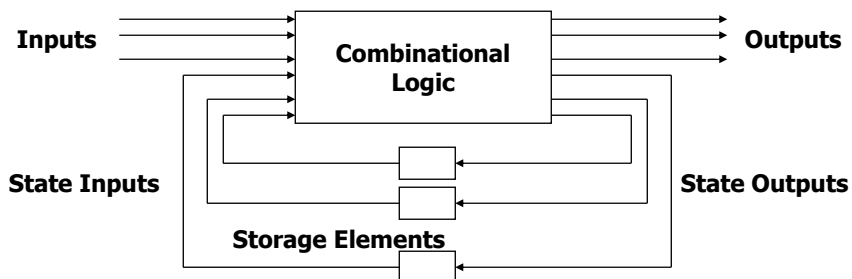


Finite State Machines

- Sequential circuits
 - primitive sequential elements
 - combinational logic
- Models for representing sequential circuits
 - finite-state machines (Moore and Mealy)
- Basic sequential circuits revisited
 - shift registers
 - counters
- Design procedure
 - state diagrams
 - state transition table
 - next state functions
- Hardware description languages

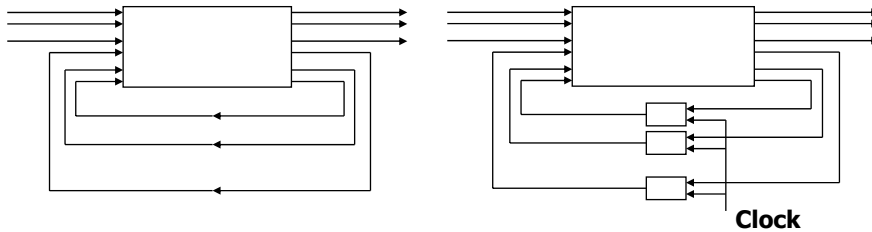
Abstraction of state elements

- Divide circuit into combinational logic and state
- Localize the feedback loops and make it easy to break cycles
- Implementation of storage elements leads to various forms of sequential logic



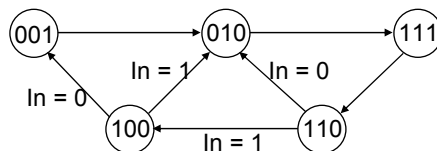
Forms of sequential logic

- Asynchronous sequential logic – state changes occur whenever state inputs change (elements may be simple wires or delay elements)
- Synchronous sequential logic – state changes occur in lock step across all storage elements (using a periodic waveform - the clock)



Finite state machine representations

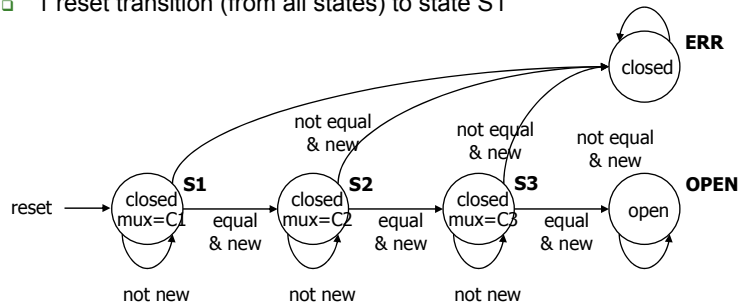
- States: determined by possible values in sequential storage elements
- Transitions: change of state
- Clock: controls when state can change by controlling storage elements
- Sequential logic
 - sequences through a series of states
 - based on sequence of values on input signals
 - clock period defines elements of sequence



Example finite state machine diagram

- Combination lock from introduction to course

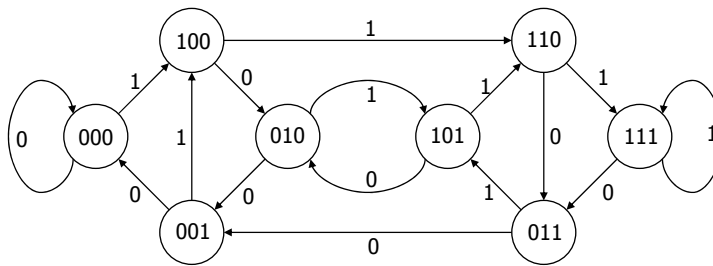
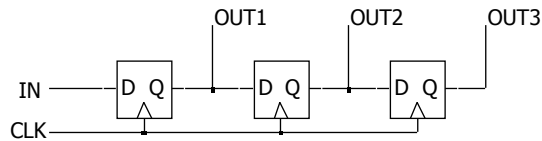
- 5 states
- 5 self-transitions
- 6 other transitions between states
- 1 reset transition (from all states) to state S1



Can any sequential system be represented with a state diagram?

