The Hardware/Software Interface

Data III & Integers I

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http://xkcd.com/257/

Relevant Course Information

- HW2 due tonight, HW3 due Friday, HW4 due next Wednesday

- Lab 1a released
  - Some later functions require *bit shifting*, covered in Reading/Lecture 5
  - Workflow:
    1) Edit `pointer.c`
    2) Run the Makefile (`make clean` followed by `make`) and check for compiler errors & warnings
    3) Run ptest (`./ptest`) and check for correct behavior
    4) Run rule/syntax checker (`./dlc.py`) and check output
  - Due Monday 10/6, will overlap a bit with Lab 1b
    - We grade just your *last* submission
    - Don’t wait until the last minute to submit – need to check autograder output

Lab Synthesis Questions

- All subsequent labs (after Lab 0) have a “synthesis question” portion
  - Can be found on the lab specs and are intended to be done *after* you finish the lab
  - You will type up your responses in a `.txt` file for submission on Gradescope
  - These will be graded “by hand” (read by TAs)

- Intended to check your understanding of what you should have learned from the lab
  - Also, great practice for short answer questions on the exams
  - Some are reflective questions – we expect a *personal* (*i.e.*, not generic) response

Lecture Outline (1/3)

Data III:

- **Bitwise and Logical Operators**

Numerical Representation:

- Numerical Encoding Design Example
- Encoding Integers

Boolean Algebra and Bitwise Operators (Review)

- Developed by George Boole in 19th Century
  - Algebraic representation of logic (True → 1, False → 0)

- Bitwise operators apply Boolean operations to bit vectors of matching length
  - Apply to any "integral" data type
    - `char`, `short`, `int`, `long`, `unsigned`
  - Examples:

```
  01101001      01101001
& 01010101    | 01010101

  01101001
^ 01010101    ~ 01010101
```

AND

Outputs 1 only when both input bits are 1:

&	0	1
0	**0**	**0**
1	**0**	**1**

OR

Outputs 1 when either input bit is 1:

|	0	1
0	**0**	**1**
1	**1**	**1**

XOR

Outputs 1 when either input is *exclusively* 1:

^	0	1
0	**0**	**1**
1	**1**	**0**

NOT

Outputs the opposite of its input:

~	
0	**1**
1	**0**

Logical Operators (Review)

- Logical operators: && (AND), || (OR), ! (NOT)
  - In C: **0** is False, **anything nonzero** is True; **always** return 0 or 1

- Examples (char data type)
  - `0xCC && 0x33 -> 0x01`
  - `0x00 || 0x33 -> 0x01`
  - `!0x33 -> 0x00`
  - `!0x00 -> 0x01`

Polling Questions (1/2)

- Compute the result of the following expressions for **char** `c = 0x81;`
  - `c ^ c`
  - `~c & 0xA9`
  - `c || 0x80`
  - `!!c`

Short-Circuit Evaluation (Review)

- If the result of a binary logical operator (&&, `||`) can be determined by its first operand, then the second operand is never evaluated
  - Also known as *early termination*
- Example: `(p && *p)` for a pointer p to "protect" the dereference
  - Dereferencing `NULL` (`0`) results in a segfault

Bitmasks

- Typically binary bitwise operators (&, |, ^) are used with one operand being the "input" and other operand being a specially-chosen *bitmask* (or *mask*) that performs a desired operation

- Operations for a bit b (answer with 0, 1, b, or $\bar{b}$):

$b \,\&\, 0 = \_\_\_$

$b \,\&\, 1 = \_\_\_$

$b \mid 0 = \_\_\_$

$b \mid 1 = \_\_\_$

$b \,\hat{}\, 0 = \_\_\_$

$b \,\hat{}\, 1 = \_\_\_$

Bitmasks Example

- Typically binary bitwise operators (&, |, ^) are used with one operand being the "input" and other operand being a specially-chosen *bitmask* (or *mask*) that performs a desired operation

