The Hardware/Software Interface

Integers II

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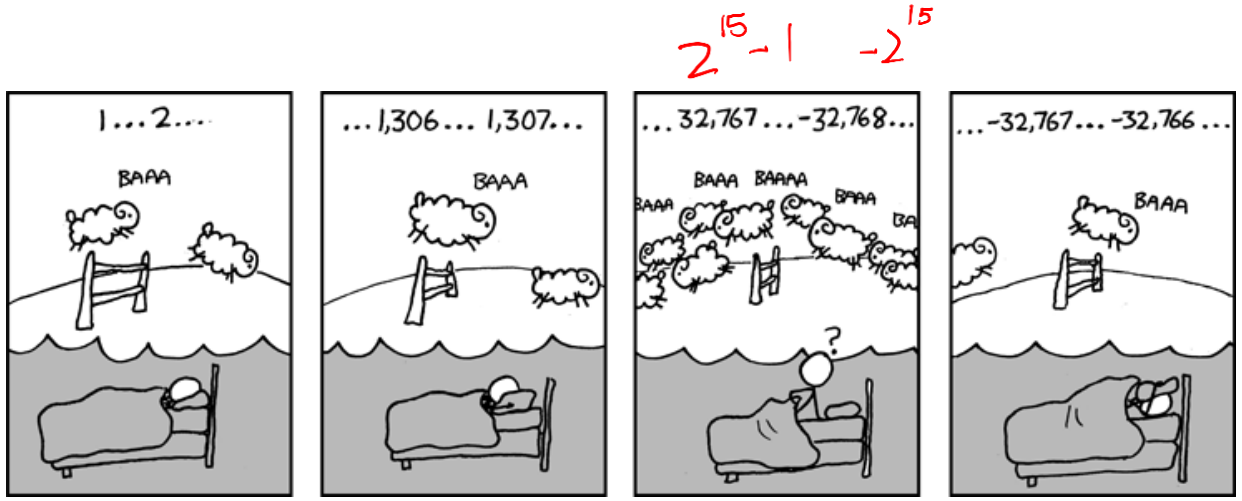


http://xkcd.com/571/

Relevant Course Information

- HW3 due tonight, HW4 due Monday, HW5 due Wednesday
- Lab 1a due Monday (10/9)
  - Use `ptest` and `dlc.py` to check your solution for correctness (on the CSE Linux environment)
  - Submit `pointer.c` and `lab1Asynthesis.txt` to Gradescope
    - Make sure you pass the File and Compilation Check – all the correct files were found and there were no compilation or runtime errors
- Lab 1b released today, due 10/16
  - Bit manipulation on a custom encoding scheme
  - Bonus slides at the end of today's lecture have relevant examples
- Reading 6 is dense, do it early if you can!

Runnable Code Snippets on Ed

- Ed allows you to embed runnable code snippets (*e.g.*, readings, homework, discussion)
  - These are *editable* and *rerunnable*!
  - Hides compiler warnings, but will show compiler errors and runtime errors
  - Code must be inside of an `int main()` function
  - To use `printf()`, you must `#include<stdio.h>`

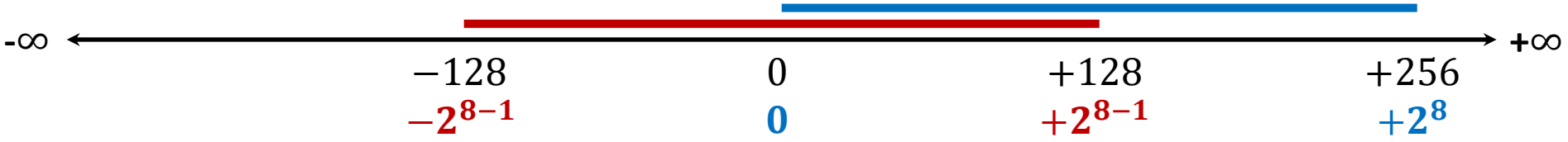
- Suggested use
  - Good for experimental questions about basic behaviors in C
  - *NOT* entirely consistent with the CSE Linux environment, so should not be used for any lab-related work

Lecture Outline (1/4)

- **Integer Limitations**
- Casting in C
- Bit Shifting
- Integer Representation Issues in Real Life

Integer Limits for w bits (Review)