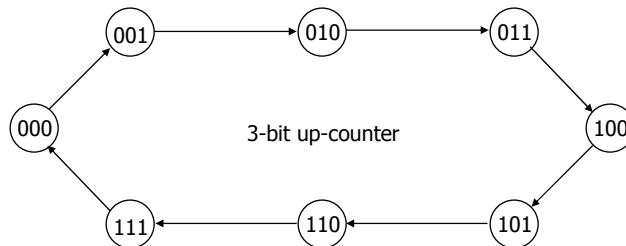
- Shift register

- input value shown on transition arcs
- output values shown within state node



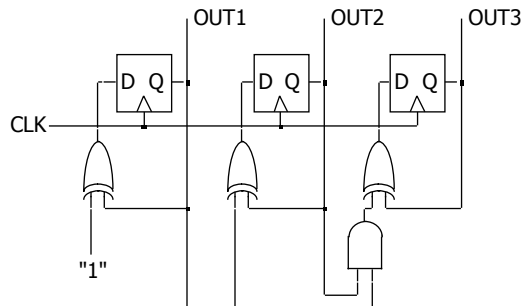
Counters are simple finite state machines

- Counters
 - proceed through well-defined sequence of states in response to enable
- Many types of counters: binary, BCD, Gray-code
 - 3-bit up-counter: 000, 001, 010, 011, 100, 101, 110, 111, 000, ...
 - 3-bit down-counter: 111, 110, 101, 100, 011, 010, 001, 000, 111, ...



How do we turn a state diagram into logic?

- Counter
 - 3 flip-flops to hold state
 - logic to compute next state
 - clock signal controls when flip-flop memory can change
 - wait long enough for combinational logic to compute new value
 - don't wait too long as that is low performance



FSM design procedure

- Start with counters
 - simple because output is just state
 - simple because no choice of next state based on input
- State diagram to state transition table
 - tabular form of state diagram
 - like a truth-table
- State encoding
 - decide on representation of states
 - for counters it is simple: just its value
- Implementation
 - flip-flop for each state bit
 - combinational logic based on encoding

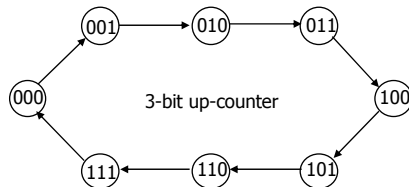
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9

FSM design procedure: state diagram to encoded state transition table

- Tabular form of state diagram
- Like a truth-table (specify output for all input combinations)
- Encoding of states: easy for counters – just use value



	current state	next state	
0	000	001	1
1	001	010	2
2	010	011	3
3	011	100	4
4	100	101	5
5	101	110	6
6	110	111	7
7	111	000	0

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10

Implementation

- D flip-flop for each state bit
- Combinational logic based on encoding

C3	C2	C1	N3	N2	N1
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	1	1
0	1	1	1	0	0
1	0	0	1	0	1
1	0	1	1	1	0
1	1	0	1	1	1
1	1	1	0	0	0

Verilog notation to show function represents an input to D-FF

```

N1 <= C1'
N2 <= C1C2' + C1'C2
    <= C1 xor C2
N3 <= C1C2C3' + C1'C3 + C2'C3
    <= (C1C2)C3' + (C1' + C2')C3
    <= (C1C2)C3' + (C1C2)'C3
    <= (C1C2) xor C3
    
```

N3

	C3	C2	C1
0	0	1	1
0	1	0	1
1	0	0	1
1	1	0	1

C1

N2

	C3	C2	C1
0	1	1	0
1	0	0	1

C1

N1

	C3	C2	C1
1	1	1	1
0	0	0	0

C1

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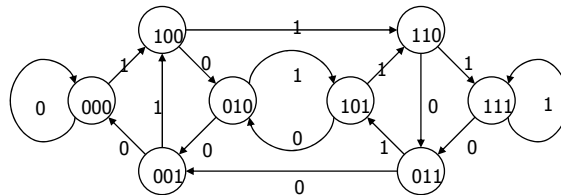
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11