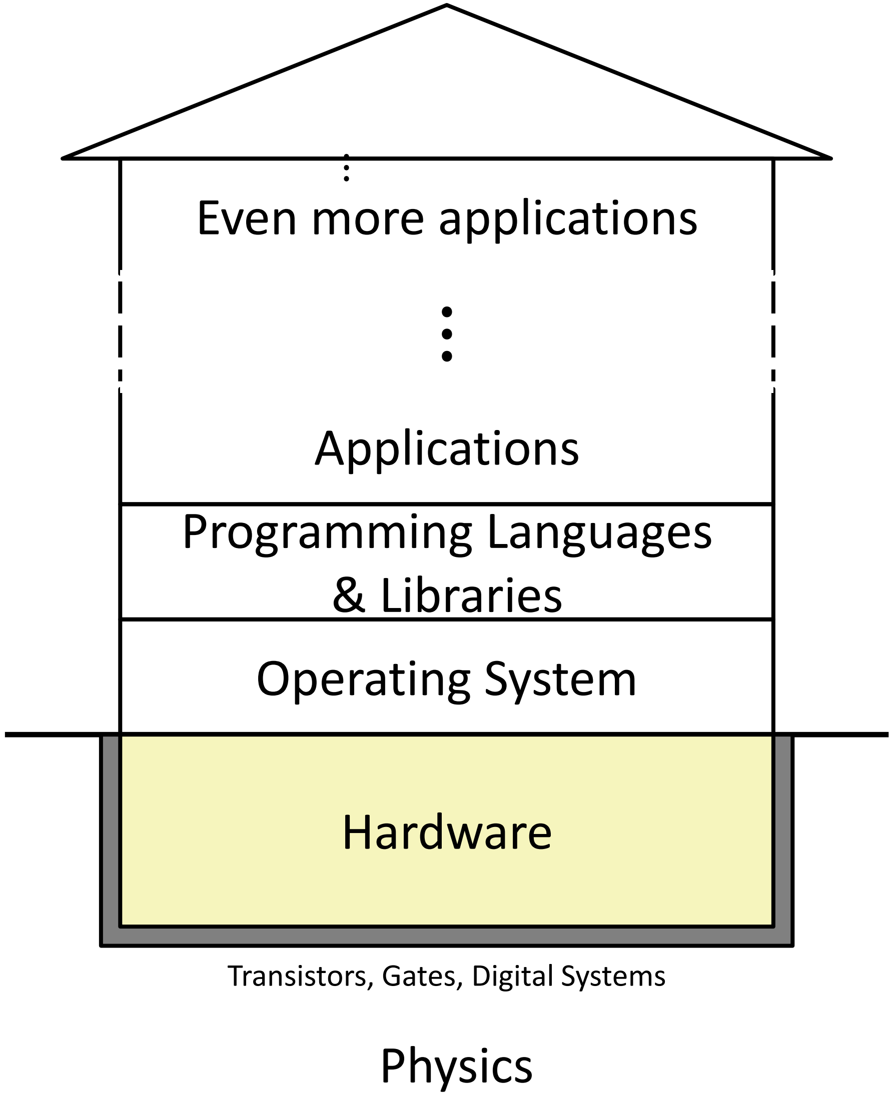
- Example: $b|0 = b,\ b|1 = 1$

```
  01010101  ← input
| 11110000  ← bitmask
  --------
  11110101
```

House of Computing Check-In

- Topic Group 1: **Data**
  - Memory, Data, **Integers**, Floating Point, Arrays, Structs

- How do we store information for other parts of the house of computing to access?
  - How do we represent data and what limitations exist?
  - What design decisions and priorities went into these encodings?



