The Hardware/Software Interface

Data III & Integers I

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http://xkcd.com/257/

Relevant Course Information

- HW2 due tonight, HW3 due Friday, HW4 due next Wednesday

- Lab 1a released
  - Some later functions require *bit shifting*, covered in Reading/Lecture 5
  - Workflow:
    1) Edit `pointer.c`
    2) Run the Makefile (`make clean` followed by `make`) and check for compiler errors & warnings
    3) Run ptest (`./ptest`) and check for correct behavior
    4) Run rule/syntax checker (`./dlc.py`) and check output
  - Due Monday 10/6, will overlap a bit with Lab 1b
    - We grade just your *last* submission
    - Don’t wait until the last minute to submit – need to check autograder output

Lab Synthesis Questions

- All subsequent labs (after Lab 0) have a “synthesis question” portion
  - Can be found on the lab specs and are intended to be done *after* you finish the lab
  - You will type up your responses in a `.txt` file for submission on Gradescope
  - These will be graded “by hand” (read by TAs)

- Intended to check your understanding of what you should have learned from the lab
  - Also, great practice for short answer questions on the exams
  - Some are reflective questions – we expect a *personal* (*i.e.*, not generic) response

Lecture Outline (1/3)

Data III:

- **Bitwise and Logical Operators**

Numerical Representation:

- Numerical Encoding Design Example
- Encoding Integers

Boolean Algebra and Bitwise Operators (Review)

- Developed by George Boole in 19th Century
  - Algebraic representation of logic (True → 1, False → 0)

- Bitwise operators apply Boolean operations to bit vectors of matching length
  - Apply to any "integral" data type
    - `char`, `short`, `int`, `long`, `unsigned`
  - Examples:

```
   ↓↓↓
   01101001          01101001
 & 01010101        | 01010101
   01000001          01111101

   01101001
 ^ 01010101        ~ 01010101
   00111100          10101010
```

AND

Outputs 1 only when both input bits are 1:

&	0	1
0	**0**	**0**
1	**0**	**1**

OR

Outputs 1 when either input bit is 1:

|	0	1
0	**0**	**1**
1	**1**	**1**

XOR

Outputs 1 when either input is *exclusively* 1:

^	0	1
0	**0**	**1**
1	**1**	**0**

NOT

Outputs the opposite of its input:

~	
0	**1**
1	**0**

Logical Operators (Review)

- Logical operators: && (AND), || (OR), ! (NOT)
  - In C: **0** is False, **anything nonzero** is True; **always** return 0 or 1
- Examples (char data type)

0xCC = 0b 1100 1100
0x33 = 0b 0011 0011

  - $\overset{T}{\texttt{0xCC}}$ `&&` $\overset{T}{\texttt{0x33}}$ `->` `0x01` vs. 0xCC & 0x33 -> 0x00
  - $\overset{F}{\texttt{0x00}}$ `||` $\overset{T}{\texttt{0x33}}$ `->` `0x01` vs. 0x00 | 0x33 -> 0x33
  - `!`$\overset{T}{\texttt{0x33}}$ `->` $\overset{F}{\texttt{0x00}}$
  - `!`$\overset{F}{\texttt{0x00}}$ `->` $\overset{T}{\texttt{0x01}}$

Polling Questions (1/2)

- Compute the result of the following expressions for **char** `c = 0x81;`

  0b 1000 0001

  - `c ^ c` = $\boxed{0x00}$

    1000 0001
    1000 0001
    0000 0000

  - `~c & 0xA9` = $\boxed{0x28}$

    0b0111 1110
    0b 1010 1001
    0010 1000

  - "true" `c` `||` "true" `0x80` = $\boxed{0x01}$ "true"

  - `!`(`!c`) = $\boxed{0x01}$ "true"

    "false"
    0x0

Short-Circuit Evaluation (Review)

- If the result of a binary logical operator (`&&`, `||`) can be determined by its ① first operand, then the ② second operand is never evaluated
  - Also known as *early termination*

- Example: `(p` ① `&&` ② `*p)` for a pointer p to "protect" the dereference
  - Dereferencing `NULL` (`0`) results in a segfault

Bitmasks

- Typically binary bitwise operators (&, |, ^) are used with one operand being the "input" and other operand being a specially-chosen *bitmask* (or *mask*) that performs a desired operation

- Operations for a bit $\overset{0/1}{b}$ (answer with 0, 1, b, or $\bar{b}$):

$b \,\&\, 0 = \underline{\ 0\ }$ "set to zero"
0 →0
1 →0

$b \mid 0 = \underline{\ b\ }$ "keep as is"
0 →0
1 →1

$b \wedge 0 = \underline{\ b\ }$ "keep as is"
0 →0
1 →1

$b \,\&\, 1 = \underline{\ b\ }$ "keep as is"
0 →0
1 →1

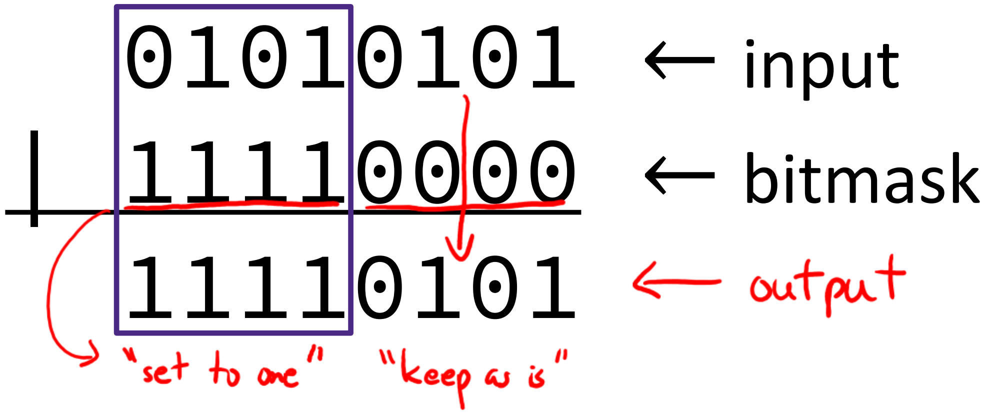
$b \mid 1 = \underline{\ 1\ }$ "set to one"
0 →1
1 →1

