# The Hardware/Software Interface

## Integers II

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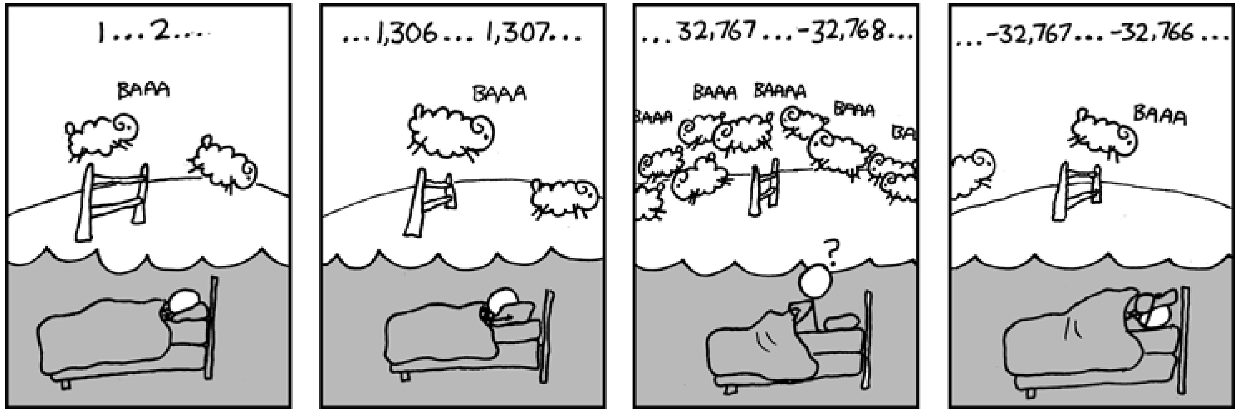



http://xkcd.com/571/

# Relevant Course Information

- HW3 due tonight, HW4 due Monday, HW5 due Wednesday
- Lab 1a due Monday (10/9)
  - Use `ptest` and `dlc.py` to check your solution for correctness (on the CSE Linux environment)
  - Submit `pointer.c` and `lab1Asynthesis.txt` to Gradescope
    - Make sure you pass the File and Compilation Check – all the correct files were found and there were no compilation or runtime errors
- Lab 1b released today, due 10/16
  - Bit manipulation on a custom encoding scheme
  - Bonus slides at the end of today's lecture have relevant examples
- Reading 6 is dense, do it early if you can!

# Runnable Code Snippets on Ed

- Ed allows you to embed runnable code snippets (*e.g.*, readings, homework, discussion)
  - These are *editable* and *rerunnable*!
  - Hides compiler warnings, but will show compiler errors and runtime errors
  - Code must be inside of an `int main()` function
  - To use `printf()`, you must `#include<stdio.h>`

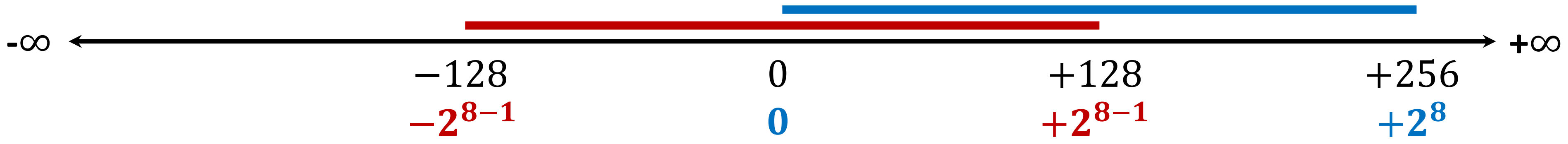
- Suggested use
  - Good for experimental questions about basic behaviors in C
  - *NOT* entirely consistent with the CSE Linux environment, so should not be used for any lab-related work

# Lecture Outline (1/4)

- **Integer Limitations**
- Casting in C
- Bit Shifting
- Integer Representation Issues in Real Life