Lecture Outline (2/3)

Data III:

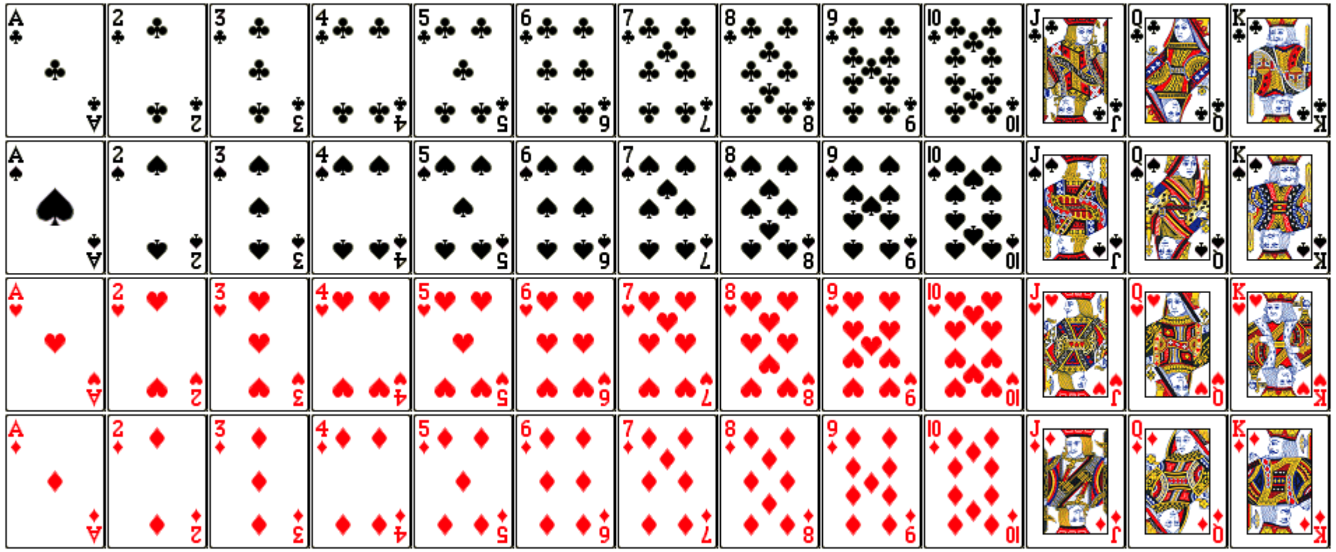
- Bitwise and Logical Operators

Numerical Representation:

- **Numerical Encoding Design Example**
- Encoding Integers

Numerical Encoding Design Example

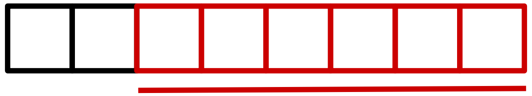
- Encode a standard 52-card deck of French-suited (4 suits) playing cards



- Operations to implement:
  - Which is the higher value card?
  - Are they the same suit?

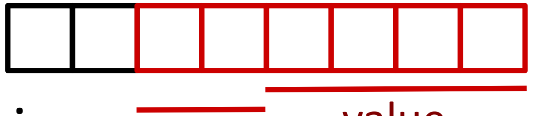
Representations and Fields

1) Binary encoding of all 52 cards – only 6 bits needed
   - $2^6 = 64 \geq 52$



   - Fits in one byte
   - How can we make value and suit comparisons easier?

2) Separate binary encodings of suit (2 bits) and value (4 bits)



   - Also fits in one byte, and easy to do comparisons

K	Q	J	. . .	3	2	A
1101	1100	1011	. . .	0011	0010	0001

♣	00
♦	01
♥	10
♠	11

Compare Card Suits

```
char hand[5];        // represents a 5-card hand
char card1, card2;   // two cards to compare
card1 = hand[0];
card2 = hand[1];
...
if ( same_suit(card1, card2) ) { ... }
```

```
#define SUIT_MASK  0x30

int same_suit(char card1, char card2) {
  return (!((card1 & SUIT_MASK) ^ (card2 & SUIT_MASK)));
  //return (card1 & SUIT_MASK) == (card2 & SUIT_MASK);
}
```