$b \wedge 1 = \underline{\ \bar{b}\ }$ "flip"
0 →1
1 →0

Bitmasks Example

- Typically binary bitwise operators (&, |, ^) are used with one operand being the “input” and other operand being a specially-chosen *bitmask* (or *mask*) that performs a desired operation

- <u>Example</u>: $b|0 = b,\ b|1 = 1$

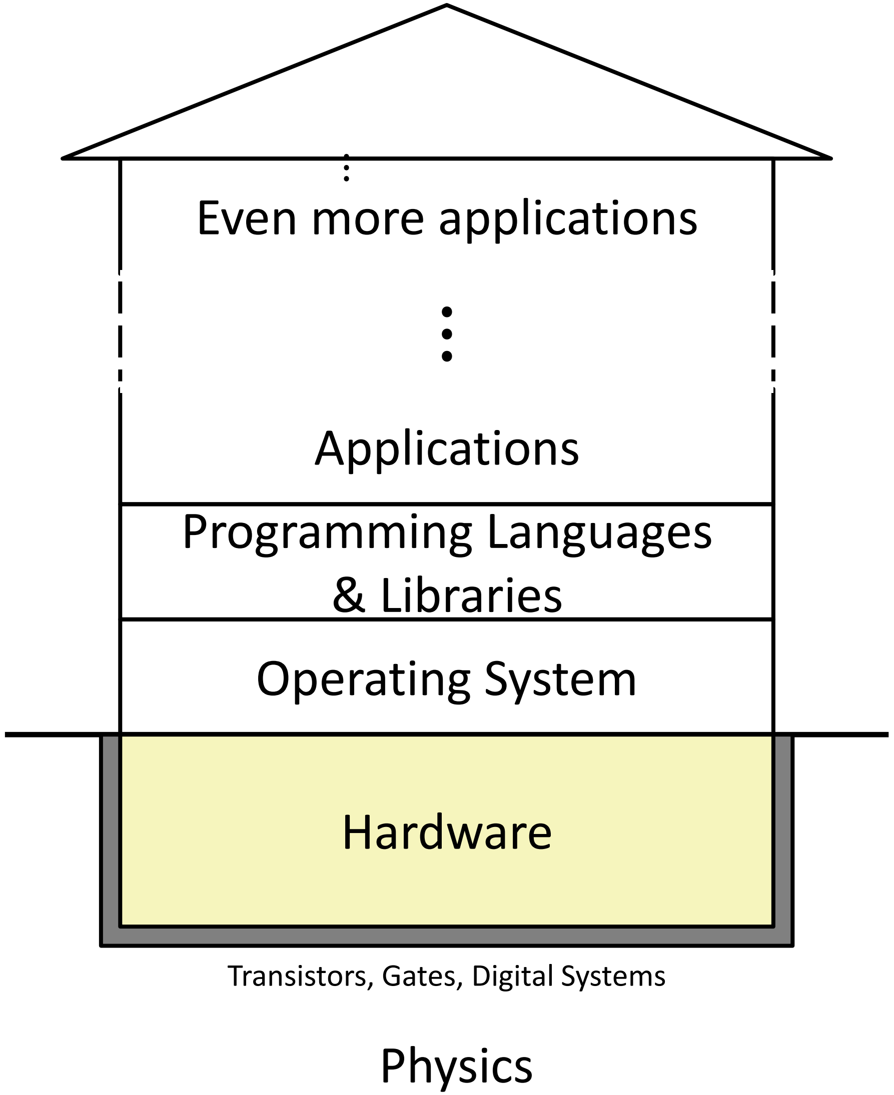


```
  01010101  ← input
| 11110000  ← bitmask
  11110101  ← output
```

"set to one" "keep as is"

House of Computing Check-In

- Topic Group 1: **Data**
  - Memory, Data, **Integers**, Floating Point, Arrays, Structs
- How do we store information for other parts of the house of computing to access?
  - How do we represent data and what limitations exist?
  - What design decisions and priorities went into these encodings?



Lecture Outline (2/3)

Data III:

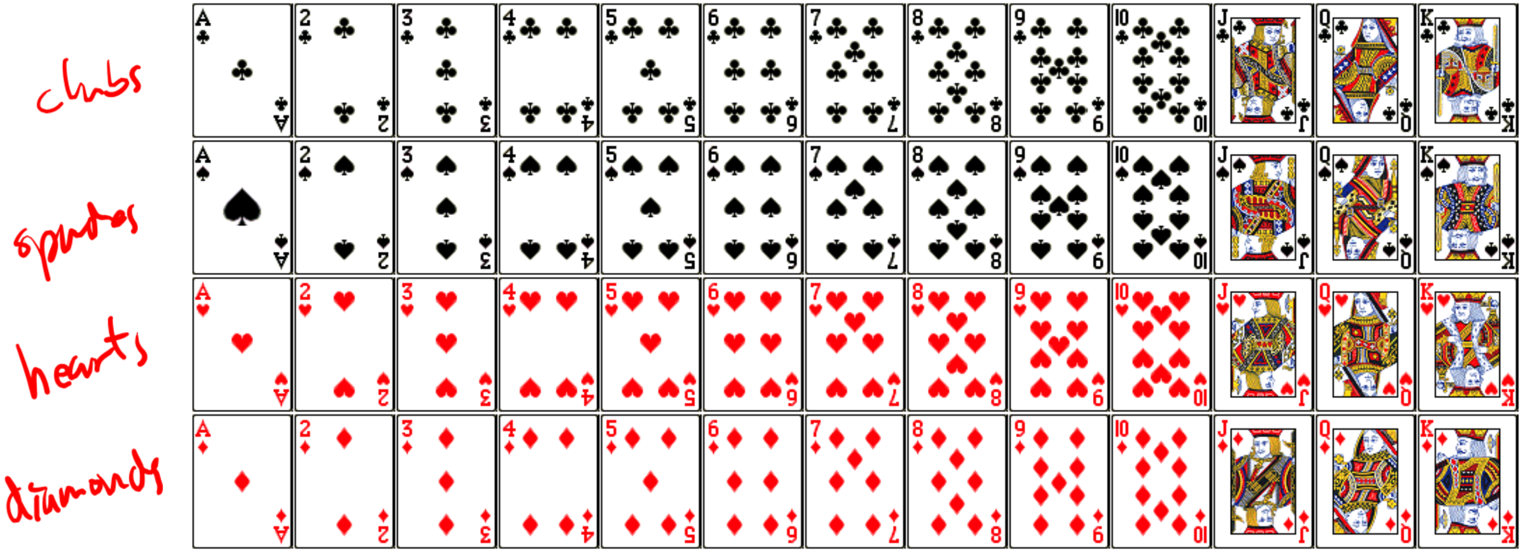
- Bitwise and Logical Operators

Numerical Representation:

- **Numerical Encoding Design Example**
- Encoding Integers

Numerical Encoding Design Example

- Encode a standard 52-card deck of French-suited (4 suits) playing cards



- Operations to implement:
  - ① Which is the higher value card?
  - ② Are they the same suit?

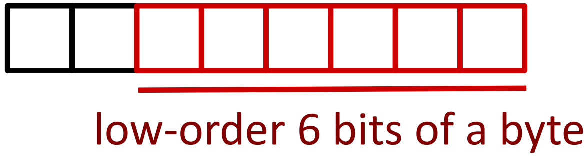
Representations and Fields

1) Binary encoding of all 52 cards – only 6 bits needed
   - $2^6 = 64 \geq 52$
   - Fits in one byte
   - How can we make value and suit comparisons easier?

$2^5 = 32 < 52$

A ♦

0b 000000

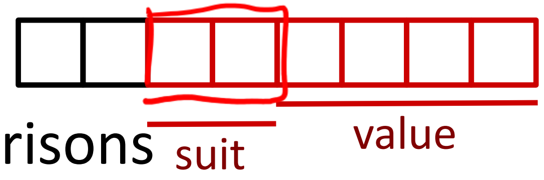

low-order 6 bits of a byte

2) Separate binary encodings of suit (2 bits) and value (4 bits)
   - Also fits in one byte, and easy to do comparisons

4

13

$2^4 = 16 \geq 13$


suit

value

K	Q	J	. . .	3	2	A
1101	1100	1011	. . .	0011	0010	0001

13 →

← 1

♣	00	C
♦	01	D
♥	10	H
♠	11	S

Compare Card Suits

```
char hand[5];         // represents a 5-card hand
char card1, card2;    // two cards to compare
card1 = hand[0];
card2 = hand[1];
...
if ( same_suit(card1, card2) ) { ... }
```

```
#define SUIT_MASK  0x30

int same_suit(char card1, char card2) {
  return (!((card1 & SUIT_MASK) ^ (card2 & SUIT_MASK)));
  //return (card1 & SUIT_MASK) == (card2 & SUIT_MASK);
}
```

text substitution

SUIT_MASK = 0x30 =

0	0	1	1	0	0	0	0

suit

value

b & 0 → 0

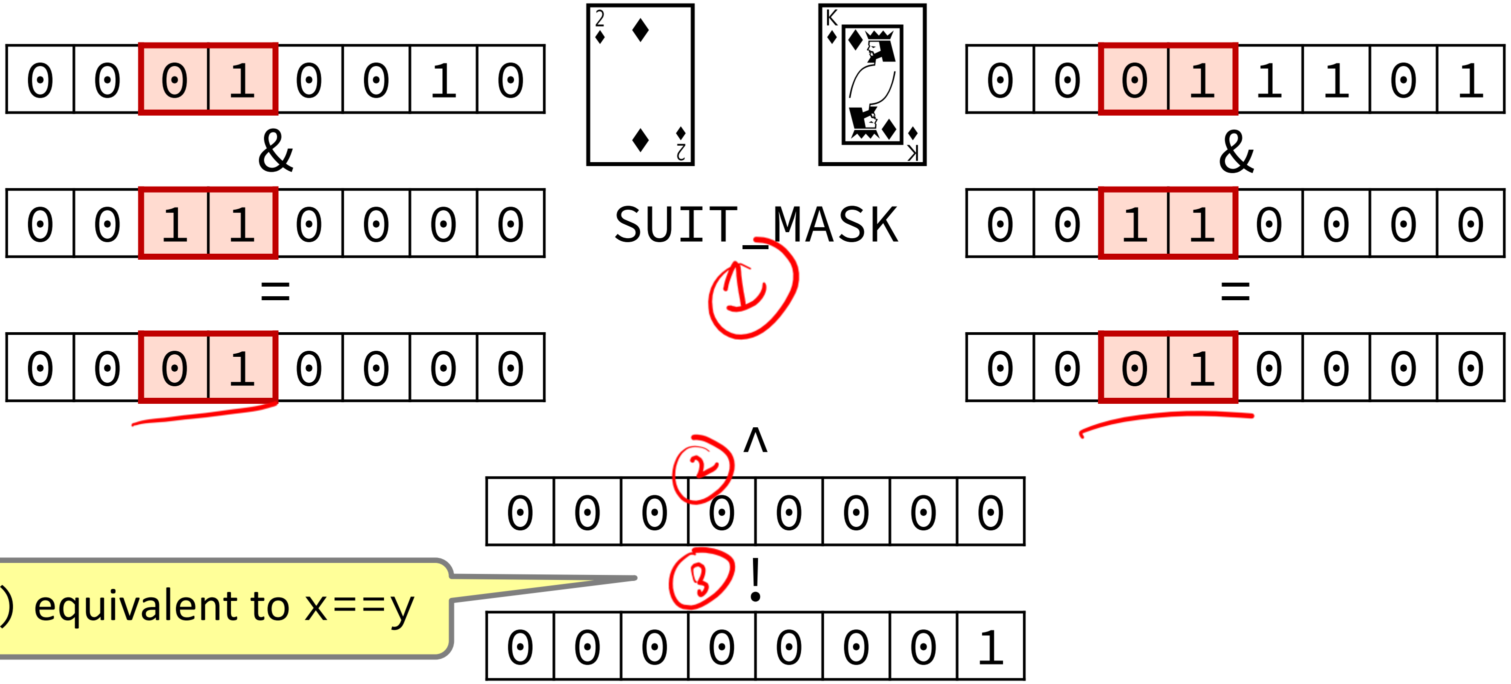
b & 1 → b

Compare Card Suits Example

```
#define SUIT_MASK  0x30

int same_suit(char card1, char card2) {
  return (!((card1 & SUIT_MASK) ^ (card2 & SUIT_MASK));
  //return (card1 & SUIT_MASK) == (card2 & SUIT_MASK);
}
```

