Sequential logic

- Sequential circuits
  - simple circuits with feedback
  - latches
  - edge-triggered flip-flops
- Timing methodologies
  - cascading flip-flops for proper operation
  - clock skew
- Asynchronous inputs
  - metastability and synchronization
- Basic registers
  - shift registers
  - simple counters
- Hardware description languages and sequential logic

Circuits with feedback

- How to control feedback?
  - what stops values from cycling around endlessly

```
X1  X2  ...  Xn
   |   |     |
   v   v     v
Z1  Z2  ...  Zn
```

switching network
Simplest circuits with feedback

- Two inverters form a static memory cell
  - will hold value as long as it has power applied

  ![Diagram of two inverters forming a static memory cell]

- How to get a new value into the memory cell?
  - selectively break feedback path
  - load new value into cell

  ![Diagram of loading a new value into a memory cell]

Memory with cross-coupled gates

- Cross-coupled NOR gates
  - similar to inverter pair, with capability to force output to 0 (reset=1) or 1 (set=1)

  ![Diagram of cross-coupled NOR gates]

- Cross-coupled NAND gates
  - similar to inverter pair, with capability to force output to 0 (reset=0) or 1 (set=0)

  ![Diagram of cross-coupled NAND gates]
Timing behavior

State behavior or R-S latch

- Truth table of R-S latch behavior

<table>
<thead>
<tr>
<th>S</th>
<th>R</th>
<th>Q</th>
<th>Q'</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>unstable</td>
<td></td>
</tr>
</tbody>
</table>
Theoretical R-S latch behavior

- State diagram
  - states: possible values
  - transitions: changes based on inputs

  possible oscillation between states 00 and 11

Observed R-S latch behavior

- Very difficult to observe R-S latch in the 1-1 state
  - one of R or S usually changes first
- Ambiguously returns to state 0-1 or 1-0
  - a so-called "race condition"
  - or non-deterministic transition
R-S latch analysis

- Break feedback path

<table>
<thead>
<tr>
<th>S</th>
<th>R</th>
<th>Q(t)</th>
<th>Q(t+Δ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>hold</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>reset</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>X</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>X</td>
</tr>
</tbody>
</table>

characteristic equation

\[ Q(t+Δ) = S + R' Q(t) \]

Activity: R-S latch using NAND gates

characteristic equation

\[ Q(t+Δ) = S + R' Q(t) \]
Gated R-S latch

- Control when R and S inputs matter
  - otherwise, the slightest glitch on R or S while enable is low could cause change in value stored

Clocks

- Used to keep time
  - wait long enough for inputs (R' and S') to settle
  - then allow to have effect on value stored
- Clocks are regular periodic signals
  - period (time between ticks)
  - duty-cycle (time clock is high between ticks - expressed as % of period)
Clocks (cont’d)

- Controlling an R-S latch with a clock
  - can't let R and S change while clock is active (allowing R and S to pass)
  - only have half of clock period for signal changes to propagate
  - signals must be stable for the other half of clock period

![Clock Diagram](image)

Cascading latches

- Connect output of one latch to input of another
- How to stop changes from racing through chain?
  - need to be able to control flow of data from one latch to the next
  - move one latch per clock period
  - have to worry about logic between latches (arrows) that is too fast

![Cascading Latches Diagram](image)
Master-slave structure

- Break flow by alternating clocks (like an air-lock)
  - use positive clock to latch inputs into one R-S latch
  - use negative clock to change outputs with another R-S latch
- View pair as one basic unit
  - master-slave flip-flop
  - twice as much logic
  - output changes a few gate delays after the falling edge of clock
    but does not affect any cascaded flip-flops

The 1s catching problem

- In first R-S stage of master-slave FF
  - 0-1-0 glitch on R or S while clock is high is "caught" by master stage
  - leads to constraints on logic to be hazard-free
**D flip-flop**

- Make S and R complements of each other
  - eliminates 1s catching problem
  - can't just hold previous value
    (must have new value ready every clock period)
  - value of D just before clock goes low is what is stored in flip-flop
  - can make R-S flip-flop by adding logic to make $D = S + R'Q$

```
D Q
```

```
master stage
```
```
slave stage
```

- 10 gates

---

**Edge-triggered flip-flops**

- More efficient solution: only 6 gates
  - sensitive to inputs only near edge of clock signal (not while high)

```
D
```
```
D'
```
```
Clk=1
```

- negative edge-triggered D flip-flop (D-FF)
  - 4-5 gate delays
  - must respect setup and hold time constraints to successfully capture input

```
d 0
```
```
d'
```
```
characteristic equation
```
```
$Q(t+1) = D$
```