- Unsigned range
  - UMin = `0b00…0` = 0
  - UMax = `0b11…1` = $2^w - 1$
- Signed (Two's Complement) values
  - TMin = `0b10…0` = -2^{w-1}
  - TMax = `0b01…1` = $2^{w-1} - 1$
- Example: w = 8 (*e.g.*, `char`)



Integer Arithmetic

- The same addition procedure works for both unsigned and signed (Two's Complement) integers
  - **Simplifies hardware:** Only one algorithm for addition!
  - **Algorithm:** Normal binary addition, discard the highest carry bit
    - Called modular addition: result is sum *modulo* 2^w

- 4-bit Examples: (HW = hardware, US = unsigned, TC = signed)

HW	US	TC
0100	4	4
+0011	3	3
= 0111	7 ✓	7 ✓

HW	US	TC
1100	12	-4
+0011	3	3
= 1111	15 ✓	-1 ✓

HW	US	TC
1101	13	-3
+0100	4	4
= ~~1~~0001	1 ?	1 ✓

Arithmetic Overflow

Bits	Unsigned	Signed
0000	0 UMin	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7 TMax
1000	8	-8 TMin
1001	9	-7
1010	10	-6
1011	11	-5
1100	12	-4
1101	13	-3
1110	14	-2
1111	15 UMax	-1

- When a calculation produces a result that can't be represented in the current encoding scheme
  - Integer range limited by fixed width UMin - UMax, TMin - TMax
  - Can occur in both the positive and negative directions
- C and Java ignore overflow exceptions
  - You end up with a bad value in your program and no warning/indication… oops!

Overflow: Unsigned

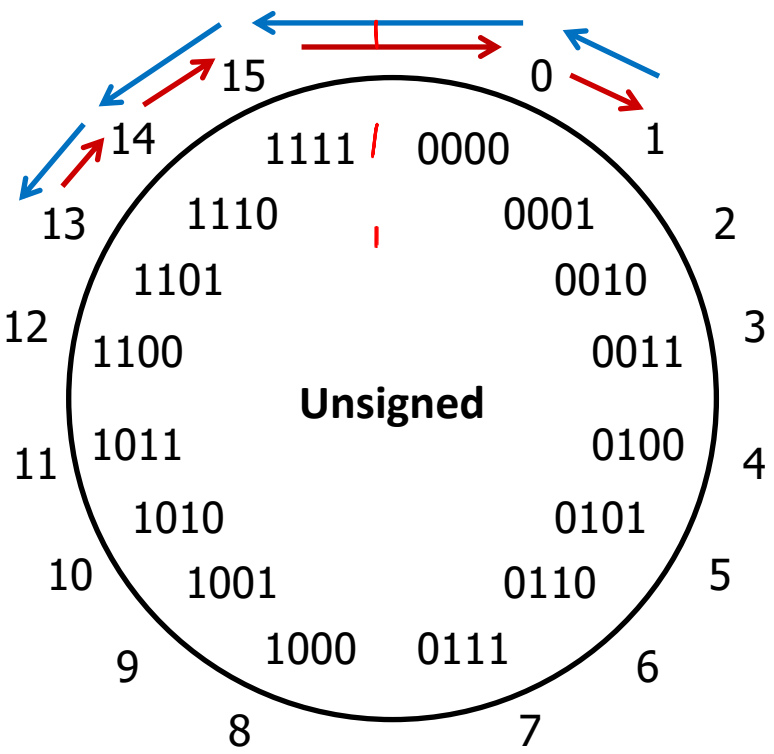
- **Addition:** drop carry bit (-2^w)

```
  13        1101
+  4      + 0100
  17       10001
   1
```

- **Subtraction:** borrow ($+2^w$)

```
   1        10001
-  4      - 0100
  -3        1101
  13
```