Back to the shift register

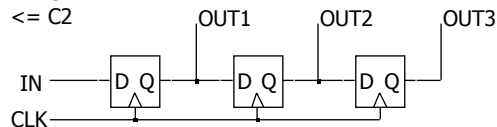
- Input determines next state

In	C1	C2	C3	N1	N2	N3
0	0	0	0	0	0	0
0	0	0	1	0	0	0
0	0	1	0	0	0	1
0	0	1	1	0	0	1
0	1	0	0	0	1	0
0	1	0	1	0	1	0
0	1	1	0	0	1	1
0	1	1	1	0	1	1
1	0	0	0	1	0	0
1	0	0	1	1	0	0
1	0	1	0	1	0	1
1	0	1	1	1	0	1
1	1	0	0	1	1	0
1	1	0	1	1	1	0
1	1	1	0	1	1	1
1	1	1	1	1	1	1



```

N1 <= In
N2 <= C1
N3 <= C2
    
```



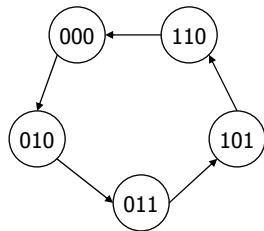
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12

More complex counter example

- Complex counter
 - repeats 5 states in sequence
 - not a binary number representation
- Step 1: derive the state transition diagram
 - count sequence: 000, 010, 011, 101, 110
- Step 2: derive the state transition table from the state transition diagram



Present State			Next State		
C	B	A	C+	B+	A+
0	0	0	0	1	0
0	0	1	-	-	-
0	1	0	0	1	1
0	1	1	1	0	1
1	0	0	-	-	-
1	0	1	1	1	0
1	1	0	0	0	0
1	1	1	-	-	-

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note the don't care conditions that arise from the unused state codes

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13

More complex counter example (cont'd)

- Step 3: K-maps for next state functions

	C			
C+	0	0	0	X
A	X	1	X	1
	B			

	C			
B+	1	1	0	X
A	X	0	X	1
	B			

	C			
A+	0	1	0	X
A	X	1	X	0
	B			

$$C+ \leq A$$

$$B+ \leq B' + A'C'$$

$$A+ \leq BC'$$

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14

Self-starting counters (cont'd)

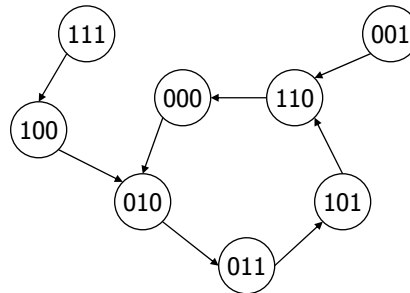
- Re-deriving state transition table from don't care assignment

C+	C			
	0	0	0	0
A	1	1	1	1
	B			

B+	C			
	1	1	0	1
A	1	0	0	1
	B			

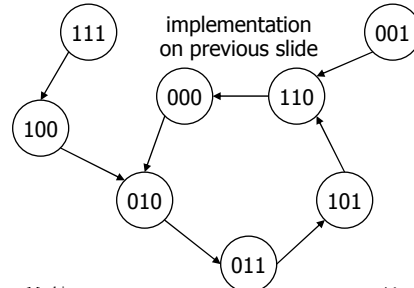
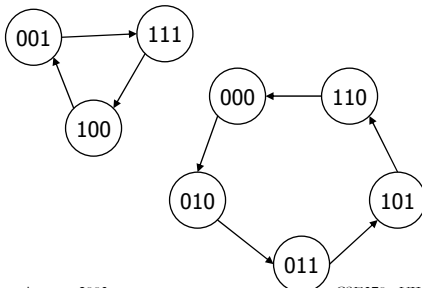
A+	C			
	0	1	0	0
A	0	1	0	0
	B			

Present State			Next State		
C	B	A	C+	B+	A+
0	0	0	0	1	0
0	0	1	1	1	0
0	1	0	0	1	1
0	1	1	1	0	1
1	0	0	0	1	0
1	0	1	1	1	0
1	1	0	0	0	0
1	1	1	1	0	0



Self-starting counters

- Start-up states
 - at power-up, counter may be in an unused or invalid state
 - designer must guarantee that it (eventually) enters a valid state
- Self-starting solution
 - design counter so that invalid states eventually transition to a valid state
 - may limit exploitation of don't cares



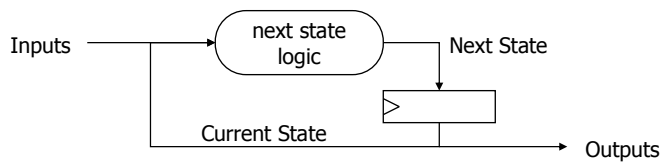
Activity

- 2-bit up-down counter (2 inputs)
 - direction: $D = 0$ for up, $D = 1$ for down
 - count: $C = 0$ for hold, $C = 1$ for count

Activity (cont'd)