---
Edge-triggered flip-flops (cont’d)

- **Step-by-step analysis**
  - When clock goes high-to-low: data is latched
  - When clock is low: data is held

- **Positive edge-triggered FF**
  - Inputs sampled on rising edge; outputs change after rising edge
- **Negative edge-triggered flip-flops**
  - Inputs sampled on falling edge; outputs change after falling edge

- **Diagram**
  - Diagram showing the operation of positive and negative edge-triggered flip-flops under different clock states.
  - Diagram illustrating the waveform of inputs and outputs, highlighting the positive edge-triggered FF and negative edge-triggered FF.
Timing methodologies

- Rules for interconnecting components and clocks
  - guarantee proper operation of system when strictly followed
- Approach depends on building blocks used for memory elements
  - we'll focus on systems with edge-triggered flip-flops
    - found in programmable logic devices
  - many custom integrated circuits focus on level-sensitive latches
- Basic rules for correct timing:
  - (1) correct inputs, with respect to time, are provided to the flip-flops
  - (2) no flip-flop changes state more than once per clocking event

Timing methodologies (cont’d)

- Definition of terms
  - clock: periodic event, causes state of memory element to change can be rising edge or falling edge or high level or low level
  - setup time: minimum time before the clocking event by which the input must be stable (Tsu)
  - hold time: minimum time after the clocking event until which the input must remain stable (Th)

There is a timing "window" around the clocking event during which the input must remain stable and unchanged in order to be recognized.
Comparison of latches and flip-flops

- **DQ**
  - **CLK**
  - positive edge-triggered flip-flop

- **DQ**
  - **CLK**
  - transparent (level-sensitive) latch

Behavior is the same unless input changes while the clock is high.

Comparison of latches and flip-flops (cont’d)

<table>
<thead>
<tr>
<th>Type</th>
<th>When inputs are sampled</th>
<th>When output is valid</th>
</tr>
</thead>
<tbody>
<tr>
<td>unclocked latch</td>
<td>always</td>
<td>propagation delay from input change</td>
</tr>
<tr>
<td>level-sensitive latch</td>
<td>clock high (Tsu/Th around falling edge of clock)</td>
<td>propagation delay from input change or clock edge (whichever is later)</td>
</tr>
<tr>
<td>master-slave flip-flop</td>
<td>clock hi-to-lo transition (Tsu/Th around falling edge of clock)</td>
<td>propagation delay from falling edge of clock</td>
</tr>
<tr>
<td>negative edge-triggered flip-flop</td>
<td>clock hi-to-lo transition (Tsu/Th around falling edge of clock)</td>
<td>propagation delay from falling edge of clock</td>
</tr>
</tbody>
</table>
Typical timing specifications

- Positive edge-triggered D flip-flop
  - setup and hold times
  - minimum clock width
  - propagation delays (low to high, high to low, max and typical)

![Timing Diagram]

all measurements are made from the clocking event (the rising edge of the clock)

Cascading edge-triggered flip-flops

- Shift register
  - new value goes into first stage
  - while previous value of first stage goes into second stage
  - consider setup/hold/propagation delays (prop must be > hold)
Cascading edge-triggered flip-flops (cont’d)

- Why this works
  - propagation delays exceed hold times
  - clock width constraint exceeds setup time
  - this guarantees following stage will latch current value before it changes to new value

Clock skew

- The problem
  - correct behavior assumes next state of all storage elements determined by all storage elements at the same time
  - this is difficult in high-performance systems because time for clock to arrive at flip-flop is comparable to delays through logic
  - effect of skew on cascaded flip-flops:
Summary of latches and flip-flops

- Development of D-FF
  - level-sensitive used in custom integrated circuits
  - can be made with 4 switches
  - edge-triggered used in programmable logic devices
  - good choice for data storage register
- Historically J-K FF was popular but now never used
  - similar to R-S but with 1-1 being used to toggle output (complement state)
  - good in days of TTL/SSI (more complex input function: \( D = J Q' + K' Q \))
  - not a good choice for PALs/PLAs as it requires 2 inputs
  - can always be implemented using D-FF
- Preset and clear inputs are highly desirable on flip-flops
  - used at start-up or to reset system to a known state

Metastability and asynchronous inputs

- Clocked synchronous circuits
  - inputs, state, and outputs sampled or changed in relation to a common reference signal (called the clock)
  - e.g., master/slave, edge-triggered
- Asynchronous circuits
  - inputs, state, and outputs sampled or changed independently of a common reference signal (glitches/hazards a major concern)
  - e.g., R-S latch
- Asynchronous inputs to synchronous circuits
  - inputs can change at any time, will not meet setup/hold times
  - dangerous, synchronous inputs are greatly preferred
  - cannot be avoided (e.g., reset signal, memory wait, user input)
Synchronization failure