# Integer Limits for *w* bits (Review)

- Unsigned range
  - UMin = `0b00…0` = $0$
  - UMax = `0b11…1` = $2^w - 1$
- Signed (Two's Complement) values
  - TMin = `0b10…0` = $-2^{w-1}$
  - TMax = `0b01…1` = $2^{w-1} - 1$
- Example: $w$ = 8 (*e.g.*, `char`)



# Integer Arithmetic

- The same addition procedure works for both unsigned and signed (Two's Complement) integers
  - **Simplifies hardware:** Only one algorithm for addition!
  - **Algorithm:** Normal binary addition, discard the highest carry bit
    - Called modular addition: result is sum *modulo* $2^w$

- 4-bit Examples: (HW = hardware, US = unsigned, TC = signed)

| HW | US | TC |
|---|---|---|
| 0100 | | |
| +0011 | | |
| = | | |

| HW | US | TC |
|---|---|---|
| 1100 | | |
| +0011 | | |
| = | | |

| HW | US | TC |
|---|---|---|
| 1101 | | |
| +0100 | | |
| = | | |

# Arithmetic Overflow

| Bits | Unsigned | Signed |
|---|---|---|
| `0000` | 0 | 0 |
| `0001` | 1 | 1 |
| `0010` | 2 | 2 |
| `0011` | 3 | 3 |
| `0100` | 4 | 4 |
| `0101` | 5 | 5 |
| `0110` | 6 | 6 |
| `0111` | 7 | 7 |
| `1000` | 8 | -8 |
| `1001` | 9 | -7 |
| `1010` | 10 | -6 |
| `1011` | 11 | -5 |
| `1100` | 12 | -4 |
| `1101` | 13 | -3 |
| `1110` | 14 | -2 |
| `1111` | 15 | -1 |

- When a calculation produces a result that can’t be represented in the current encoding scheme
  - Integer range limited by fixed width
  - Can occur in both the positive and negative directions
- C and Java ignore overflow exceptions
  - You end up with a bad value in your program and no warning/indication… oops!

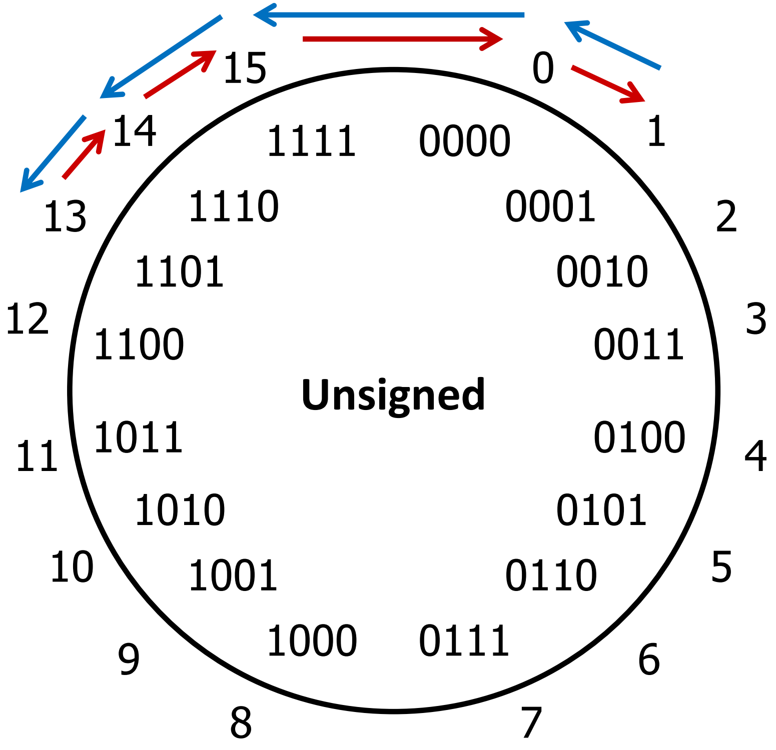
# Overflow: Unsigned

- **Addition:** drop carry bit ($-2^w$)

$$\begin{array}{r} 13 \\ +\ \ 4 \\ \hline \cancel{17} \\ \color{red}{1} \end{array} \qquad \begin{array}{r} \texttt{1101} \\ +\ \ \texttt{0100} \\ \hline \cancel{\texttt{1}}\texttt{0001} \end{array}$$

