Working with combinational logic

- **Simplification**
  - two-level simplification
  - exploiting don’t cares
  - algorithm for simplification
- **Logic realization**
  - two-level logic and canonical forms realized with NANDs and NORs
  - multi-level logic, converting between ANDs and ORs
- **Time behavior**
- **Hardware description languages**

Design example: two-bit comparator

- **Block diagram and truth table**
- we'll need a 4-variable Karnaugh map for each of the 3 output functions
Design example: two-bit comparator (cont’d)

\[
\text{LT} = \quad A' B' D' + A' C + B' C D
\]

\[
\text{EQ} = \quad A' B' C' D' + A' B C' D + A B C D + A B' C D' = (A \text{xnor } C) \cdot (B \text{xnor } D)
\]

\[
\text{GT} = \quad B C' D' + A C' + A B D'
\]

LT and GT are similar (flip A/C and B/D)

Design example: two-bit comparator (cont’d)

Two alternative implementations of EQ with and without XOR.

XNOR is implemented with at least 3 simple gates.
Design example: 2x2-bit multiplier

Block diagram and truth table

### K-map for P8

<table>
<thead>
<tr>
<th>A2</th>
<th>A1</th>
<th>B2</th>
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<th>P8</th>
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P8 = A2A1B2B1

### K-map for P4

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### K-map for P2

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### Design example: 2x2-bit multiplier (cont’d)

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### K-map for P1

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P1 = A1B1
Design example: BCD increment by 1

Design example: BCD increment by 1 (cont’d)
Definition of terms for two-level simplification

- **Implicant**
  - single element of ON-set or DC-set or any group of these elements that can be combined to form a subcube

- **Prime implicant**
  - implicant that can't be combined with another to form a larger subcube

- **Essential prime implicant**
  - prime implicant is essential if it alone covers an element of ON-set
  - will participate in ALL possible covers of the ON-set
  - DC-set used to form prime implicants but not to make implicant essential

- **Objective:**
  - grow implicant into prime implicants
    (minimize literals per term)
  - cover the ON-set with as few prime implicants as possible
    (minimize number of product terms)

Examples to illustrate terms

- **Table 1:**
  - 6 prime implicants: \( A'B'D, BC', AC, A'C'D, AB, B'CD \)
  - minimum cover: \( AC + BC' + A'B'D \)
  - essential

- **Table 2:**
  - 5 prime implicants: \( BD, ABC', ACD, A'BC, A'C'D \)
  - minimum cover: 4 essential implicants
Algorithm for two-level simplification

- Algorithm: minimum sum-of-products expression from a Karnaugh map
  - Step 1: choose an element of the ON-set
  - Step 2: find "maximal" groupings of 1s and Xs adjacent to that element
    - consider top/bottom row, left/right column, and corner adjacencies
    - this forms prime implicants (number of elements always a power of 2)
  - Repeat Steps 1 and 2 to find all prime implicants
  - Step 3: revisit the 1s in the K-map
    - if covered by single prime implicant, it is essential, and participates in final cover
    - 1s covered by essential prime implicant do not need to be revisited
  - Step 4: if there remain 1s not covered by essential prime implicants
    - select the smallest number of prime implicants that cover the remaining 1s

---

Algorithm for two-level simplification (example)

- **3 primes around AB'C'D'**
- **2 primes around A'BC'D'**
- **2 essential primes**
- **minimum cover (3 primes)**
Activity

- List all prime implicants for the following K-map:

```
  A 0 0
  B 1 X
  C X X
  D 1 1
```

- Which are essential prime implicants?
- What is the minimum cover?