③ ① ② ①



0 0 0 1 0 0 1 0   0 0 0 1 1 1 0 1

&   &

0 0 1 1 0 0 0 0   SUIT_MASK ①   0 0 1 1 0 0 0 0

=   =

0 0 0 1 0 0 0 0   0 0 0 1 0 0 0 0

② ^

0 0 0 0 0 0 0 0

③ !

0 0 0 0 0 0 0 1

!(x^y) equivalent to x==y

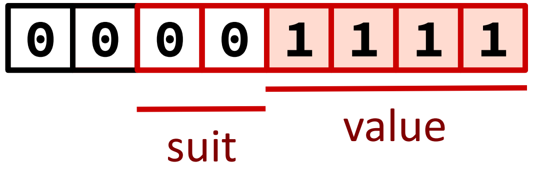
Compare Card Values

```
char hand[5];          // represents a 5-card hand
char card1, card2;     // two cards to compare
card1 = hand[0];
card2 = hand[1];
...
if ( greater_value(card1, card2) ) { ... }
```

```
#define VALUE_MASK  0x0F

int greater_value(char card1, char card2) {
  return ((unsigned int)(card1 & VALUE_MASK) >
          (unsigned int)(card2 & VALUE_MASK));
}
```

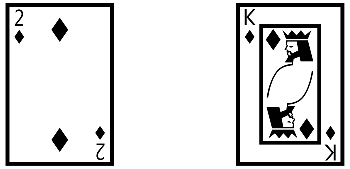
VALUE_MASK = 0x0F =



Compare Card Values Example

```
#define VALUE_MASK  0x0F

int greater_value(char card1, char card2) {
  return ((unsigned int)(card1 & VALUE_MASK) >
          (unsigned int)(card2 & VALUE_MASK));
}
```



0 0 1 0 0 0 1 0
&
0 0 0 0 1 1 1 1
=
0 0 0 0 0 0 1 0

VALUE_MASK

0 0 1 0 1 1 0 1
&
0 0 0 0 1 1 1 1
=
0 0 0 0 1 1 0 1

$2_{10} > 13_{10}$

0 (false)

Takeaways

- Custom encodings may need to be created when dealing with custom data types or if you’re trying to be very space efficient

- There may be many valid encodings but your choices matter
  - *e.g.*, space efficiency, ease of implementation
  - Can separate encoding into multiple fields

- Bitwise and logical operators can be useful for manipulating data

Lecture Outline (3/3)

Data III:

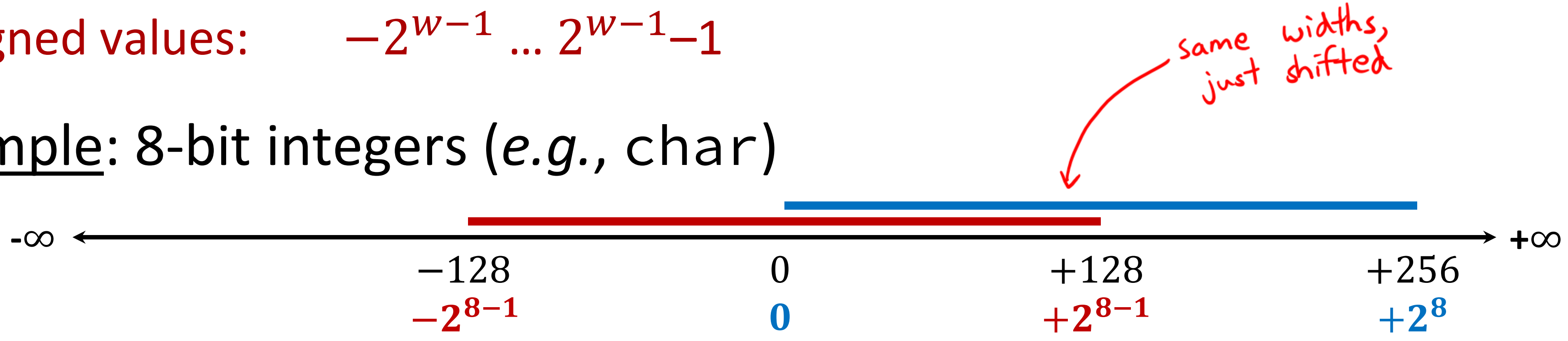
- Bitwise and Logical Operators

Numerical Representation:

- Numerical Encoding Design Example
- **Encoding Integers**

Encoding Integers

- The hardware (and C) supports two flavors of integers
  - *unsigned* – only the non-negatives
  - *signed* – both negatives and non-negatives
- Cannot represent all integers with w bits
  - Only 2^w distinct bit patterns
  - Unsigned values: $0 \ldots 2^w - 1$
  - Signed values: $-2^{w-1} \ldots 2^{w-1} - 1$
- Example: 8-bit integers (*e.g.*, `char`)


same widths, just shifted

Unsigned Integers (Review)

- Unsigned values follow the standard base 2 system
  - $\mathrm{b_7b_6b_5b_4b_3b_2b_1b_0} = \mathrm{b_7}2^7 + \mathrm{b_6}2^6 + \cdots + \mathrm{b_1}2^1 + \mathrm{b_0}2^0$
- Add and subtract using the normal "carry" and "borrow" rules, just in binary:

$$\begin{array}{r} {\scriptstyle 1\,1} \\ 63 \\ +\underline{\ \ 8} \\ 7\underline{1} \end{array} \iff \begin{array}{r} {\scriptstyle 1\,1\,1\,1} \\ 00111111_2 \\ +\underline{00001000}_2 \\ 01000111 \end{array} \qquad \begin{array}{r} {\scriptstyle 10} \\ {\scriptstyle 4\ \curvearrowright} \\ \cancel{5}5 \\ -\underline{\ \ 8\ } \\ 47 \end{array} \iff \begin{array}{r} {\scriptstyle 2} \\ {\scriptstyle 0\curvearrowright} \\ 001\cancel{1}0111_2 \\ -\underline{00001000}_2 \\ 00101111 \end{array}$$