- **Subtraction:** borrow ($+2^w$)

$$\begin{array}{r} 1 \\ -\ \ 4 \\ \hline \cancel{-3} \\ \color{blue}{13} \end{array} \qquad \begin{array}{r} \texttt{10001} \\ -\ \ \texttt{0100} \\ \hline \texttt{1101} \end{array}$$



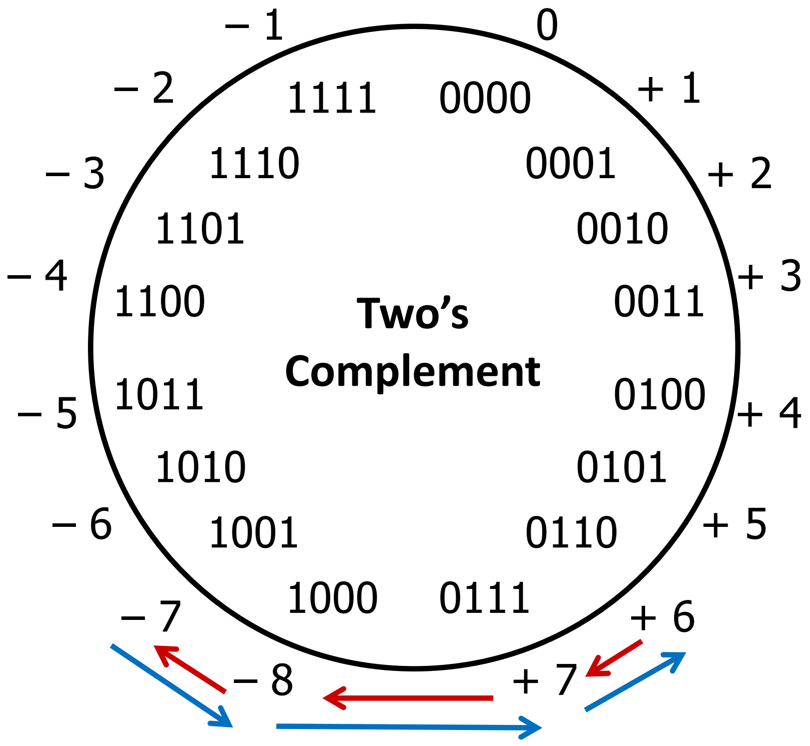
# Overflow: Two's Complement

- **Addition:** (+) + (+) = (−) result?

| | |
|---|---|
| 6 | 0110 |
| + 3 | + 0011 |
| ~~9~~ | 1001 |
| −7 | |

- **Subtraction:** (−) + (−) = (+)?

| | |
|---|---|
| −7 | 1001 |
| − 3 | − 0011 |
| ~~−10~~ | 0110 |
| 6 | |



# Arithmetic Overflow Summary

- Error is always a multiple of $\pm 2^w$ because of modular arithmetic
  - *Unsigned overflow* occurs if result falls outside of [UMin, UMax]
    - There is a carryout from the MSB
  - *Signed overflow* occurs if result falls outside of [TMin, TMax]
    - Signs of both inputs to addition are the same, but the sign of the output is different

- *Independent* properties of the arithmetic operation
  - All four combinations of signed OF and unsigned OF are possible!

| HW | US | TC |
|---|---|---|
| 1101 | 13 | -3 |
| +0100 | + 4 | + 4 |
| =~~1~~0001 | = 1 | = 1 |

✓ unsigned overflow
✗ signed overflow

# Polling Questions (1/2)

- What is the value (and encoding) of **`TMin`** for a fictional 6-bit wide integer data type?