UMax ; UMin



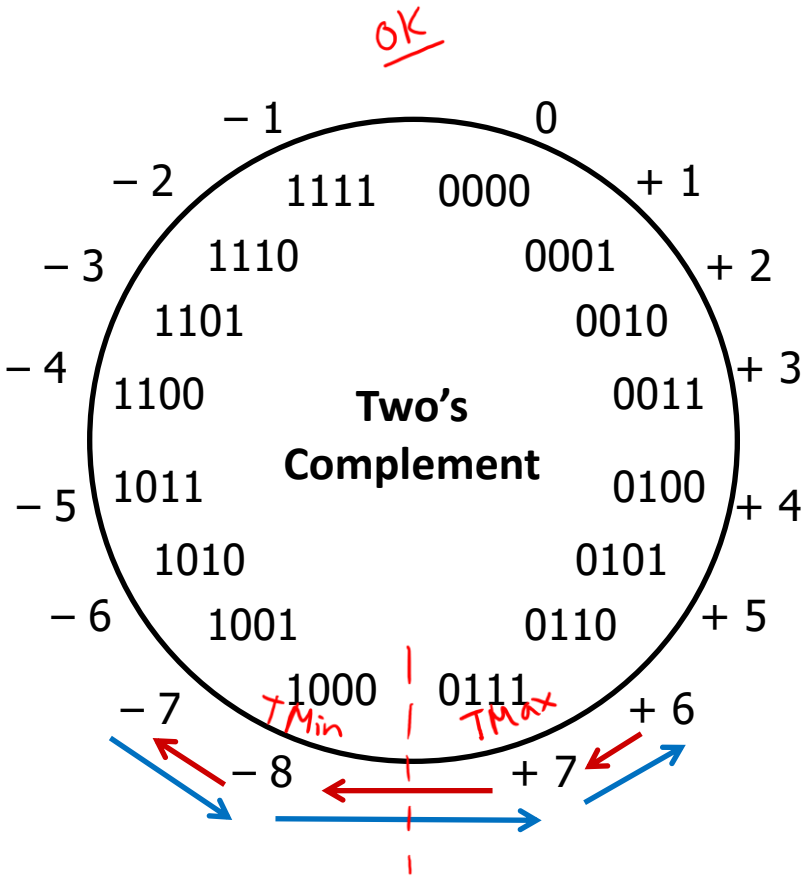
Overflow: Two's Complement

- **Addition:** (+) + (+) = (−) result?

```
   6        0110
 + 3      + 0011
 ---      ------
   9        1001
  -7
```

- **Subtraction:** (−) + (−) = (+)?

```
  -7        1001
 - 3      - 0011
 ---      ------
 -10        0110
   6
```



Arithmetic Overflow Summary

- Error is always a multiple of $\pm 2^w$ because of modular arithmetic
  - *Unsigned overflow* occurs if result falls outside of [UMin, UMax]
    - There is a carryout from the MSB
  - *Signed overflow* occurs if result falls outside of [TMin, TMax]
    - Signs of both inputs to addition are the same, but the sign of the output is different

- *Independent* properties of the arithmetic operation
  - All four combinations of signed OF and unsigned OF are possible!

HW	US	TC
1101	13	−3
+0100	+ 4	+ 4
=~~1~~0001	= 1	= 1

✓ unsigned overflow

✗ signed overflow

Polling Questions (1/2)

signed

most negative

represent $2^6 = 64$ numbers

- What is the value (and encoding) of **`TMin`** for a fictional 6-bit wide integer data type?

$0b\ \underset{-2^5}{1}\ \underset{2^4}{0}\ \underset{2^3}{0}\ \underset{2^2}{0}\ \underset{2^1}{0}\ \underset{2^0}{0}$

$-2^{n-1} = -2^5 = \boxed{-32}$

- For the following 8-bit integer additions, did signed and/or unsigned overflow occur?
  - [TMin, TMax] = [-128, 127]
  - [UMin, UMax] = [0, 255]

Numeral	Signed	Unsigned
0x27	39	39
0xD9	-39	217
0x7F	127	127
0x81	-127	129

a) **`0x27 + 0x81`**

signed: $39 + (-127) = -88$ no signed overflow

unsigned: $39 + 129 = 168$ no unsigned overflow

b) **`0x7F + 0xD9`**

signed: $127 + (-39) = 88$ no signed overflow

unsigned: $127 + 217 = 344$ unsigned overflow

Lecture Outline (2/4)

- Integer Limitations
- **Casting in C**
- Bit Shifting
- Integer Representation Issues in Real Life

Data Types

- How does a data type affect a variable?
  - Size of allocation (*e.g.*, `char` vs. `long`)
  - How to interpret the bits (*e.g.*, `int` vs. `unsigned`)
  - Valid operators/operations and their behavior (*e.g.*, can't use subscript notation `[]` on integral types, normal vs. pointer arithmetic)

- What does it mean or what are the consequences of *changing* your data type?