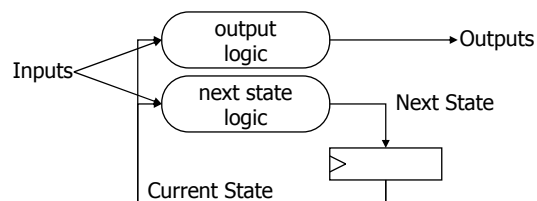
Counter/shift-register model

- Values stored in registers represent the state of the circuit
- Combinational logic computes:
 - next state
 - function of current state and inputs
 - outputs
 - values of flip-flops



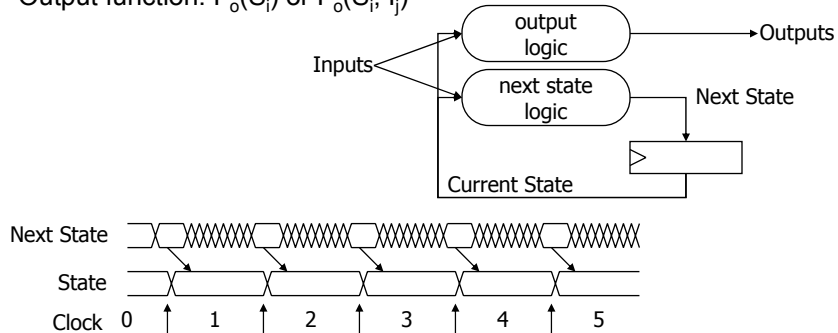
General state machine model

- Values stored in registers represent the state of the circuit
- Combinational logic computes:
 - next state
 - function of current state and inputs
 - outputs
 - function of current state and inputs (Mealy machine)
 - function of current state only (Moore machine)



State machine model (cont'd)

- States: S_1, S_2, \dots, S_k
- Inputs: I_1, I_2, \dots, I_m
- Outputs: O_1, O_2, \dots, O_n
- Transition function: $F_s(S_i, I_j)$
- Output function: $F_o(S_i)$ or $F_o(S_i, I_j)$



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21

Comparison of Mealy and Moore machines

- Mealy machines tend to have less states
 - different outputs on arcs (n^2) rather than states (n)
- Moore machines are safer to use
 - outputs change at clock edge (always one cycle later)
 - in Mealy machines, input change can cause output change as soon as logic is done – a big problem when two machines are interconnected – asynchronous feedback may occur if one isn't careful
- Mealy machines react faster to inputs
 - react in same cycle – don't need to wait for clock
 - in Moore machines, more logic may be necessary to decode state into outputs – more gate delays after clock edge

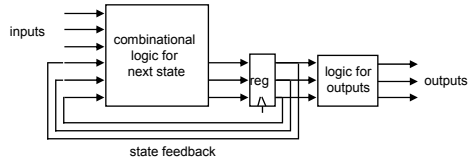
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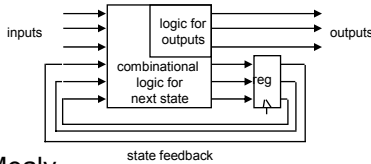
22

Comparison of Mealy and Moore machines (cont'd)

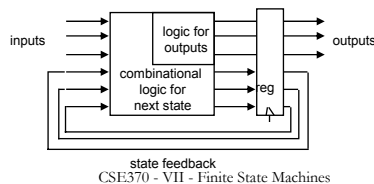
- Moore



- Mealy



- Synchronous Mealy



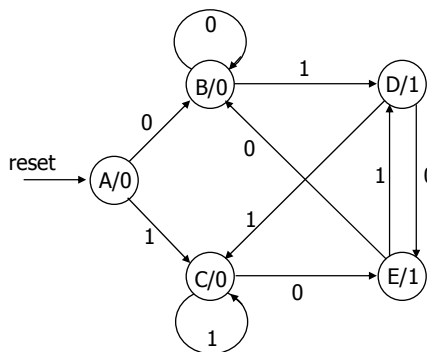
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23

Specifying outputs for a Moore machine

- Output is only function of state
 - specify in state bubble in state diagram
 - example: sequence detector for 01 or 10



reset	input	current state	next state	output
1	-	-	A	
0	0	A	B	0
0	1	A	C	0
0	0	B	B	0
0	1	B	D	0
0	0	C	E	0
0	1	C	C	0
0	0	D	E	1
0	1	D	C	1
0	0	E	B	1
0	1	E	D	1

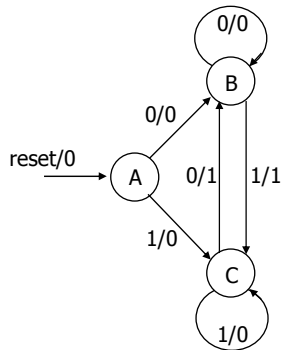
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24