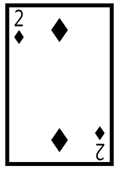
SUIT_MASK = 0x30 = | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |

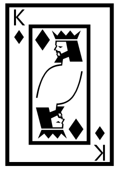
suit (bits 1 1) value (bits 0 0 0 0)

Compare Card Suits Example

```
#define SUIT_MASK  0x30

int same_suit(char card1, char card2) {
  return (!((card1 & SUIT_MASK) ^ (card2 & SUIT_MASK)));
  //return (card1 & SUIT_MASK) == (card2 & SUIT_MASK);
}
```





0	0	0	1	0	0	1	0

&

0	0	1	1	0	0	0	0

=

0	0	0	1	0	0	0	0

SUIT_MASK

0	0	0	1	1	1	0	1

&

0	0	1	1	0	0	0	0

=

0	0	0	1	0	0	0	0

^

0	0	0	0	0	0	0	0

!

0	0	0	0	0	0	0	1

`!(x^y)` equivalent to `x==y`

Compare Card Values

```
char hand[5];         // represents a 5-card hand
char card1, card2;    // two cards to compare
card1 = hand[0];
card2 = hand[1];
...
if ( greater_value(card1, card2) ) { ... }
```

```
#define VALUE_MASK  0x0F

int greater_value(char card1, char card2) {
  return ((unsigned int)(card1 & VALUE_MASK) >
          (unsigned int)(card2 & VALUE_MASK));
}
```

VALUE_MASK = 0x0F =

0	0	0	0	1	1	1	1
		suit		value			

Compare Card Values Example

```
#define VALUE_MASK  0x0F

int greater_value(char card1, char card2) {
  return ((unsigned int)(card1 & VALUE_MASK) >
          (unsigned int)(card2 & VALUE_MASK));
}
```

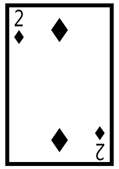
0	0	1	0	0	0	1	0

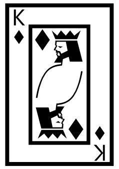
&

0	0	0	0	1	1	1	1

=

0	0	0	0	0	0	1	0





VALUE_MASK

0	0	1	0	1	1	0	1

&

0	0	0	0	1	1	1	1

=

0	0	0	0	1	1	0	1

$2_{10} > 13_{10}$

0 (false)

Takeaways

- Custom encodings may need to be created when dealing with custom data types or if you're trying to be very space efficient

- There may be many valid encodings but your choices matter
  - *e.g.*, space efficiency, ease of implementation
  - Can separate encoding into multiple fields

- Bitwise and logical operators can be useful for manipulating data

Lecture Outline (3/3)

Data III:

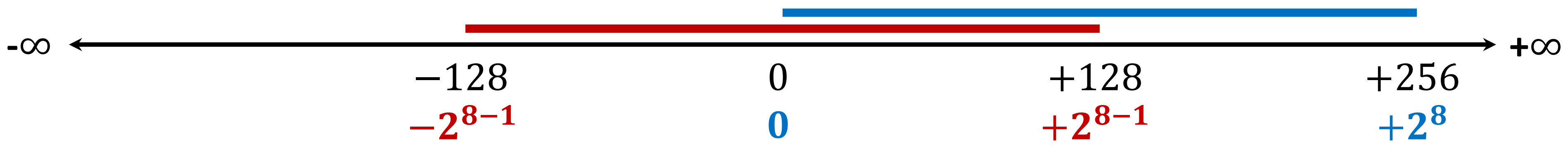
- Bitwise and Logical Operators

Numerical Representation:

- Numerical Encoding Design Example
- **Encoding Integers**

Encoding Integers

- The hardware (and C) supports two flavors of integers
  - *unsigned* – only the non-negatives
  - *signed* – both negatives and non-negatives
- Cannot represent all integers with w bits
  - Only 2^w distinct bit patterns
  - Unsigned values: $0 \ldots 2^w - 1$
  - Signed values: $-2^{w-1} \ldots 2^{w-1} - 1$
- <u>Example</u>: 8-bit integers (*e.g.*, `char`)