- For the following 8-bit integer additions, did signed and/or unsigned overflow occur?
  - [TMin, TMax] = [-128, 127]
  - [UMin, UMax] = [0, 255]

a) **`0x27 + 0x81`**

b) **`0x7F + 0xD9`**

| Numeral | Signed | Unsigned |
|---|---|---|
| `0x27` | 39 | 39 |
| `0xD9` | -39 | 217 |
| `0x7F` | 127 | 127 |
| `0x81` | -127 | 129 |

# Lecture Outline (2/4)

- Integer Limitations
- **Casting in C**
- Bit Shifting
- Integer Representation Issues in Real Life

# Data Types

- How does a data type affect a variable?
  - Size of allocation (*e.g.*, `char` vs. `long`)
  - How to interpret the bits (*e.g.*, `int` vs. `unsigned`)
  - Valid operators/operations and their behavior (*e.g.*, can't use subscript notation `[]` on integral types, normal vs. pointer arithmetic)

- What does it mean or what are the consequences of *changing* your data type?

# Literals

- Constants/literals in your code also have "types"
  - Affect the stored/manipulated data and the behavior of operators
  - In C:
    - By default, literals (decimal or hex) are treated as *signed integers*
    - Use "U" (or "u") suffix to explicitly force *unsigned* (*e.g.*, 100U, 4294967259u)
    - Integer literals generally have an assumed size of 4 bytes unless longer is needed
    - We will learn about floating point literals next lesson

- Can be confusing if types don't match
  - Example: `signed char c = 255u; printf("%d", c);`
  - Example: `int* ip = 0x40210 + 1;`

# Type Casting: Implicit (Review)

- ***Casting*** converts data of one data type into a different data type
  - Different programming languages may not allow casting or only in certain cases
- C is known for having very flexible casts, with different effects:
  - Changes in bit width (*e.g.*, `short` to `int`)
  - Changes in interpretations (*e.g.*, `int` to `unsigned int`, `long int` to `char*`)
  - Full changes in representations (*e.g.*, `int` to `float`)
- An ***implicit cast*** is done automatically by the compiler to fix type mismatches
  - Needs to be a well-defined conversion between the two types
  - Examples: `int int_var = short_var;`, `printf("%c", short_var);`

# Type Casting: Explicit (Review)

- ***Casting*** converts data of one data type into a different data type
  - Different programming languages may not allow casting or only in certain cases
- C is known for having very flexible casts, with different effects:
  - Changes in bit width (*e.g.*, `short` to `int`)
  - Changes in interpretations (*e.g.*, `int` to `unsigned int`, `long int` to `char*`)
  - Full changes in representations (*e.g.*, `int` to `float`)
- An ***explicit cast*** can be performed by the programmer by using the syntax: `(data_type)expression`
  - Suppress compiler warnings for implicit casts
  - Forcibly cause changes in interpretation or representation

# Casting: Bit Width Change (Mostly Review)

- Longer to shorter
  - *e.g.*, `long` → `int` → `short` → `char`
  - ***Truncation*** (*i.e.*, drop upper bytes)

```
short s = 0x0351;
char c = s;
```