Specifying outputs for a Mealy machine

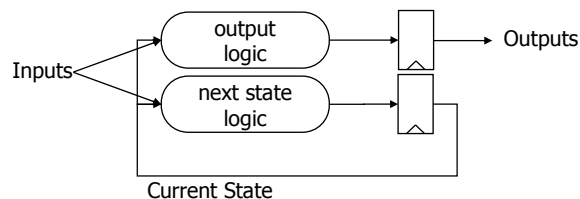
- Output is function of state and inputs
 - specify output on transition arc between states
 - example: sequence detector for 01 or 10



reset	input	current state	next state	output
1	–	–	A	0
0	0	A	B	0
0	1	A	C	0
0	0	B	B	0
0	1	B	C	1
0	0	C	B	1
0	1	C	C	0

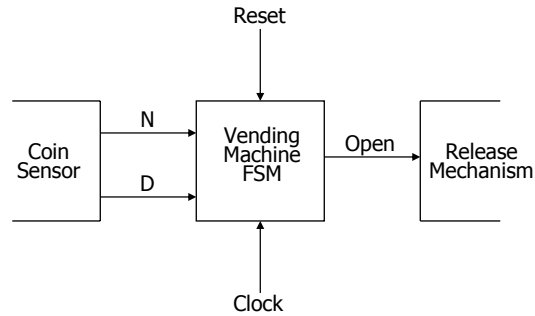
Registered Mealy machine (really Moore)

- Synchronous (or registered) Mealy machine
 - registered state AND outputs
 - avoids 'glitchy' outputs
 - easy to implement in PLDs
- Moore machine with no output decoding
 - outputs computed on transition to next state rather than after entering
 - view outputs as expanded state vector



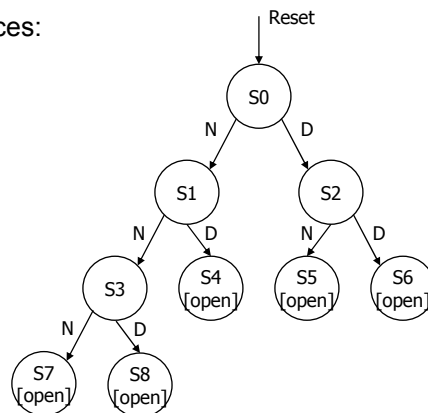
Example: vending machine

- Release item after 15 cents are deposited
- Single coin slot for dimes, nickels
- No change



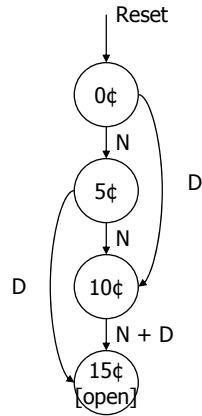
Example: vending machine (cont'd)

- Suitable abstract representation
 - tabulate typical input sequences:
 - 3 nickels
 - nickel, dime
 - dime, nickel
 - two dimes
 - draw state diagram:
 - inputs: N, D, reset
 - output: open chute
 - assumptions:
 - assume N and D asserted for one cycle
 - each state has a self loop for $N = D = 0$ (no coin)



Example: vending machine (cont'd)

- Minimize number of states - reuse states whenever possible



present state	inputs		next state	output open
	D	N		
0¢	0	0	0¢	0
	0	1	5¢	0
	1	0	10¢	0
	1	1	—	—
5¢	0	0	5¢	0
	0	1	10¢	0
	1	0	15¢	0
	1	1	—	—
10¢	0	0	10¢	0
	0	1	15¢	0
	1	0	15¢	0
	1	1	—	—
15¢	—	—	15¢	1

symbolic state table

Example: vending machine (cont'd)

- Uniquely encode states

present state O1 O0	inputs		next state D1 D0	output open
	D	N		
0 0	0	0	0 0	0
	0	1	0 1	0
	1	0	1 0	0
	1	1	—	—
0 1	0	0	0 1	0
	0	1	1 0	0
	1	0	1 1	0
	1	1	—	—
1 0	0	0	1 0	0
	0	1	1 1	0
	1	0	1 1	0
	1	1	—	—
1 1	—	—	1 1	1