Unsigned Integers (Review)

- Unsigned values follow the standard base 2 system
  - $b_7b_6b_5b_4b_3b_2b_1b_0 = b_7 2^7 + b_6 2^6 + \cdots + b_1 2^1 + b_0 2^0$
- Add and subtract using the normal "carry" and "borrow" rules, just in binary:

$$\begin{array}{r} 63 \\ +\underline{\ \ 8} \end{array} \Leftrightarrow \begin{array}{r} 00111111_2 \\ +\underline{00001000}_2 \end{array} \qquad \begin{array}{r} 55 \\ -\underline{\ \ 8\ } \end{array} \Leftrightarrow \begin{array}{r} 00110111_2 \\ -\underline{00001000}_2 \end{array}$$

- In C, add "`unsigned`" keyword in front of any integral type
  - *e.g.*, `unsigned char`, `unsigned short`, `unsigned int`, `unsigned long`

Sign and Magnitude

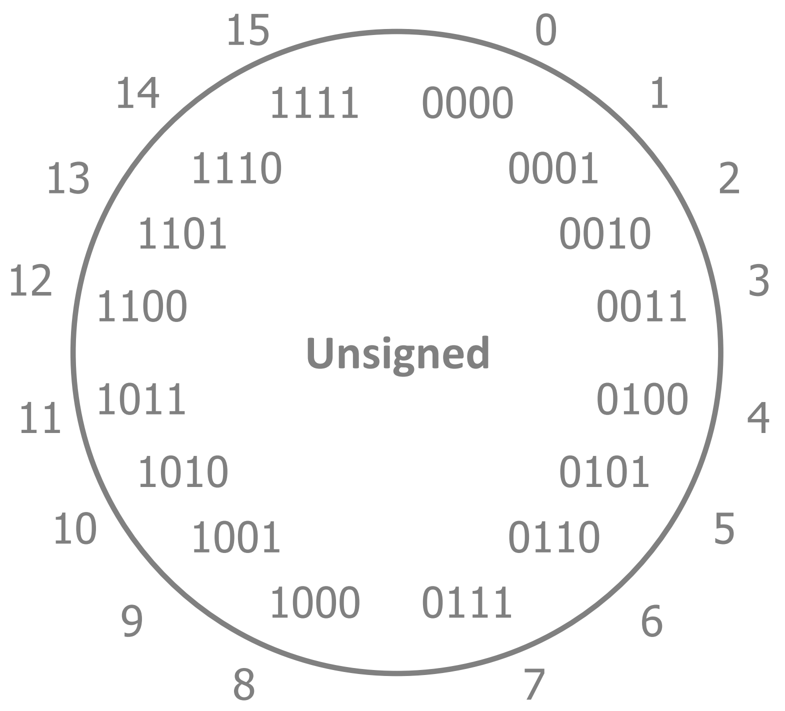
Not used in practice for integers!