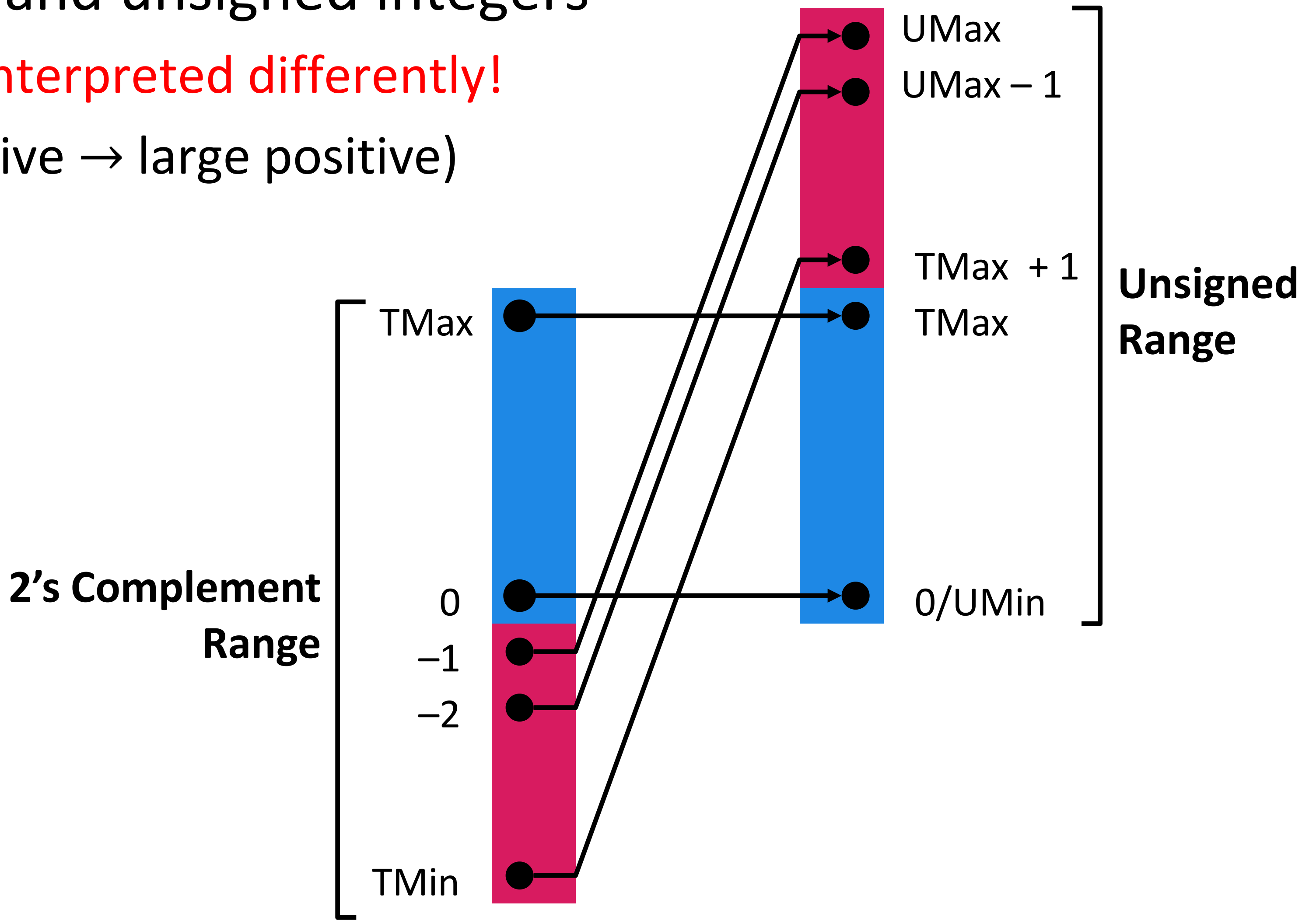
- Shorter to longer
  - *e.g.*, `char` → `short` → `int` → `long`
  - ***Zero extension***: Add all zeros
    - In C, done for unsigned data
  - ***Sign extension***: Add all {old sign bit/MSB}
    - In C, done for signed data to preserve value

```
unsigned char uc = 0xFF;
unsigned short us = uc;

signed char sc = 0xFF;
short ss = sc;
sc = 0x10; ss = sc;
```

# Casting: Interpretation Change

- Casting between signed and unsigned integers
  - Bits are unchanged, just interpreted differently!
  - Ordering Inversion (negative → large positive)



# Data Types: Operator Behavior (Review)

- Expression Evaluation
  - When you mixed unsigned and signed in a single expression, then **signed values are implicitly cast to <u>unsigned</u>**
  - Including comparison operators <, >, ==, <=, >=