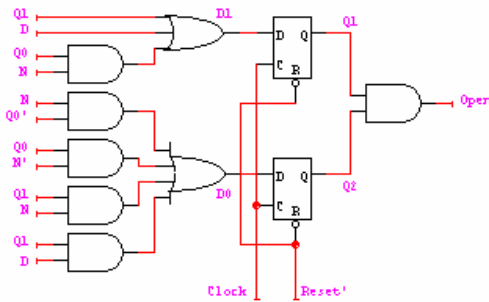
Example: Moore implementation

- Mapping to logic

		Q1		N
		D1	D0	
D	0	0	1	X
	0	1	1	1
	X	X	1	X
	1	1	1	1
		Q0		

		Q1		N
		D0	Open	
D	0	1	1	0
	1	0	1	1
	X	X	1	X
	0	1	1	1
		Q0		

		Q1		N
		Open		
D	0	0	1	0
	0	0	1	0
	X	X	1	X
	0	0	1	0
		Q0		



$$D1 = Q1 + D + Q0 N$$

$$D0 = Q0' N + Q0 N' + Q1 N + Q1 D$$

$$OPEN = Q1 Q0$$

Example: vending machine (cont'd)

- One-hot encoding

present state		inputs		next state output						
Q3	Q2	Q1	Q0	D	N	D3	D2	D1	D0	open
0	0	0	1	0	0	0	0	1	0	
				0	1	0	0	1	0	0
				1	0	0	1	0	0	0
				1	1	-	-	-	-	-
0	0	1	0	0	0	0	0	1	0	0
				0	1	0	1	0	0	0
				1	0	1	0	0	0	0
				1	1	-	-	-	-	-
0	1	0	0	0	0	0	1	0	0	0
				0	1	1	0	0	0	0
				1	0	1	0	0	0	0
				1	1	-	-	-	-	-
1	0	0	0	-	-	1	0	0	0	1

$$D0 = Q0 D' N'$$

$$D1 = Q0 N + Q1 D' N'$$

$$D2 = Q0 D + Q1 N + Q2 D' N'$$

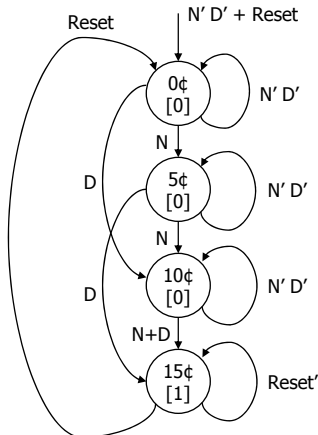
$$D3 = Q1 D + Q2 D + Q2 N + Q3$$

$$OPEN = Q3$$

Equivalent Mealy and Moore state diagrams

- Moore machine

- outputs associated with state

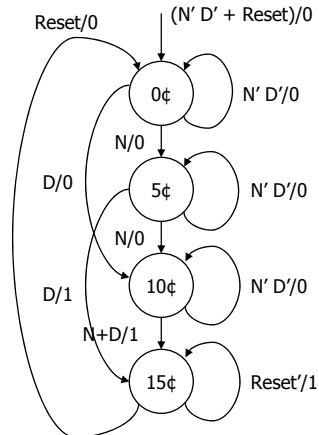


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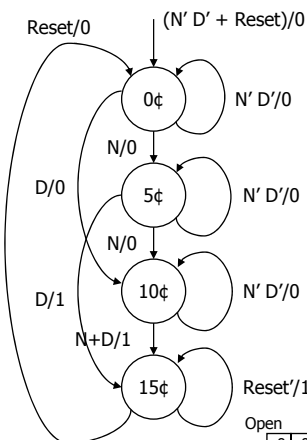
- Mealy machine

- outputs associated with transitions



33

Example: Mealy implementation



present state		inputs		next state		output	
Q1	Q0	D	N	D1	D0	open	
0	0	0	0	0	0	0	
		0	1	0	1	0	
		1	0	1	0	0	
		1	1	-	-	-	
0	1	0	0	0	1	0	
		0	1	1	0	0	
		1	0	1	1	1	
		1	1	-	-	-	
1	0	0	0	1	0	0	
		0	1	1	1	1	
		1	0	1	1	1	
		1	1	-	-	-	
1	1	-	-	1	1	1	

	Q1			
Open	0	0	1	0
	0	0	1	1
D	X	X	1	X
	0	1	1	1

$$D0 = Q0'N + Q0N' + Q1N + Q1D$$

$$D1 = Q1 + D + Q0N$$

$$OPEN = Q1Q0 + Q1N + Q1D + Q0D$$

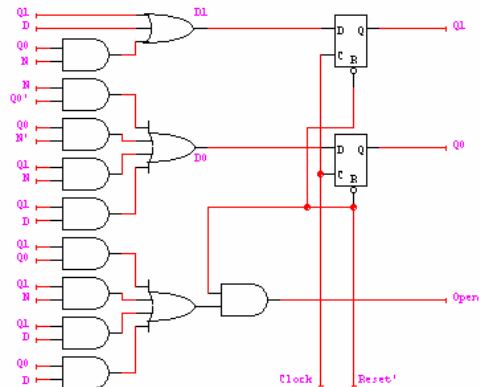
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34