- In C, add "`unsigned`" keyword in front of any integral type
  - *e.g.*, `unsigned` $\overset{1B}{\texttt{char}}$, `unsigned` $\overset{2B}{\texttt{short}}$, `unsigned` $\overset{4B}{\texttt{int}}$, `unsigned` $\overset{8B}{\texttt{long}}$

Sign and Magnitude

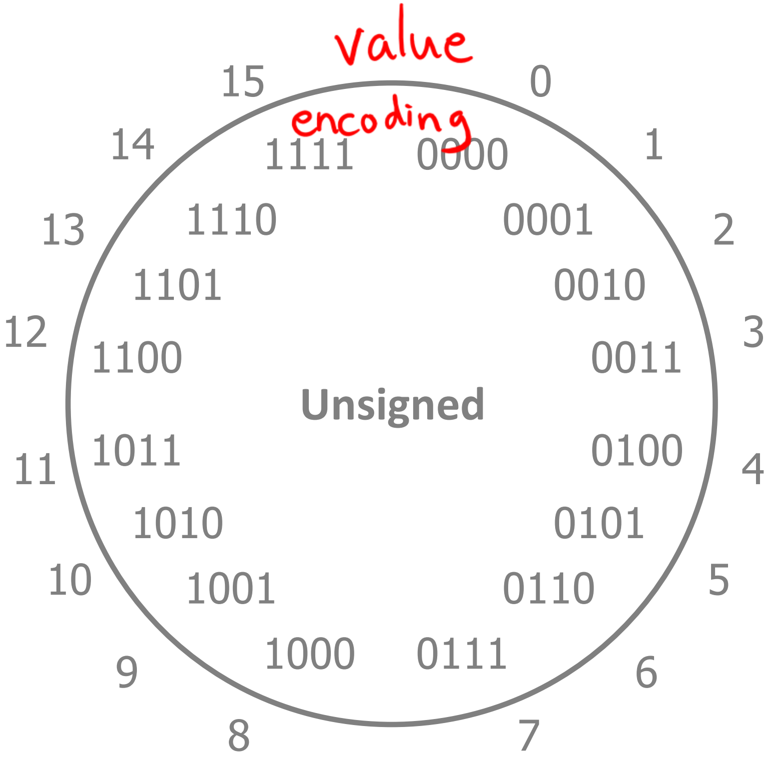
☆ Not used in practice for integers!

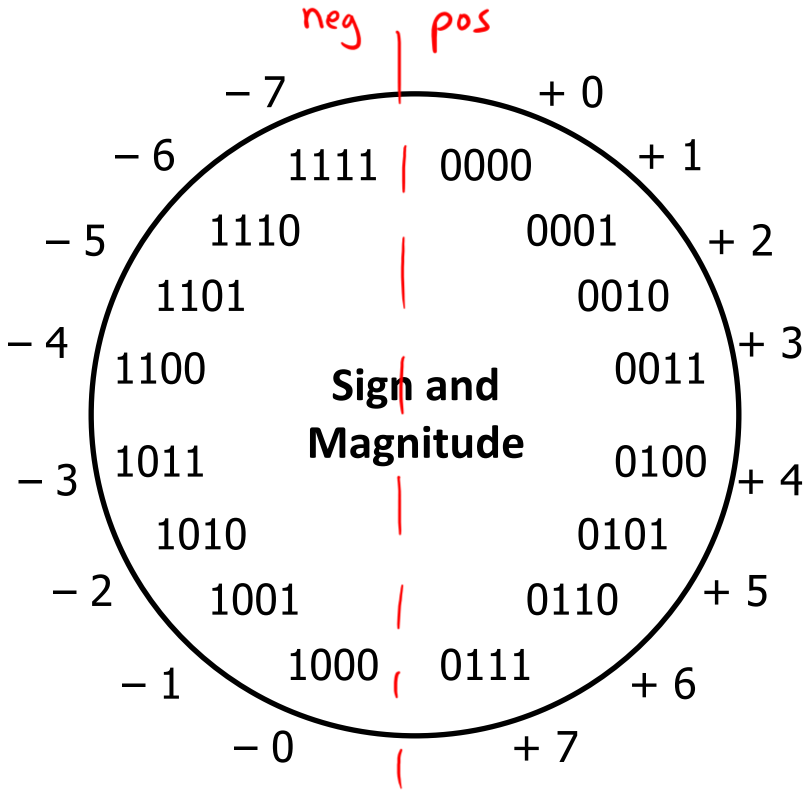
- Designate the high-order bit (MSB) as the "sign bit"
  - `sign=0`: positive number; `sign=1`: negative number
- Benefits:
  - Using MSB as sign bit matches positive numbers with unsigned ✓
    unsigned: $0b\,0010 = 2^1 = 2$; sign+mag: $0b\,0010 = +2^1 = 2$ ☑
  - All zeros encoding is still = 0
- Examples (8 bits):
  - 0x00 = $\overset{\oplus}{0}0000000_2$ is non-negative, because the sign bit is 0
  - 0x7F = $\overset{\oplus}{0}\underline{1111111}_2$ is non-negative ($+127_{10}$) 2^7-1
  - 0x85 = $\overset{\ominus}{1}0000\underline{101}_2$ is negative (-5_{10})
  - 0x80 = 10000000_2 is negative… zero???

Sign and Magnitude Visualization

Not used in practice for integers!

- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks?