- <u>Examples</u>: For 8-bit data, what will the following expressions evaluate to?
  - `127 < 128u`
  - `127 < (signed char) 128u`

# Lecture Outline (3/4)

- Integer Limitations
- Casting in C
- **Bit Shifting**
- Integer Representation Issues in Real Life

# Shift Operations (Review, 1/2)

- Throw away (drop) extra bits that "fall off" the end

- Left shift (`x<<n`) bit vector `x` by `n` positions
  - Fill with `0`'s on right
- Right shift (`x>>n`) bit-vector `x` by `n` positions
  - Logical shift (for unsigned values)
    - Fill with `0`'s on left
  - Arithmetic shift (for signed values)
    - Replicate most significant bit on left (maintains sign of `x`)

# Shift Operations (Review, 2/2)

- Arithmetic:
  - Left shift (`x<<n`) is equivalent to multiply by $2^n$
  - Right shift (`x>>n`) is equivalent to divide by $2^n$
  - Shifting is faster than general multiply and divide operations!

- Notes:
  - Shifts by `n<0` or `n`≥`w` (`w` is bit width of `x`) are *undefined*
  - **In C:** behavior of `>>` is determined by the compiler
    - In gcc / C lang, depends on data type of `x` (signed/unsigned)
  - **In Java:** logical shift is `>>>` and arithmetic shift is `>>`

# Left Shifting 8-bit Example

- No difference in left shift operation for unsigned and signed numbers (just manipulates bits)
  - Difference comes during interpretation: x*2^n?

| | | | Signed | Unsigned |
|---|---|---|---|---|
| `x = 25;` | 00011001 | = | 25 | 25 |
| `L1 = x<<2;` | 0001100100 | = | 100 | 100 |
| `L2 = x<<3;` | 00011001000 | = | -56 | 200 |
| `L3 = x<<4;` | 000110010000 | = | -112 | 144 |

signed overflow

unsigned overflow

# Logical Right Shifting 8-bit Example

- **Reminder:** C operator >> does *logical* shift on unsigned values and *arithmetic* shift on signed values
  - Logical Shift: $x/2^n$?

```
 xu = 240u;   11110000      = 240
R1u = xu>>3; 00011110000    =  30
R2u = xu>>5; 0000011110000  =   7
```

rounding (down)

# Arithmetic Right Shifting 8-bit Negative Example

- **Reminder:** C operator >> does *logical* shift on unsigned values and *arithmetic* shift on signed values
  - Arithmetic Shift: x / $2^n$?

```
 xs = -16;     11110000          = -16
R1s = xs>>3; 11111110000         =  -2
R2s = xs>>5; 1111111110000       =  -1
```

rounding (down)

# Arithmetic Right Shifting 8-bit Positive Example

- **Reminder:** C operator >> does *logical* shift on unsigned values and *arithmetic* shift on signed values
  - Arithmetic Shift: $x/2^n$?

```
 xs = 112;    01110000       = 112
R3s = xs>>3;  00001110000    =  14
R4s = xs>>5;  0000001110000  =   3
```

rounding (down)

# Polling Questions (2/2)

- For `unsigned char uc = 0xA1;`, what are the produced data for the cast **`(unsigned short)uc`**?

- What is the result of the following expressions?
  - **`(signed char)uc >> 2`**
  - **`(unsigned char)uc >> 3`**

# Lecture Outline (4/4)

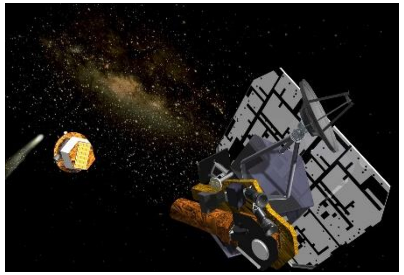
- Integer Limitations
- Casting in C
- Bit Shifting
- **Integer Representation Issues in Real Life**

# Integer Representation Issues in Real Life

- **1985**: Therac-25 radiation therapy machine
  - Overdoses of radiation due to arithmetic overflow of incrementing a 1-byte safety flag variable