Example: Mealy implementation

$$\begin{aligned} D0 &= Q0'N + Q0N' + Q1N + Q1D \\ D1 &= Q1 + D + Q0N \\ \text{OPEN} &= Q1Q0 + Q1N + Q1D + Q0D \end{aligned}$$



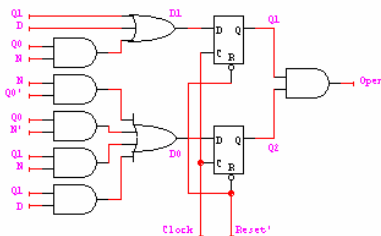
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35

Vending machine: Moore to synch. Mealy

- OPEN = Q1Q0 creates a combinational delay after Q1 and Q0 change in Moore implementation
- This can be corrected by retiming, i.e., move flip-flops and logic through each other to improve delay
- $\text{OPEN}.d = (Q1 + D + Q0N)(Q0'N + Q0N' + Q1N + Q1D)$
 $= Q1Q0N' + Q1N + Q1D + Q0'ND + Q0N'D$
- Implementation now looks like a synchronous Mealy machine
 - it is common for programmable devices to have FF at end of logic



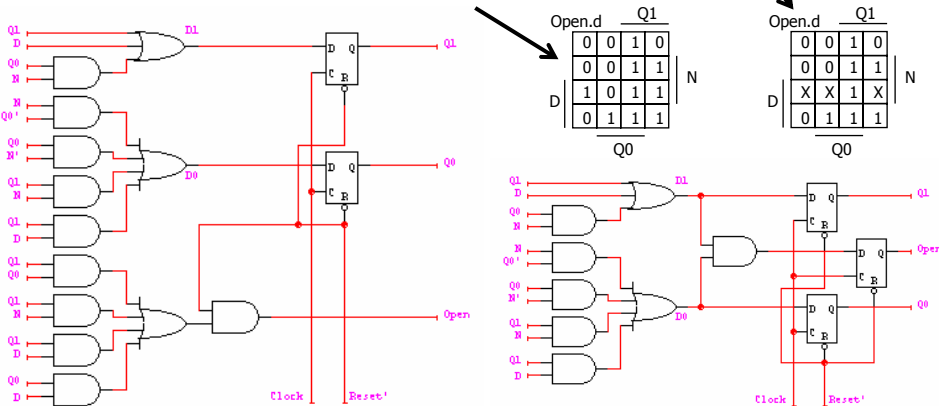
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36

Vending machine: Mealy to synch. Mealy

- $OPEN.d = Q1Q0 + Q1N + Q1D + Q0D$
- $OPEN.d = (Q1 + D + Q0N)(Q0'N + Q0N' + Q1N + Q1D)$
 $= Q1Q0N' + Q1N + Q1D + Q0'ND + Q0N'D$



Open.d	Q1		
0	0	1	0
0	0	1	1
1	0	1	1
0	1	1	1

Open.d	Q1		
0	0	1	0
0	0	1	1
X	X	X	X
0	1	1	1

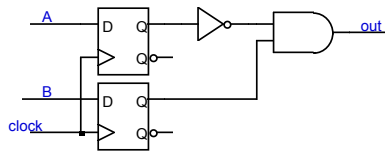
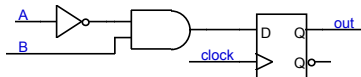
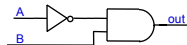
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37

Mealy and Moore examples

- Recognize A,B = 0,1
 - Mealy or Moore?



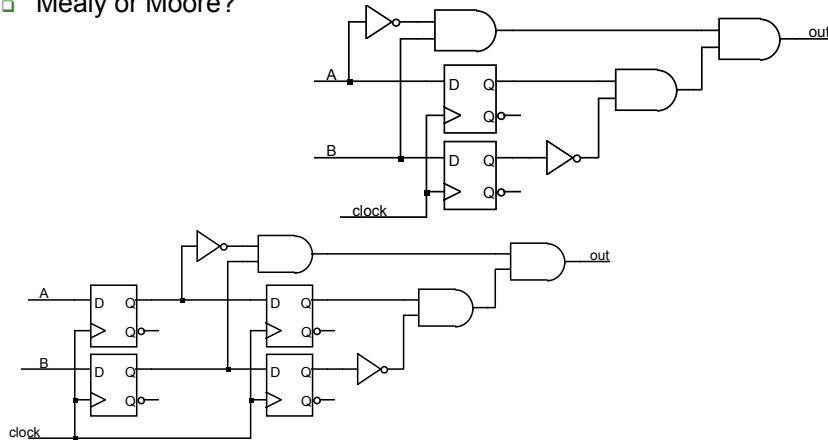
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38

Mealy and Moore examples (cont'd)

- Recognize A,B = 1,0 then 0,1
 - Mealy or Moore?