Literals

- Constants/literals in your code also have "types"
  - Affect the stored/manipulated data and the behavior of operators
  - In C:
    - By default, literals (decimal or hex) are treated as *signed integers*
    - Use "U" (or "u") suffix to explicitly force *unsigned* (*e.g.*, 100U, 4294967259u)
    - Integer literals generally have an assumed size of 4 bytes unless longer is needed
    - We will learn about floating point literals next lesson

- Can be confusing if types don't match
  - Example: `signed char c = 255u; printf("%d", c);` stored as 0xFF; prints as -1
  - Example: `int* ip = 0x40210 + 1;` no pointer arithmetic! type mismatch error

Type Casting: Implicit (Review)

- ***Casting*** converts data of one data type into a different data type
  - Different programming languages may not allow casting or only in certain cases
- C is known for having very flexible casts, with different effects:
  - Changes in bit width (*e.g.*, `short` to `int`)
  - Changes in interpretations (*e.g.*, `int` to `unsigned int`, `long int` to `char*`)
  - Full changes in representations (*e.g.*, `int` to `float`)

now

next lesson

- An ***implicit cast*** is done automatically by the compiler to fix type mismatches
  - Needs to be a well-defined conversion between the two types
  - Examples: `int int_var = short_var;`, `printf("%c", short_var);`

print as character

Type Casting: Explicit (Review)

- ***Casting*** converts data of one data type into a different data type
  - Different programming languages may not allow casting or only in certain cases
- C is known for having very flexible casts, with different effects:
  - Changes in bit width (*e.g.*, `short` to `int`)
  - Changes in interpretations (*e.g.*, `int` to `unsigned int`, `long int` to `char*`)
  - Full changes in representations (*e.g.*, `int` to `float`)
- An ***explicit cast*** can be performed by the programmer by using the syntax: `(data_type)expression`
  - Suppress compiler warnings for implicit casts
  - Forcibly cause changes in interpretation or representation

very powerful, very dangerous!

Casting: Bit Width Change (Mostly Review)

- Longer to shorter
  - *e.g.*, `long` → `int` → `short` → `char`
  - ***Truncation*** (*i.e.*, drop upper bytes)

```
short s = 0x0351;
char c = s;    // 0x51
```