- Occurs when FF input changes close to clock edge
  - the FF may enter a metastable state – neither a logic 0 nor 1 –
  - it may stay in this state an indefinite amount of time
  - this is not likely in practice but has some probability

Dealing with synchronization failure

- Probability of failure can never be reduced to 0, but it can be reduced
  - (1) slow down the system clock
    this gives the synchronizer more time to decay into a steady state;
    synchronizer failure becomes a big problem for very high speed systems
  - (2) use fastest possible logic technology in the synchronizer
    this makes for a very sharp "peak" upon which to balance
  - (3) cascade two synchronizers
    this effectively synchronizes twice (both would have to fail)
Handling asynchronous inputs

- Never allow asynchronous inputs to fan-out to more than one flip-flop
  - Synchronize as soon as possible and then treat as synchronous signal

Handling asynchronous inputs (cont’d)

- What can go wrong?
  - Input changes too close to clock edge (violating setup time constraint)

In is asynchronous and fans out to D0 and D1

One FF catches the signal, one does not

Inconsistent state may be reached!
Flip-flop features

- **Reset** (set state to 0) – R
  - synchronous: \( D_{\text{new}} = R' \cdot D_{\text{old}} \) (when next clock edge arrives)
  - asynchronous: doesn't wait for clock, quick but dangerous
- **Preset or set** (set state to 1) – S (or sometimes P)
  - synchronous: \( D_{\text{new}} = D_{\text{old}} + S \) (when next clock edge arrives)
  - asynchronous: doesn't wait for clock, quick but dangerous
- **Both reset and preset**
  - \( D_{\text{new}} = R' \cdot D_{\text{old}} + S \) (set-dominant)
  - \( D_{\text{new}} = R' \cdot D_{\text{old}} + R'S \) (reset-dominant)
- **Selective input capability** (input enable or load) – LD or EN
  - multiplexor at input: \( D_{\text{new}} = LD' \cdot Q + LD \cdot D_{\text{old}} \)
  - load may or may not override reset/set (usually R/S have priority)
- **Complementary outputs** – Q and Q'

Registers

- **Collections of flip-flops with similar controls and logic**
  - stored values somehow related (for example, form binary value)
  - share clock, reset, and set lines
  - similar logic at each stage
- **Examples**
  - shift registers
  - counters

![Registers Diagram]
Shift register

- Holds samples of input
  - store last 4 input values in sequence
  - 4-bit shift register:

![Shift register diagram](image)

Universal shift register

- Holds 4 values
  - serial or parallel inputs
  - serial or parallel outputs
  - permits shift left or right
  - shift in new values from left or right

![Universal shift register diagram](image)
Design of universal shift register

- Consider one of the four flip-flops
  - new value at next clock cycle:

<table>
<thead>
<tr>
<th>clear s0</th>
<th>s1</th>
<th>new value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-</td>
<td>0 output</td>
</tr>
<tr>
<td>0 0</td>
<td>0</td>
<td>output value of FF to left (shift right)</td>
</tr>
<tr>
<td>0 1 0</td>
<td></td>
<td>output value of FF to right (shift left)</td>
</tr>
<tr>
<td>0 1 1</td>
<td></td>
<td>input</td>
</tr>
</tbody>
</table>

Shift register application

- Parallel-to-serial conversion for serial transmission
**Pattern recognizer**

- Combinational function of input samples
  - in this case, recognizing the pattern 1001 on the single input signal

**Counters**

- Sequences through a fixed set of patterns
  - in this case, 1000, 0100, 0010, 0001
  - if one of the patterns is its initial state (by loading or set/reset)
Activity

How does this counter work?

Counts through the sequence: 1000, 1100, 1110, 1111, 0111, 0011, 0001, 0000

Known as Mobius (or Johnson) counter

Binary counter

Logic between registers (not just multiplexer)
- XOR decides when bit should be toggled
- always for low-order bit, only when first bit is true for second bit, and so on
### Four-bit binary synchronous up-counter

- Standard component with many applications
  - positive edge-triggered FFs w/ synchronous load and clear inputs
  - parallel load data from D, C, B, A
  - enable inputs: must be asserted to enable counting
  - RCO: ripple-carry out used for cascading counters
    - high when counter is in its highest state 1111
    - implemented using an AND gate

![Counter Diagram](image)

- (1) Low order 4-bits = 1111
- (2) RCO goes high
- (3) High order 4-bits are incremented

### Offset counters

- Starting offset counters – use of synchronous load
  - e.g., 0110, 0111, 1000, 1001, 1010, 1011, 1100, 1101, 1111, 0110, ...