- **2000**: Y2K problem
  - Limited representation (two-digit decimal year)


- **2013**: Deep Impact spacecraft lost
  - Suspected integer overflow from storing time as tenth-seconds in unsigned int: 8/11/2013, 00:38:49.6



- **2038**: Unix epoch time rollover (seconds since 1/1/1970)
  - Signed 32-bit integer representation rolls over to TMin in 2038

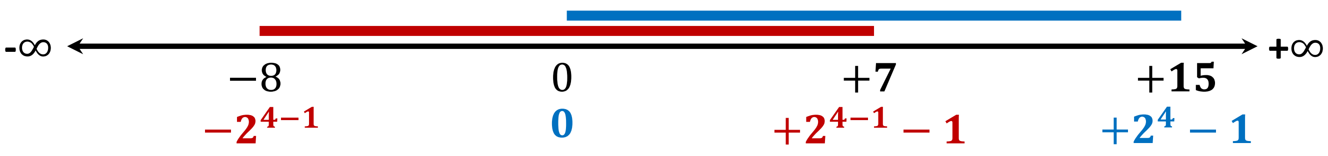
# Discussion Question

- Discuss the following question(s) in groups of 3-4 students
  - I will call on a few groups afterwards so please be prepared to share out
  - Be respectful of others' opinions and experiences

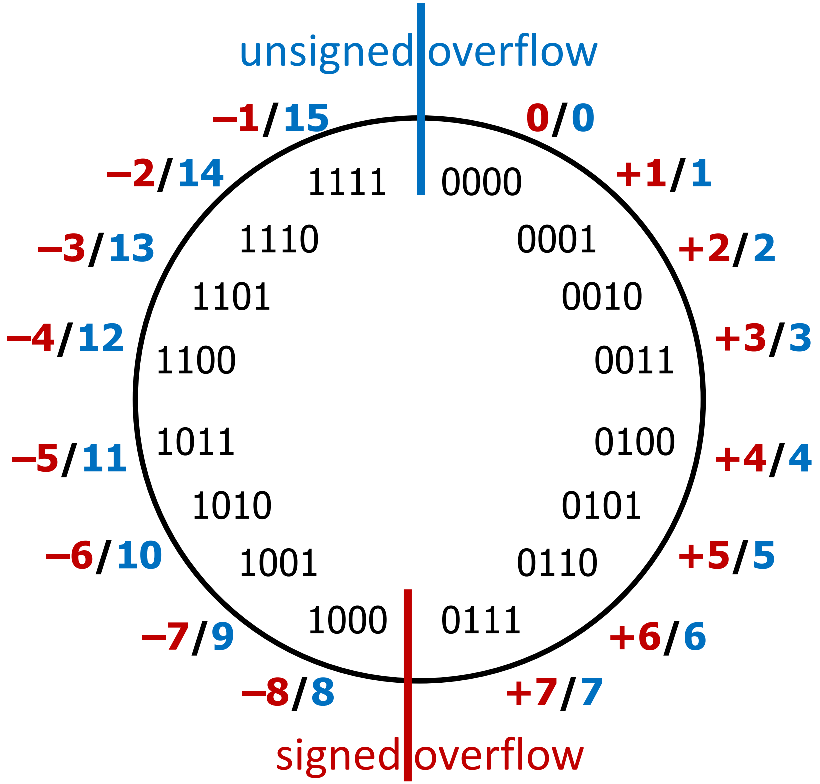
- Given that **arithmetic overflow** is a well-known property of integers in computing, what do you think are some of the *causes* and *pressures* that perpetuate these issues?
  - Think broadly! Ideas could be technical, economic, societal, etc.