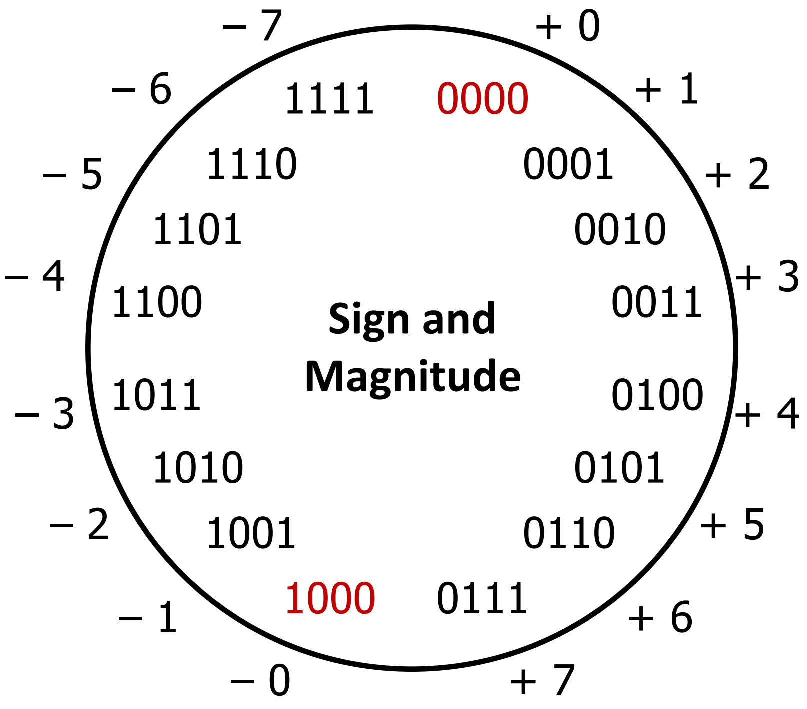




Sign and Magnitude Drawbacks (1/2)

Not used in practice for integers!

- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks:
  - Two representations of 0 (bad for checking equality)



Sign and Magnitude Drawbacks (2/2)

Not used in practice for integers!

- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks:
  - Two representations of 0 (bad for checking equality)
  - Arithmetic is cumbersome
    - Example: `4-3 != 4+(-3)`