Implementations of two-level logic

- Sum-of-products
  - AND gates to form product terms (minterms)
  - OR gate to form sum

- Product-of-sums
  - OR gates to form sum terms (maxterms)
  - AND gates to form product
Two-level logic using NAND gates

- Replace minterm AND gates with NAND gates
- Place compensating inversion at inputs of OR gate

Two-level logic using NAND gates (cont’d)

- OR gate with inverted inputs is a NAND gate
  - de Morgan’s: \( A' + B' = (A \cdot B)' \)
- Two-level NAND-NAND network
  - inverted inputs are not counted
  - in a typical circuit, inversion is done once and signal distributed
Two-level logic using NOR gates

- Replace maxterm OR gates with NOR gates
- Place compensating inversion at inputs of AND gate

AND gate with inverted inputs is a NOR gate
  - de Morgan’s: $A' \cdot B' = (A + B)'$
- Two-level NOR-NOR network
  - inverted inputs are not counted
  - in a typical circuit, inversion is done once and signal distributed
Two-level logic using NAND and NOR gates

- NAND-NAND and NOR-NOR networks
  - de Morgan's law: \((A + B)' = A' \cdot B'\) \quad \((A \cdot B)' = A' + B'\)
  - written differently: \(A + B = (A' \cdot B')'\) \quad \((A \cdot B) = (A' + B')'\)

- In other words —
  - OR is the same as NAND with complemented inputs
  - AND is the same as NOR with complemented inputs
  - NAND is the same as OR with complemented inputs
  - NOR is the same as AND with complemented inputs

Conversion between forms

- Convert from networks of ANDs and ORs to networks of NANDs and NORs
  - introduce appropriate inversions ("bubbles")

Each introduced "bubble" must be matched by a corresponding "bubble"

- conservation of inversions
- do not alter logic function

- Example: AND/OR to NAND/NAND

A \rightarrow B \rightarrow C \rightarrow D \rightarrow Z
A \rightarrow B \rightarrow C \rightarrow D \rightarrow Z

A \rightarrow B \rightarrow C \rightarrow D \rightarrow Z
A \rightarrow B \rightarrow C \rightarrow D \rightarrow Z
Conversion between forms (cont’d)

Example: verify equivalence of two forms

\[
Z = \overline{(A \cdot B)} \cdot \overline{(C \cdot D)}' \\
= \overline{(A' + B')} \cdot \overline{(C' + D')} \\
= \overline{(A' + B')'} + \overline{(C' + D')'} \\
= (A \cdot B) + (C \cdot D)
\]

Conversion between forms (cont’d)

Example: map AND/OR network to NOR/NOR network

Step 1: conserve "bubbles"

Step 2: conserve "bubbles"
Conversion between forms (cont’d)

- Example: verify equivalence of two forms

\[
Z = \{ (A' + B')' + (C' + D')' \}' \\
= \{ (A' + B') \cdot (C' + D') \}' \\
= (A' + B')' + (C' + D')' \\
= (A \cdot B) + (C \cdot D)
\]

Multi-level logic

- \( x = A D F + A E F + B D F + B E F + C D F + C E F + G \)
  - reduced sum-of-products form – already simplified
  - 6 x 3-input AND gates + 1 x 7-input OR gate (that may not even exist!)
  - 25 wires (19 literals plus 6 internal wires)
- \( x = (A + B + C) (D + E) F + G \)
  - factored form – not written as two-level S-o-P
  - 1 x 3-input OR gate, 2 x 2-input OR gates, 1 x 3-input AND gate
  - 10 wires (7 literals plus 3 internal wires)
Conversion of multi-level logic to NAND gates

\[ F = A \left( B + C \overline{D} \right) + \overline{B} C' \]

Conversion of multi-level logic to NORs

\[ F = A \left( B + C D \right) + \overline{B} C' \]
Conversion between forms

- Example

(a) Original circuit

(b) Add double bubbles at inputs

(c) Distribute bubbles, some mismatches

(d) Insert inverters to fix mismatches

AND-OR-invert gates

- AOI function: three stages of logic — AND, OR, Invert
  - multiple gates "packaged" as a single circuit block