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39

Hardware Description Languages and Sequential Logic

- Flip-flops
 - representation of clocks - timing of state changes
 - asynchronous vs. synchronous
- FSMs
 - structural view (FFs separate from combinational logic)
 - behavioral view (synthesis of sequencers – not in this course)
- Data-paths = data computation (e.g., ALUs, comparators) + registers
 - use of arithmetic/logical operators
 - control of storage elements

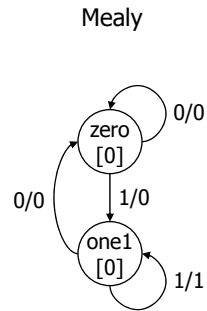
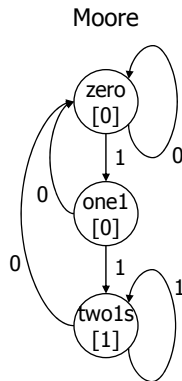
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40

Example: reduce-1-string-by-1

- Remove one 1 from every string of 1s on the input



Verilog FSM - Reduce 1s example

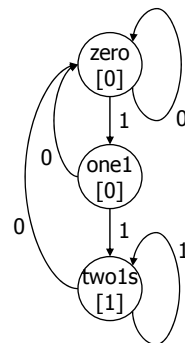
- Moore machine

```
parameter zero = 2'b00;
parameter one1 = 2'b01;
parameter twos = 2'b10;
```

← state assignment (easy to change, if in one place)

```
module reduce (clk, reset, in, out);
  input clk, reset, in;
  output out;
  reg out;
  reg [2:1] state; // state variables
  reg [2:1] next_state;

  always @(posedge clk)
    if (reset) state = zero;
    else state = next_state;
```



Moore Verilog FSM (cont'd)

```

always @(in or state)
    case (state)
    zero:
        // last input was a zero
        begin
            if (in) next_state = one1;
            else next_state = zero;
        end
    one1:
        // we've seen one 1
        begin
            if (in) next_state = twos;
            else next_state = zero;
        end
    twos:
        // we've seen at least 2 ones
        begin
            if (in) next_state = twos;
            else next_state = zero;
        end
    endcase

```

crucial to include
all signals that are
input to state determination

note that output
depends only on state

```

always @(state)
    case (state)
    zero: out = 0;
    one1: out = 0;
    twos: out = 1;
    endcase

```

```
endmodule
```

Mealy Verilog FSM

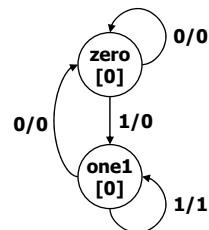
```

module reduce (clk, reset, in, out);
    input clk, reset, in;
    output out;
    reg out;
    reg state; // state variables
    reg next_state;

    always @(posedge clk)
        if (reset) state = zero;
        else state = next_state;

    always @(in or state)
        case (state)
        zero: // last input was a zero
            begin
                out = 0;
                if (in) next_state = one;
                else next_state = zero;
            end
        one: // we've seen one 1
            if (in) begin
                next_state = one; out = 1;
            end else begin
                next_state = zero; out = 0;
            end
        endcase
    endmodule

```



Synchronous Mealy Machine

```
module reduce (clk, reset, in, out);
  input clk, reset, in;
  output out;
  reg out;
  reg state; // state variables

  always @(posedge clk)
    if (reset) state = zero;
    else
      case (state)
        zero: // last input was a zero
          begin
            out = 0;
            if (in) state = one;
            else state = zero;
          end
        one: // we've seen one 1
          if (in) begin
            state = one; out = 1;
          end else begin
            state = zero; out = 0;
          end
      endcase
endmodule
```

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45

Finite state machines summary

- Models for representing sequential circuits
 - abstraction of sequential elements
 - finite state machines and their state diagrams
 - inputs/outputs
 - Mealy, Moore, and synchronous Mealy machines
- Finite state machine design procedure
 - deriving state diagram
 - deriving state transition table
 - determining next state and output functions
 - implementing combinational logic
- Hardware description languages

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46