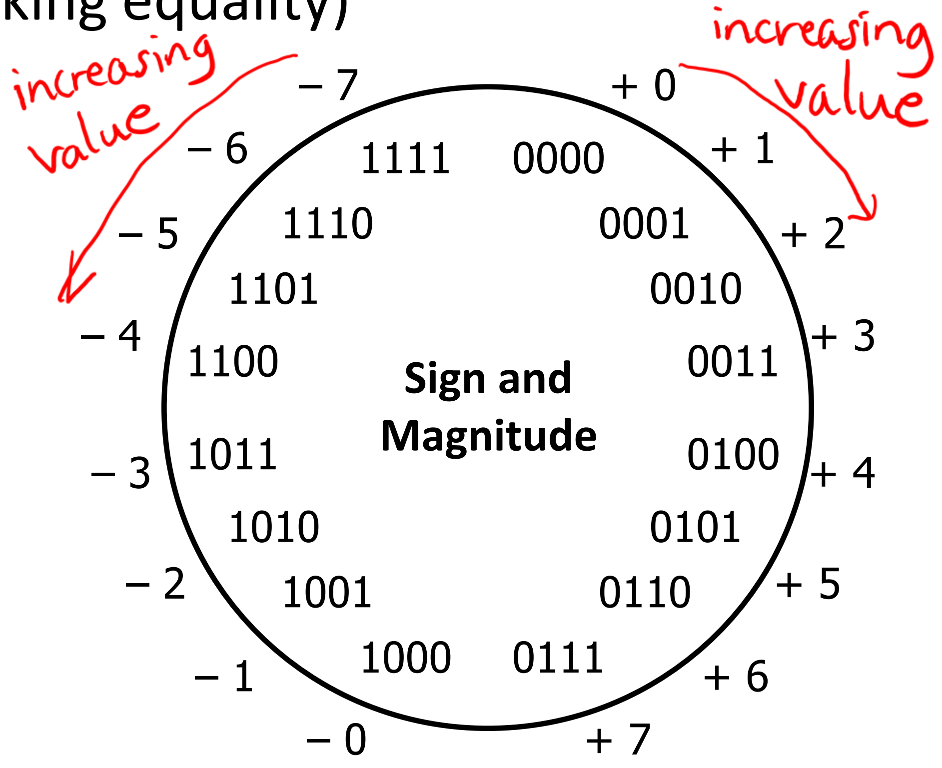
4	0100
– 3	– 0011
1	0001

✓

4	0100
+ -3	+ 1011
-7	1111

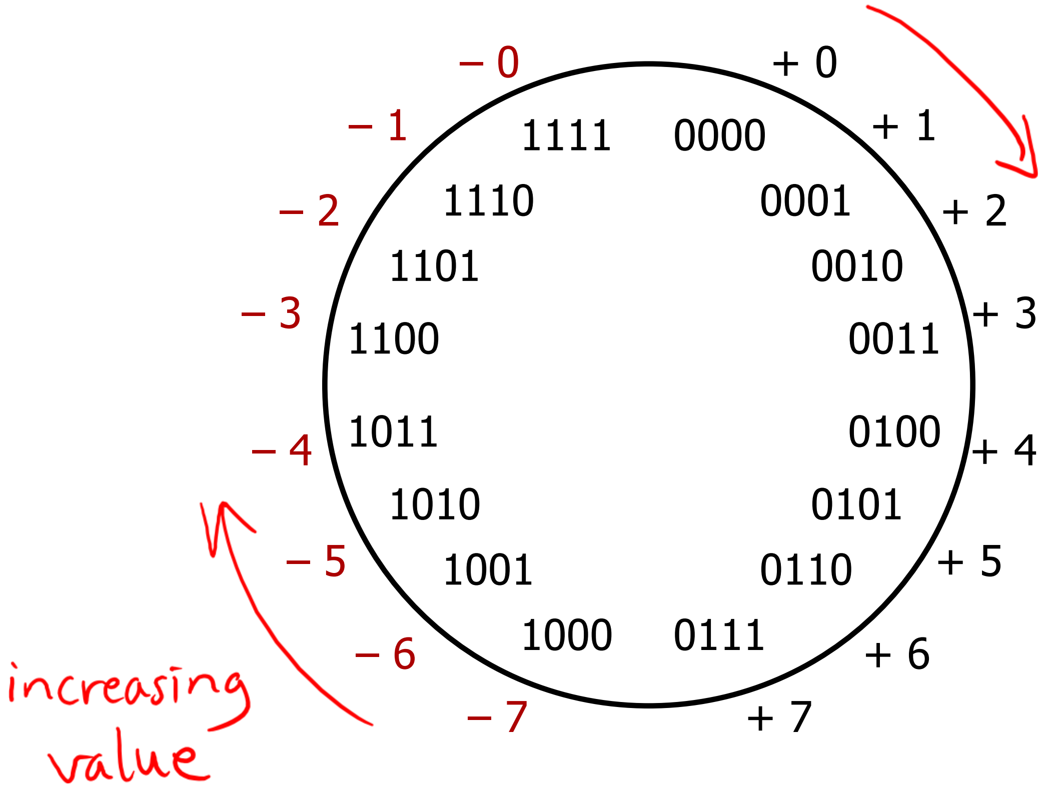
✗

    - Negatives "increment" in wrong direction!



Two's Complement Development (1/2)

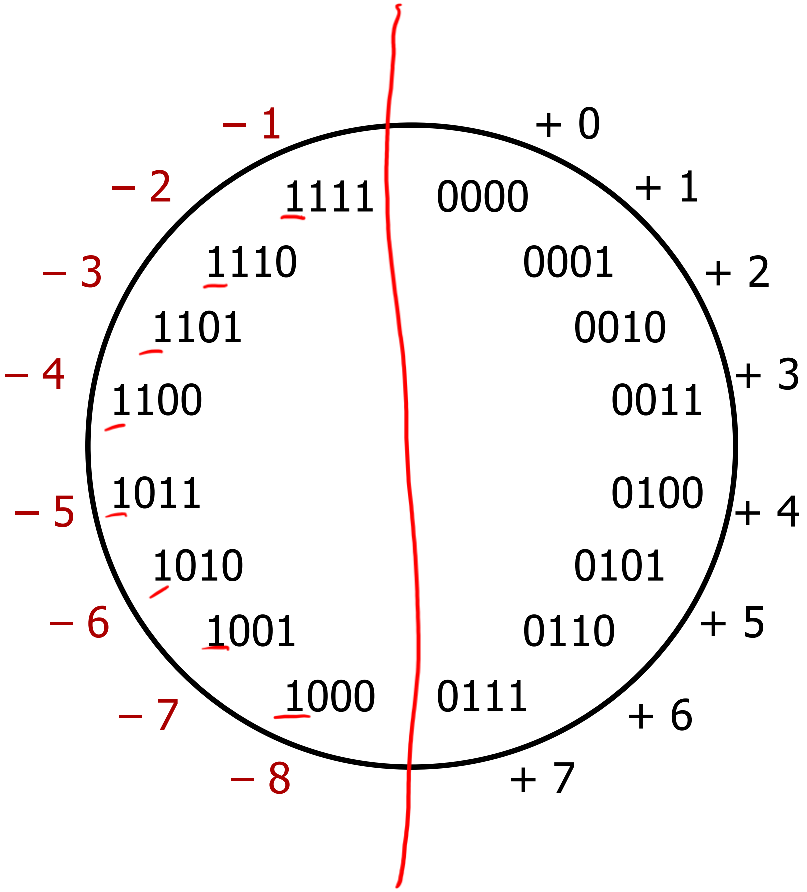
- Let's fix these problems:
  1) "Flip" negative encodings so incrementing works
  2) "Shift" negative numbers to eliminate –0



Two’s Complement Development (2/2)

- Let’s fix these problems:
  1) “Flip” negative encodings so incrementing works
  2) “Shift” negative numbers to eliminate –0

- MSB *still* indicates sign!
  - This is why we represent one more negative than positive number (-2^{N-1} to 2^{N-1} $-$1)



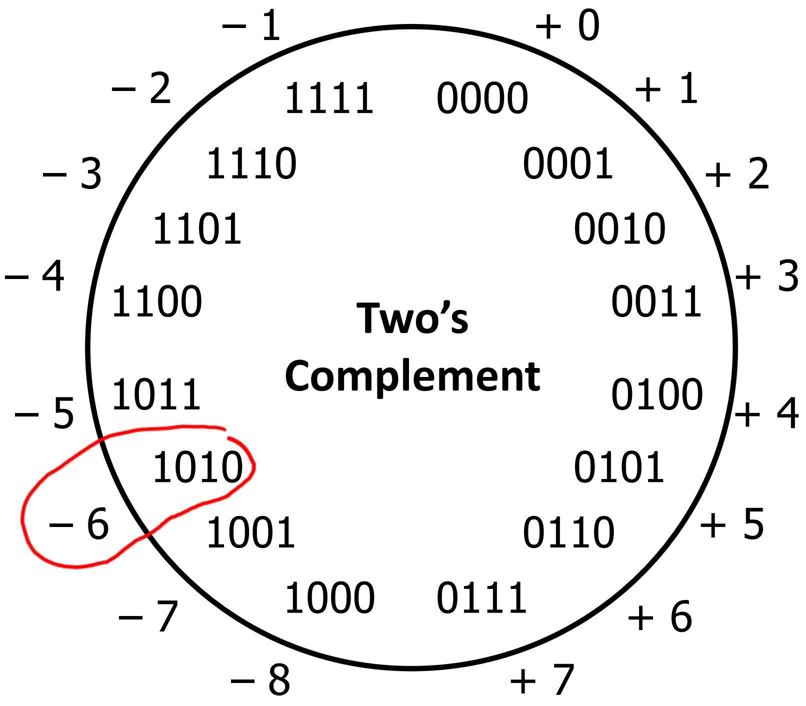
Two’s Complement Negatives (Review)

- Accomplished with one neat mathematical trick!

b_{w-1} has weight -2^{w-1}, other bits have usual weights $+2^i$

- <u>4-bit Example</u>:
  8 2
  - 1010_2 unsigned:
    $1*2^3+0*2^2+1*2^1+0*2^0$ = **10**