Logical concept

Possible implementation

2x2 AOI gate symbol

3x2 AOI gate symbol
Conversion to AOI forms

- General procedure to place in AOI form
  - compute the complement of the function in sum-of-products form
  - by grouping the 0s in the Karnaugh map
- Example: XOR implementation
  - $A \text{ xor } B = A' \cdot B + A \cdot B'$
  - AOI form:
    - $F = (A' \cdot B' + A \cdot B)'$

Examples of using AOI gates

- Example:
  - $F = A \cdot B + A' \cdot C + B \cdot C'$
  - $F = (A' \cdot B' + A' \cdot C + B' \cdot C)'$
  - Implemented by 2-input 3-stack AOI gate

  - $F = (A + B) \cdot (A + C') \cdot (B + C')$
  - $F = [(A' + B') \cdot (A' + C) \cdot (B' + C)]'$
  - Implemented by 2-input 3-stack OAI gate

- Example: 4-bit equality function
  - $Z = (A_0 \cdot B_0 + A_0' \cdot B_0')(A_1 \cdot B_1 + A_1' \cdot B_1')(A_2 \cdot B_2 + A_2' \cdot B_2')(A_3 \cdot B_3 + A_3' \cdot B_3')$
  - each implemented in a single 2x2 AOI gate
Examples of using AOI gates (cont’d)

- Example: AOI implementation of 4-bit equality function

```
A0 & B0 & +
A1 & B1 & +
A2 & B2 & +
A3 & B3 & +

NOR Z
```

- high if A0 ≠ B0
- low if A0 = B0
- conservation of bubbles
- if all inputs are low then Ai = Bi, i=0,...,3 output Z is high

Summary for multi-level logic

- Advantages
  - circuits may be smaller
  - gates have smaller fan-in
  - circuits may be faster
- Disadvantages
  - more difficult to design
  - tools for optimization are not as good as for two-level
  - analysis is more complex
Time behavior of combinational networks

- Waveforms
  - visualization of values carried on signal wires over time
  - useful in explaining sequences of events (changes in value)
- Simulation tools are used to create these waveforms
  - input to the simulator includes gates and their connections
  - input stimulus, that is, input signal waveforms
- Some terms
  - gate delay — time for change at input to cause change at output
    - min delay – typical/nominal delay – max delay
    - careful designers design for the worst case
  - rise time — time for output to transition from low to high voltage
  - fall time — time for output to transition from high to low voltage
  - pulse width — time that an output stays high or stays low between changes

F is not always 0
pulse 3 gate-delays wide
D remains high for three gate delays after A changes from low to high
F is not always 0 pulse 3 gate-delays wide

Momentary changes in outputs

- Can be useful — pulse shaping circuits
- Can be a problem — incorrect circuit operation (glitches/hazards)
- Example: pulse shaping circuit
  - A' • A = 0
  - delays matter
Oscillatory behavior

- Another pulse shaping circuit

Hardware description languages

- Describe hardware at varying levels of abstraction
  - Structural description
    - textual replacement for schematic
    - hierarchical composition of modules from primitives
  - Behavioral/functional description
    - describe what module does, not how
    - synthesis generates circuit for module
  - Simulation semantics
HDLs

- Abel (circa 1983) - developed by Data-I/O
  - targeted to programmable logic devices
  - not good for much more than state machines
- ISP (circa 1977) - research project at CMU
  - simulation, but no synthesis
- Verilog (circa 1985) - developed by Gateway (absorbed by Cadence)
  - similar to Pascal and C
  - delays is only interaction with simulator
  - fairly efficient and easy to write
  - IEEE standard
- VHDL (circa 1987) - DoD sponsored standard
  - similar to Ada (emphasis on re-use and maintainability)
  - simulation semantics visible
  - very general but verbose
  - IEEE standard