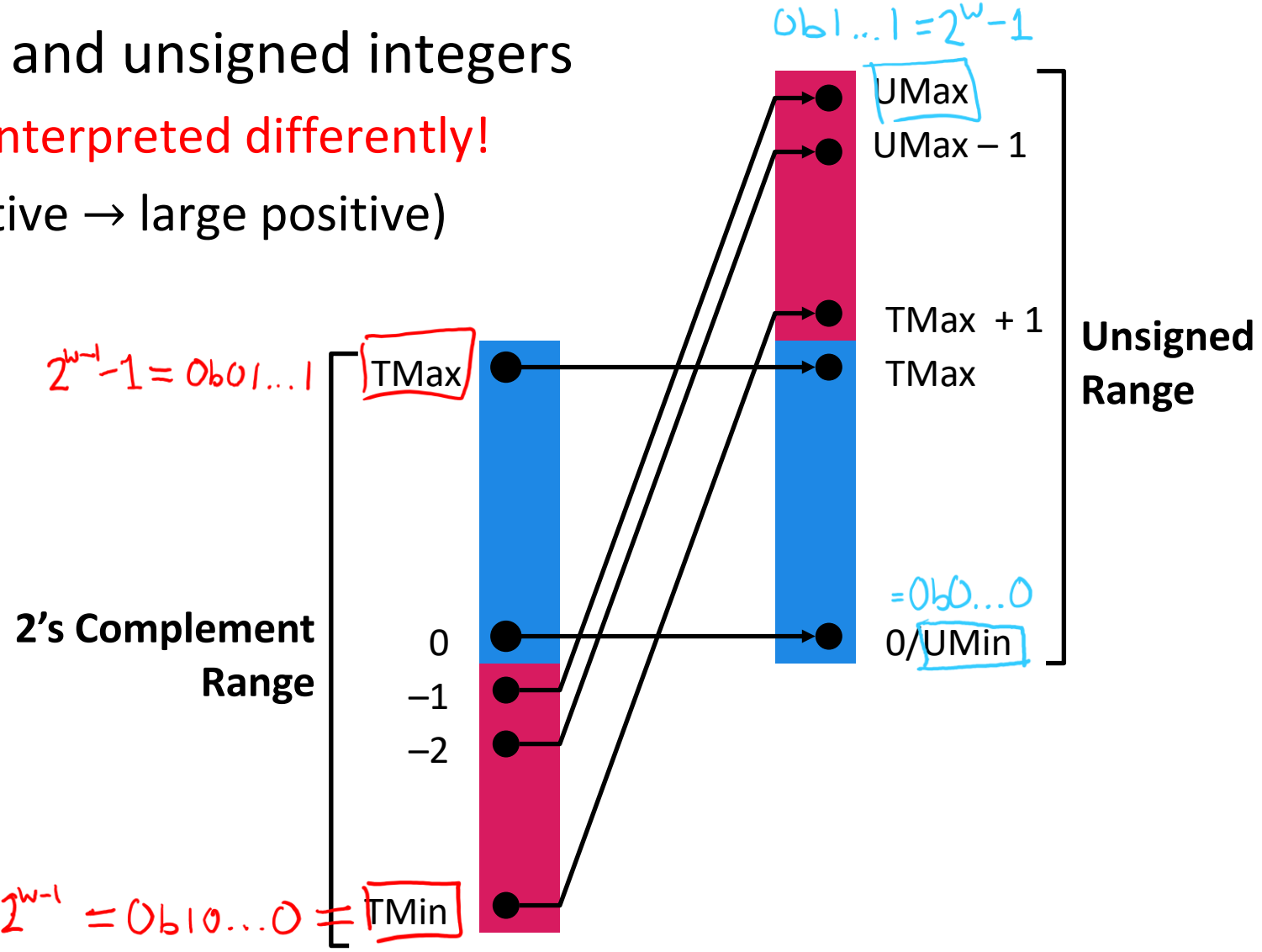
- Shorter to longer
  - *e.g.*, `char` → `short` → `int` → `long`
  - ***Zero extension***: Add all zeros
    - In C, done for unsigned data
  - ***Sign extension***: Add all {old sign bit/MSB}
    - In C, done for signed data to preserve value

```
unsigned char uc = 0xFF;
unsigned short us = uc;  // 0x00FF

signed char sc = 0xFF;
short ss = sc;   // 0xFFFF
sc = 0x10; ss = sc; // 0x0010
```

Casting: Interpretation Change

- Casting between signed and unsigned integers
  - Bits are unchanged, just interpreted differently!
  - Ordering Inversion (negative → large positive)



Data Types: Operator Behavior (Review)

- Expression Evaluation
  - When you mixed unsigned and signed in a single expression, then **signed values are implicitly cast to unsigned**
  - Including comparison operators <, >, ==, <=, >=

("unsigned "dominates")

- Examples: For 8-bit data, what will the following expressions evaluate to?
  - `127 < 128u`

signed: 0b0111 1111 unsigned: 0b1000 0000

unsigned comparison: 0b0111 1111 (+127) < (True) 0b1000 0000 (+128)

  - `127 < (signed char) 128u`

signed: 0b0111 1111; signed: (signed char) 0b1000 0000

signed comparison: 0b0111 1111 (+127) < (False) 0b1000 0000 (-128)

Lecture Outline (3/4)

- Integer Limitations
- Casting in C
- **Bit Shifting**
- Integer Representation Issues in Real Life

Shift Operations (Review, 1/2)

- Throw away (drop) extra bits that "fall off" the end

- Left shift (x<<n) bit vector x by n positions
  - Fill with 0's on right
- Right shift (x>>n) bit-vector x by n positions
  - Logical shift (for unsigned values)
    - Fill with 0's on left
  - Arithmetic shift (for signed values)
    - Replicate most significant bit on left (maintains sign of x)