- Ending offset counter – comparator for ending value
  - e.g., 0000, 0001, 0010, ..., 1100, 1101, 0000

- Combinations of the above (start and stop value)
Hardware Description Languages and Sequential Logic

- Flip-flops
  - representation of clocks - timing of state changes
  - asynchronous vs. synchronous
- Shift registers
- Simple counters

Flip-flop in Verilog

- Use always block’s sensitivity list to wait for clock edge

```verilog
define dff (clk, d, q);
    input clk, d;
    output q;
    reg q;

    always @(posedge clk)
        q = d;

endmodule
```
More Flip-flops

- Synchronous/asynchronous reset/set
  - single thread that waits for the clock
  - three parallel threads – only one of which waits for the clock

### Synchronous

```verilog
module dff (clk, s, r, d, q);
  input  clk, s, r, d;
  output q;
  reg    q;
  always @(posedge clk)
    if (r)      q = 1'b0;
    else if (s) q = 1'b1;
    else        q = d;
endmodule
```

### Asynchronous

```verilog
module dff (clk, s, r, d, q);
  input  clk, s, r, d;
  output q;
  reg    q;
  always @(posedge r)
    q = 1'b0;
  always @(posedge s)
    q = 1'b1;
  always @(posedge clk)
    q = d;
endmodule
```

Incorrect Flip-flop in Verilog

- Use always block's sensitivity list to wait for clock to change

```verilog
module dff (clk, d, q);
  input  clk, d;
  output q;
  reg    q;
  always @(clk)
    q = d;
endmodule
```

Not correct! Q will change whenever the clock changes, not just on an edge.
Blocking and Non-Blocking Assignments

- **Blocking assignments (X=A)**
  - completes the assignment before continuing on to next statement
- **Non-blocking assignments (X<=A)**
  - completes in zero time and doesn’t change the value of the target until a blocking point (delay/wait) is encountered
- **Example:** swap

```verbatim
always @(posedge CLK)
begin
  temp = B;
  B = A;
  A = temp;
end
```

Register-transfer-level (RTL) Assignment

- **Non-blocking assignment is also known as an RTL assignment**
  - if used in an always block triggered by a clock edge
  - all flip-flops change together

```verbatim
// B,C,D all get the value of A
always @(posedge clk)
begin
  B = A;
  C = B;
  D = C;
end
```

```verbatim
// implements a shift register too
always @(posedge clk)
begin
  B <= A;
  C <= B;
  D <= C;
end
```
Shift register in Verilog

```
module shift_register (clk, in, out);
  input  clk;
  input  in;
  output [0:3] out;
  reg [0:3] out;
  initial begin
    out = 0;  // out[0:3] = {0, 0, 0, 0};
  end
  always @(posedge clk) begin
    out = {in, out [0:2]};
  end
endmodule
```

Mobius Counter in Verilog

```
initial begin
  A = 1’b0;
  B = 1’b0;
  C = 1’b0;
  D = 1’b0;
end

always @(posedge clk)
begin
  A <= ~D;
  B <= A;
  C <= B;
  D <= C;
  {A, B, C, D} <= {~D, A, B, C};
end
```
**Binary Counter in Verilog**

```verilog
module binary_counter (clk, c8, c4, c2, c1);
  input clk;
  output c8, c4, c2, c1;
  reg [3:0] count;
  initial begin
    count = 0;
  end
  always @(posedge clk) begin
    count = count + 4'b0001;
  end
  assign c8 = count[3];
  assign c4 = count[2];
  assign c2 = count[1];
  assign c1 = count[0];
endmodule
```

```verilog
module binary_counter (clk, c8, c4, c2, c1, rco);
  input clk;
  output c8, c4, c2, c1, rco;
  reg [3:0] count;
  reg rco;
  initial begin . . . end
  always @(posedge clk) begin . . . end
  assign c8 = count[3];
  assign c4 = count[2];
  assign c2 = count[1];
  assign c1 = count[0];
  assign rco = (count == 4'b1111);
endmodule
```

**Sequential logic summary**

- Fundamental building block of circuits with state
  - latch and flip-flop
  - R-S latch, R-S master/slave, D master/slave, edge-triggered D flip-flop
- Timing methodologies
  - use of clocks
  - cascaded FFs work because propagation delays exceed hold times
  - beware of clock skew
- Asynchronous inputs and their dangers
  - synchronizer failure: what it is and how to minimize its impact
- Basic registers
  - shift registers
  - counters
- Hardware description languages and sequential logic