Verilog

- Supports structural and behavioral descriptions
  - Structural
    - explicit structure of the circuit
    - e.g., each logic gate instantiated and connected to others
  - Behavioral
    - program describes input/output behavior of circuit
    - many structural implementations could have same behavior
    - e.g., different implementation of one Boolean function
- We'll mostly be using behavioral Verilog in Aldec ActiveHDL
  - rely on schematic when we want structural descriptions
Structural model

```verilog
module xor_gate (out, a, b);
    input     a, b;
    output    out;
    wire      abar, bbar, t1, t2;

    inverter invA (abar, a);
    inverter invB (bbar, b);
    and_gate and1 (t1, a, bbar);
    and_gate and2 (t2, b, abar);
    or_gate  orl (out, t1, t2);
endmodule
```

Simple behavioral model

- Continuous assignment

```verilog
module xor_gate (out, a, b);
    input      a, b;
    output     out;
    reg        out;

    assign #6 out = a ^ b;
endmodule
```
Simple behavioral model

- always block

```verilog
module xor_gate (out, a, b);
  input     a, b;
  output    out;
  reg       out;

  always @(a or b) begin
    #6 out = a ^ b;
  end
endmodule
```

Driving a simulation through a “testbench”

```verilog
module testbench (x, y);
  output    x, y;
  reg [1:0] cnt;

  initial begin
    cnt = 0;
    repeat (4) begin
      #10 cnt = cnt + 1;
      $display("@ time=%d, x=%b, y=%b, cnt=%b",
             $time, x, y, cnt);
    end
    #10 $finish;
  end

  assign x = cnt[1];
  assign y = cnt[0];
endmodule
```
Complete simulation

- Instantiate stimulus component and device to test in a schematic

```
module Compare1 (Equal, Alarger, Blarger, A, B);
  input A, B;
  output Equal, Alarger, Blarger;
  assign #5 Equal = (A & B) | (~A & ~B);
  assign #3 Alarger = (A & ~B);
  assign #3 Blarger = (~A & B);
endmodule
```

Comparator example
More complex behavioral model

```verilog
class Life
module life (n0, n1, n2, n3, n4, n5, n6, n7, self, out);
  input n0, n1, n2, n3, n4, n5, n6, n7, self;
  output out;
  reg out;
  reg [7:0] neighbors;
  reg [3:0] count;
  reg [3:0] i;
  assign neighbors = {n7, n6, n5, n4, n3, n2, n1, n0};
  always @(neighbors or self) begin
    count = 0;
    for (i = 0; i < 8; i = i+1) count = count + neighbors[i];
    out = (count == 3);
    out = out | ((self == 1) & (count == 2));
  end
endmodule
```

Hardware description languages vs. programming languages

- **Program structure**
  - instantiation of multiple components of the same type
  - specify interconnections between modules via schematic
  - hierarchy of modules (only leaves can be HDL in Aldec ActiveHDL)
- **Assignment**
  - continuous assignment (logic always computes)
  - propagation delay (computation takes time)
  - timing of signals is important (when does computation have its effect)
- **Data structures**
  - size explicitly spelled out - no dynamic structures
  - no pointers
- **Parallelism**
  - hardware is naturally parallel (must support multiple threads)
  - assignments can occur in parallel (not just sequentially)
Hardware description languages and combinational logic

- Modules - specification of inputs, outputs, bidirectional, and internal signals
- Continuous assignment - a gate’s output is a function of its inputs at all times (doesn’t need to wait to be "called")
- Propagation delay - concept of time and delay in input affecting gate output
- Composition - connecting modules together with wires
- Hierarchy - modules encapsulate functional blocks

Working with combinational logic summary

- Design problems
  - filling in truth tables
  - incompletely specified functions
  - simplifying two-level logic
- Realizing two-level logic
  - NAND and NOR networks
  - networks of Boolean functions and their time behavior
- Time behavior
- Hardware description languages
- Later
  - combinational logic technologies
  - more design case studies