Shift Operations (Review, 2/2)

digit $d_i \times 2^i$ changes power of 2 by n because it moved positions

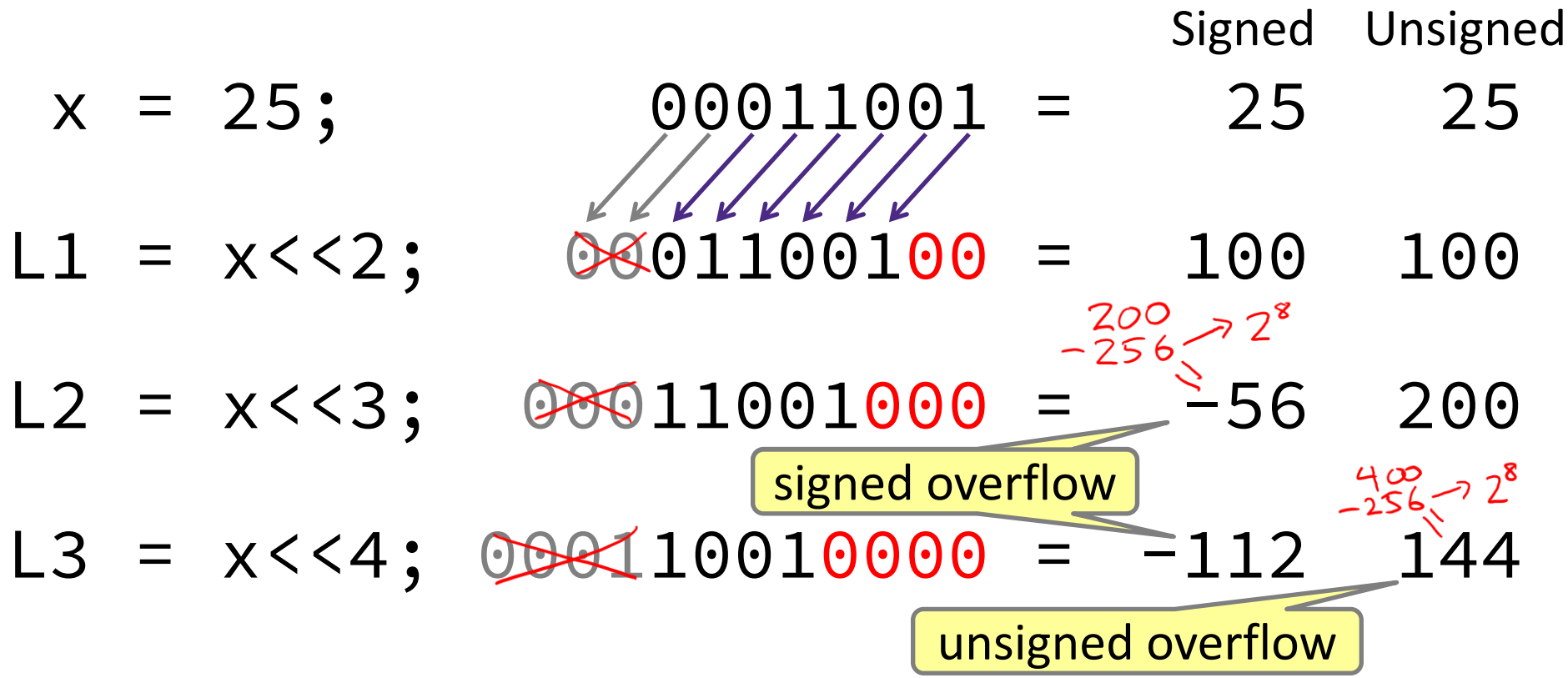
- Arithmetic:
  - Left shift (x<<n) is equivalent to multiply by 2^n
  - Right shift (x>>n) is equivalent to divide by 2^n
  - Shifting is faster than general multiply and divide operations!

(compiler will try to optimize for you)

- Notes:
  - Shifts by n<0 or n≥w (w is bit width of x) are *undefined* behavior not guaranteed
  - **In C:** behavior of >> is determined by the compiler arithmetic/logical
    - In gcc / C lang, depends on data type of x (signed/unsigned)
  - **In Java:** logical shift is >>> and arithmetic shift is >>

Left Shifting 8-bit Example

- No difference in left shift operation for unsigned and signed numbers (just manipulates bits)
  - Difference comes during interpretation: $x*2^n$?