# Summary (1/3)

- We can only represent a limited range of numbers in $w$ bits ($2^w$ things)
  - Unsigned: [UMin, UMax]
  - Signed: [TMin, TMax]




- Integer arithmetic is the same in hardware regardless of interpretation
  - When we exceed the limits, *arithmetic overflow* occurs following the rules of modular arithmetic
    - Signed vs. unsigned overflow depends on interpretation of numbers:



# Summary (2/3)

- Data types determine size, interpretations, and operator behaviors
- Casting (implicit or explicit) can convert values between different data types
  - Be careful of the possible consequences of casting (truncation, zero/sign extension, change in interpreted value, change in operator behaviors like comparisons and shifting)

```
int i = -1;
long c = i;             // changed size (sign extension)
unsigned int ui = i;    // changed interpretation

// i < 1 (True) is different than ui < 1 (False)
```

# Summary (3/3)

- Shifting is a useful bitwise operator
  - Throw away (drop) extra bits that "fall off" the end
  - Left shifting always fills with 0's
  - Right shifting can be arithmetic (fill with copies of sign bit) or logical (fill with 0's)
  - Shifts by `n<0` or `n≥w` (`w` is bit width) are *undefined*

- Common use cases: constant multiplication, bit masking
  - `x = x << 3;            // equivalent to 8*x`
  - `x = (x >> 8) << 8;     // zeros out lowest byte of x`

# BONUS SLIDES

Some examples of using shift operators in combination with bitmasks, which you may find helpful for Lab 1b.

- Extract the 2nd most significant byte of an `int`
- Extract the sign bit of a signed `int`
- Conditionals as Boolean expressions

# Using Shifts and Masks

- Extract the 2nd most significant *byte* of an `int`:
  - First shift, then mask: `(x>>16) & 0xFF`

| | |
|---|---|
| **x** | 00000001 00000010 00000011 00000100 |
| **x>>16** | 00000000 00000000 00000001 00000010 |
| **0xFF** | 00000000 00000000 00000000 11111111 |
| **(x>>16) & 0xFF** | 00000000 00000000 00000000 00000010 |

  - Or first mask, then shift: `(x & 0xFF0000)>>16`

| | |
|---|---|
| **x** | 00000001 00000010 00000011 00000100 |
| **0xFF0000** | 00000000 11111111 00000000 00000000 |
| **x & 0xFF0000** | 00000000 00000010 00000000 00000000 |
| **(x&0xFF0000)>>16** | 00000000 00000000 00000000 00000010 |

# Using Shifts and Masks

- Extract the sign bit of a signed int:
  - First shift, then mask: (x>>31) & 0x1
    - Assuming arithmetic shift here, but this works in either case
    - Need mask to clear 1s possibly shifted in

| | |
|---|---|
| `x` | 00000001 00000010 00000011 00000100 |
| `x>>31` | 00000000 00000000 00000000 00000000 |
| `0x1` | 00000000 00000000 00000000 00000001 |
| `(x>>31) & 0x1` | 00000000 00000000 00000000 00000000 |

| | |
|---|---|
| `x` | 10000001 00000010 00000011 00000100 |
| `x>>31` | 11111111 11111111 11111111 11111111 |
| `0x1` | 00000000 00000000 00000000 00000001 |
| `(x>>31) & 0x1` | 00000000 00000000 00000000 00000001 |

# Using Shifts and Masks

- Conditionals as Boolean expressions
  - For **`int`** `x`, what does `(x<<31)>>31` do?

| | |
|---|---|
| `x=!!123` | `00000000 00000000 00000000 00000001` |
| `x<<31` | `10000000 00000000 00000000 00000000` |
| `(x<<31)>>31` | `11111111 11111111 11111111 11111111` |
| `!x` | `00000000 00000000 00000000 00000000` |
| `!x<<31` | `00000000 00000000 00000000 00000000` |
| `(!x<<31)>>31` | `00000000 00000000 00000000 00000000` |

  - Can use in place of conditional:
    - In C: `if(x) {a=y;} else {a=z;}` equivalent to `a=x?y:z;`
    - `a=(((!!x<<31)>>31)&y) | (((!x<<31)>>31)&z);`