  -8
  - 1010_2 two’s complement:
    $-1*2^3+0*2^2+1*2^1+0*2^0$ = **–6**

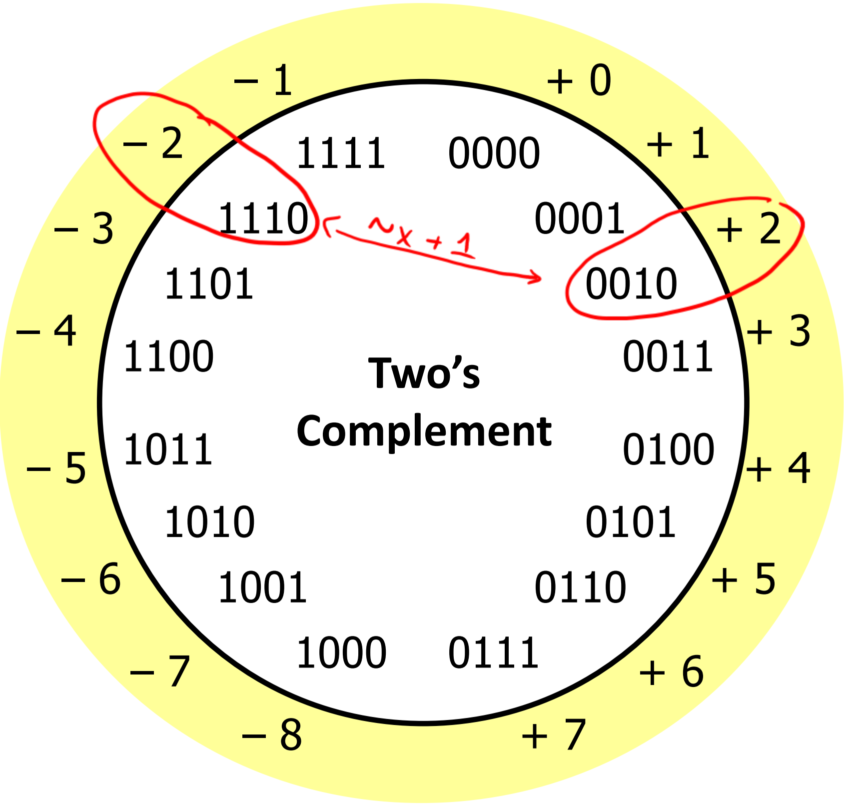


Two’s Complement is Great (Review)

- Roughly same number of (+) and (–) numbers
- Positive number encodings match unsigned
- Single zero (with all `0`’s encoding)

2 = 0b0010
−2 = 0b1101 +1 = 0b1110 ☑
2 = 0b0001+1 = 0b0010 ☑

- Simple negation procedure:
  - Get negative representation of any integer by taking bitwise complement and then adding one!
    `( ~x + 1 == -x )`



Polling Questions (2/2)

MSB

- Take the 4-bit number encoding **x = 0b1011**
- Which of the following numbers is NOT a valid interpretation of x using any of the number representation schemes discussed today?
  - Unsigned, Sign and Magnitude, Two's Complement

A. $\boxed{-4}$

~~B. -5~~

~~C. 11~~

~~D. -3~~

E. We're lost…

unsigned: 8 + 2 + 1 = 11

sign + mag: 1011 → −(2+1) = -3

two's: −8 +2+1=−5

−x = 0b 0100+1 = 5 → x=-5

Integer Hardware

- In practice, all modern system use **unsigned** and **two's complement** encoding schemes for integers
  - Sign and magnitude for integers is a historical artifact, but useful context for design decision and for floating point (next unit)
  - Much of the same hardware can be used for both encoding schemes (*e.g.*, $+$, $-$)

- Fun fact: Java was designed to only support <u>signed</u> data types
  - *Assumed easier for beginners to understand* than having unsigned as well (*i.e.*, eliminate potential sources of error)
  - Unsigned operation support provided with Unsigned Integer API (starting with Java SE 8 in 2014)

Summary (1/2)

- Bit-level operators allow for fine-grained manipulation
  - Bitwise AND (&), OR (|), XOR (^) and NOT (~) operate on the individual bits of the data
  - Especially useful with bitmasks, chosen bit vectors used with &, |, or ^
    - `b & 0 = 0, b & 1 = b` (set to zero or keep as-is)
    - `b | 0 = b, b | 1 = 1` (keep as-is or set to one)
    - `b ^ 0 = b, b & 1 = ~b` (keep as-is or flip the bit)

AND

Outputs 1 only when both input bits are 1:

&	0	1
0	**0**	**0**
1	**0**	**1**

OR

Outputs 1 when either input bit is 1:

|	0	1
0	**0**	**1**
1	**1**	**1**

XOR

Outputs 1 when either input is *exclusively* 1:

^	0	1
0	**0**	**1**
1	**1**	**0**

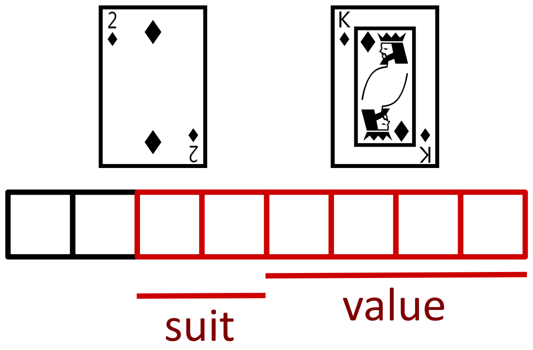
NOT

Outputs the opposite of its input:

~	
0	**1**
1	**0**

- Logical operators work on "truthiness" of data
  - 0 = False, anything else = True
  - Logical AND (&&), OR (| |), and NOT (!) → always evaluate to 1 for True

Summary (2/2)



- Choice of *encoding scheme* is important
  - Tradeoffs based on size requirements and desired operations

- Integers represented using unsigned and two's complement representations (sign and magnitude not used in practice)
  - Limited by fixed bit width, satisfy desirable arithmetic properties