Logical Right Shifting 8-bit Example

- **Reminder:** C operator >> does *logical* shift on unsigned values and *arithmetic* shift on signed values
  - Logical Shift: $x / 2^n$?

```
  xu = 240u;   11110000      = 240
R1u = xu>>3;   000 11110 000 =  30
R2u = xu>>5;   00000111 10000 =   7
```

240/8 = 30

30/4 = 7.5

rounding (down)

Arithmetic Right Shifting 8-bit Negative Example

- **Reminder:** C operator >> does *logical* shift on unsigned values and *arithmetic* shift on signed values
  - Arithmetic Shift: $x / 2^n$?

```
  xs = -16;     11110000          = -16
R1s = xs>>3;    11111110000       =  -2/4 = -0.5
R2s = xs>>5;    11111111 10000    =  -1
```

rounding (down)

Arithmetic Right Shifting 8-bit Positive Example

- **Reminder:** C operator >> does *logical* shift on unsigned values and *arithmetic* shift on signed values
  - Arithmetic Shift: $x / 2^n$?

```
  xs = 112;    01110000      = 112
R3s = xs>>3;   00001110000   =  14
R4s = xs>>5;   000000111000  =   3
```

14/4 = 3.5

rounding (down)

Polling Questions (2/2)

- For `unsigned char uc = 0xA1;`, what are the produced data for the cast **`(unsigned short)uc`**?

2 bytes

unsigned → zero extension 0x00A1

- What is the result of the following expressions?
  - **`(signed char)uc >> 2`**
  - **`(unsigned char)uc >> 3`**

signed: 0b 1010 0001 —arithmetic→ 0b 1110 1000 = 0xE8

unsigned: 0b 1010 0001 —logical→ 0b 0001 0100 = 0x14

Lecture Outline (4/4)

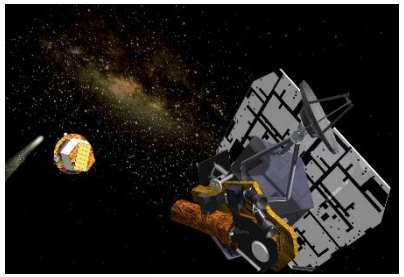
- Integer Limitations
- Casting in C
- Bit Shifting
- **Integer Representation Issues in Real Life**

Integer Representation Issues in Real Life

- **1985**: Therac-25 radiation therapy machine
  - Overdoses of radiation due to arithmetic overflow of incrementing a 1-byte safety flag variable

- **2000**: Y2K problem
  - Limited representation (two-digit decimal year)

- **2013**: Deep Impact spacecraft lost
  - Suspected integer overflow from storing time as tenth-seconds in unsigned int: 8/11/2013, 00:38:49.6



- **2038**: Unix epoch time rollover (seconds since 1/1/1970)
  - Signed 32-bit integer representation rolls over to TMin in 2038

Discussion Question

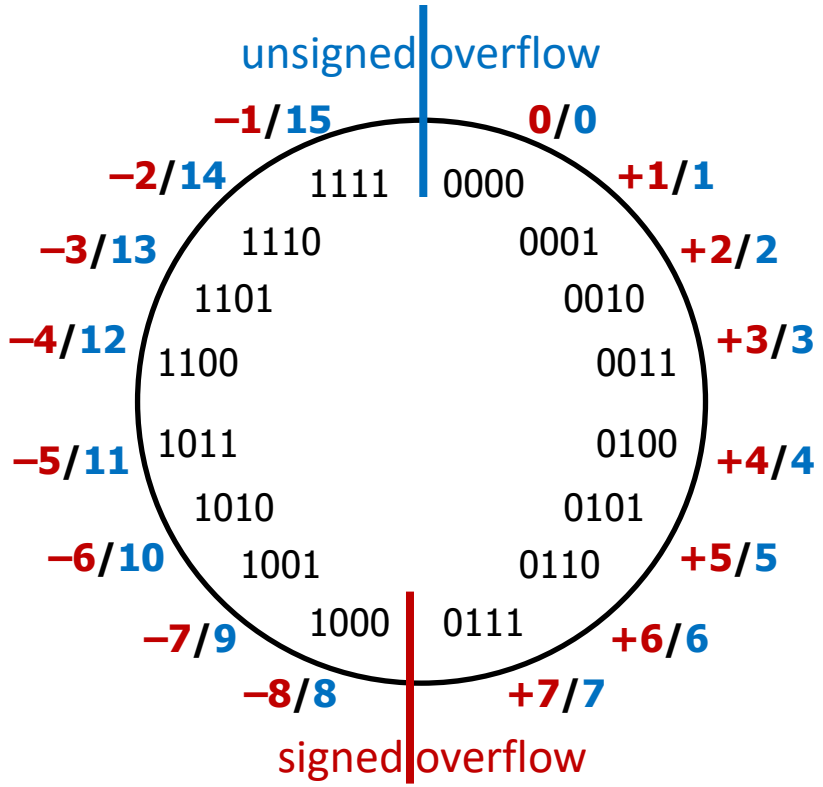
- Discuss the following question(s) in groups of 3-4 students
  - I will call on a few groups afterwards so please be prepared to share out
  - Be respectful of others' opinions and experiences

- Given that **arithmetic overflow** is a well-known property of integers in computing, what do you think are some of the *causes* and *pressures* that perpetuate these issues?
  - Think broadly! Ideas could be technical, economic, societal, etc.

