in 2's Complement

- Flip the bits and add 1



Addition in 2's Complement

-
```
  0011 (3)          1101 (-3)
 +0101 (5)         +0101 (5)
  1000 (-8)         0010 (2)

  0011 (3)          1101 (-3)
 +1011 (-5)        +1011 (-5)
  1110 (-2)         1000 (-8)
```

- Subtraction is just addition with the second operand negated first

Later in the Course

- Efficient circuits for implementing arithmetic
- Detecting overflow/underflow
- Changing the width of numbers without changing the number

Fractional Numbers

- We might want to represent non-integral numbers
- Two popular approaches:
  - Fixed-point
  - Floating-point
- Not covered in 370

Thank You for Your Attention

- Lab 1 has changed slightly, I'll post an update soon (and send a mail to the class mailing list)
- Continue reading the book
- Continue/start homework 1
- Next time: the fundamentals of Boolean logic