Summary (1/3)

- We can only represent a limited range of numbers in w bits (2^w things)
  - Unsigned: [UMin, UMax]
  - Signed: [TMin, TMax]

- Integer arithmetic is the same in hardware regardless of interpretation
  - When we exceed the limits, *arithmetic overflow* occurs following the rules of modular arithmetic
    - Signed vs. unsigned overflow depends on interpretation of numbers:



Summary (2/3)

- Data types determine size, interpretations, and operator behaviors
- Casting (implicit or explicit) can convert values between different data types
  - Be careful of the possible consequences of casting (truncation, zero/sign extension, change in interpreted value, change in operator behaviors like comparisons and shifting)

```
int i = -1;
long c = i;              // changed size (sign extension)
unsigned int ui = i;  // changed interpretation

// i < 1 (True) is different than ui < 1 (False)
```

Summary (3/3)

- Shifting is a useful bitwise operator
  - Throw away (drop) extra bits that "fall off" the end
  - Left shifting always fills with 0's
  - Right shifting can be arithmetic (fill with copies of sign bit) or logical (fill with 0's)
  - Shifts by `n<0` or `n≥w` (`w` is bit width) are *undefined*

- Common use cases: constant multiplication, bit masking
  - `x = x << 3;  // equivalent to 8*x`
  - `x = (x >> 8) << 8;  // zeros out lowest byte of x`

BONUS SLIDES

Some examples of using shift operators in combination with bitmasks, which you may find helpful for Lab 1b.

- Extract the 2nd most significant byte of an `int`
- Extract the sign bit of a signed `int`
- Conditionals as Boolean expressions

Using Shifts and Masks

- Extract the 2nd most significant *byte* of an `int`:
  - First shift, then mask: `(x>>16) & 0xFF`

x	00000001 00000010 00000011 00000100
x>>16	00000000 00000000 00000001 00000010
0xFF	00000000 00000000 00000000 11111111
(x>>16) & 0xFF	00000000 00000000 00000000 00000010

  - Or first mask, then shift: `(x & 0xFF0000)>>16`

x	00000001 00000010 00000011 00000100
0xFF0000	00000000 11111111 00000000 00000000
x & 0xFF0000	00000000 00000010 00000000 00000000
(x&0xFF0000)>>16	00000000 00000000 00000000 00000010

Using Shifts and Masks

- Extract the sign bit of a signed int:
  - First shift, then mask: (x>>31) & 0x1
    - Assuming arithmetic shift here, but this works in either case
    - Need mask to clear 1s possibly shifted in

x	**0**0000001 00000010 00000011 00000100
x>>31	00000000 00000000 00000000 0000000**0**
0x1	00000000 00000000 00000000 00000001
(x>>31) & 0x1	00000000 00000000 00000000 00000000

x	**1**0000001 00000010 00000011 00000100
x>>31	11111111 11111111 11111111 1111111**1**
0x1	00000000 00000000 00000000 00000001
(x>>31) & 0x1	00000000 00000000 00000000 00000001

Using Shifts and Masks

- Conditionals as Boolean expressions
  - For **`int`** x, what does `(x<<31)>>31` do?

x=!!123	00000000 00000000 00000000 0000000**1**
x<<31	**1**0000000 00000000 00000000 00000000
(x<<31)>>31	11111111 11111111 11111111 11111111
!x	00000000 00000000 00000000 0000000**0**
!x<<31	**0**0000000 00000000 00000000 00000000
(!x<<31)>>31	00000000 00000000 00000000 00000000

  - Can use in place of conditional:
    - In C: `if(x) {a=y;} else {a=z;}` equivalent to `a=x?y:z;`
    - `a=(((!!x<<31)>>31)&y) | (((!x<<31)>>31)&z);`