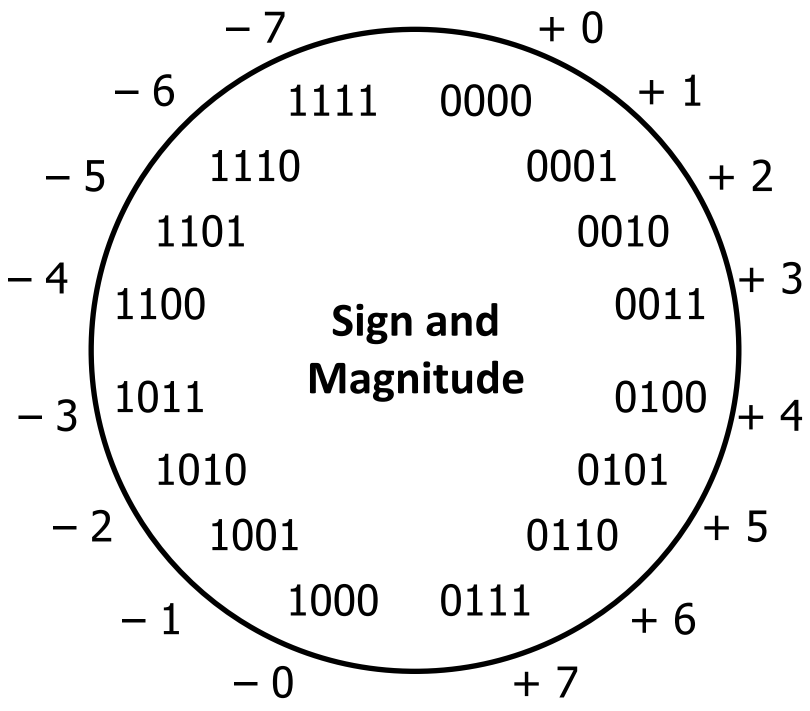
- Designate the high-order bit (MSB) as the "sign bit"
  - `sign=0`: positive number; `sign=1`: negative number
- Benefits:
  - Using MSB as sign bit matches positive numbers with unsigned
  - All zeros encoding is still = 0
- Examples (8 bits):
  - 0x00 = 00000000_2 is non-negative, because the sign bit is 0
  - 0x7F = 01111111_2 is non-negative ($+127_{10}$)
  - 0x85 = 10000101_2 is negative (-5_{10})
  - 0x80 = 10000000_2 is negative... zero???

Sign and Magnitude Visualization

Not used in practice for integers!

- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks?

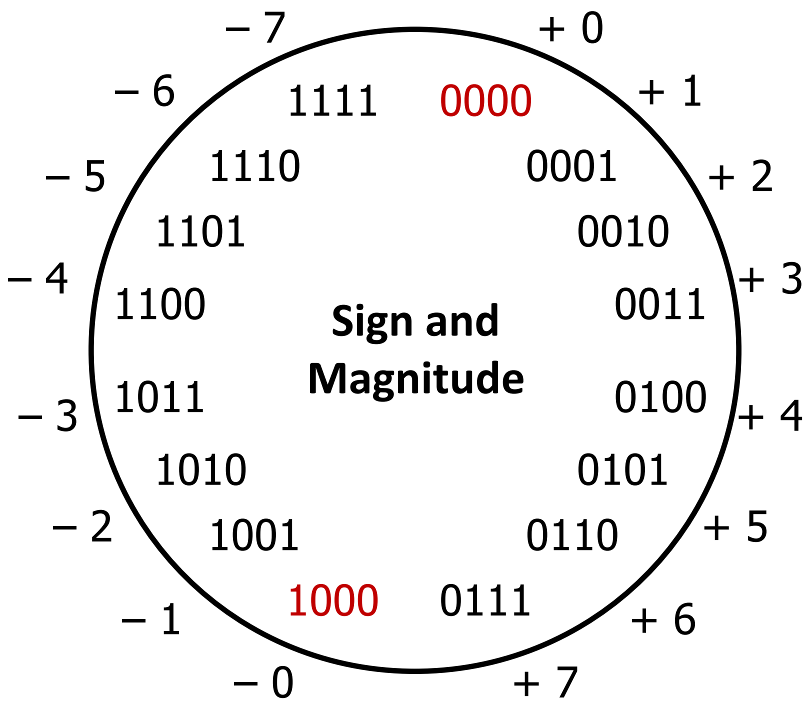




Sign and Magnitude Drawbacks (1/2)

Not used in practice for integers!

- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks:
  - Two representations of 0 (bad for checking equality)



Sign and Magnitude Drawbacks (2/2)

Not used in practice for integers!

- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks:
  - Two representations of 0 (bad for checking equality)
  - Arithmetic is cumbersome
    - Example: `4-3 != 4+(-3)`

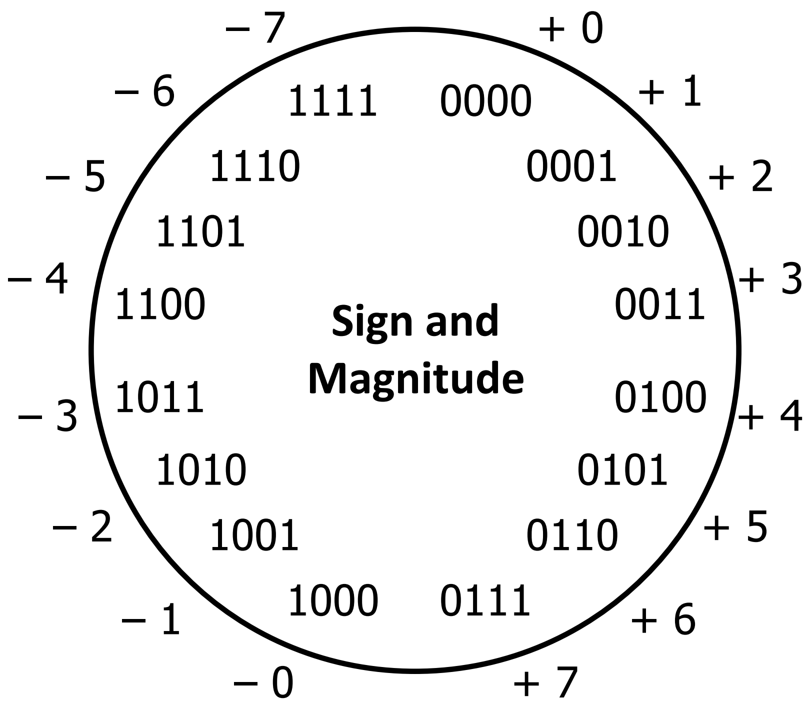
4	0100
- 3	- 0011
1	0001

✓

4	0100
+ -3	+ 1011
-7	1111

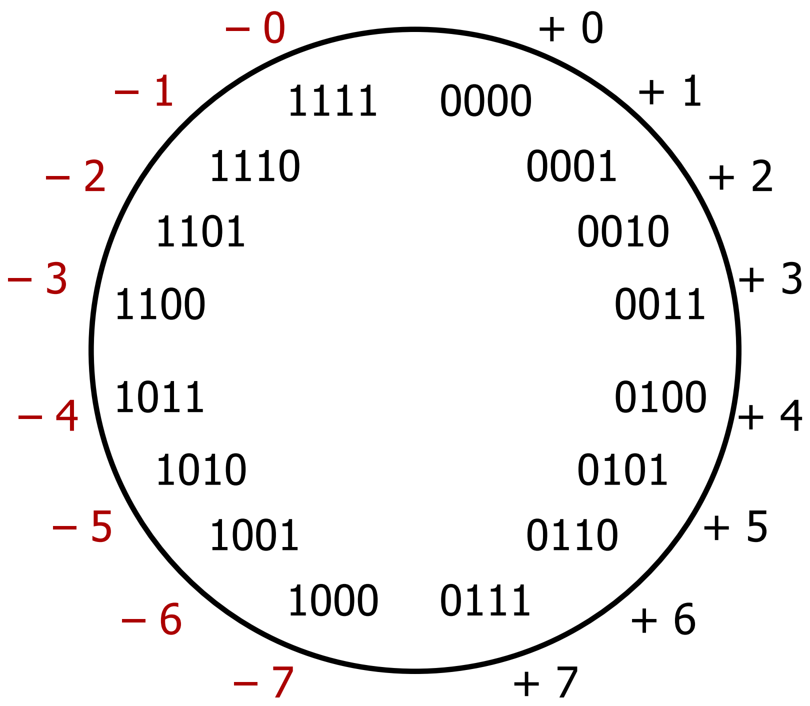
✗

    - Negatives "increment" in wrong direction!



Two’s Complement Development (1/2)

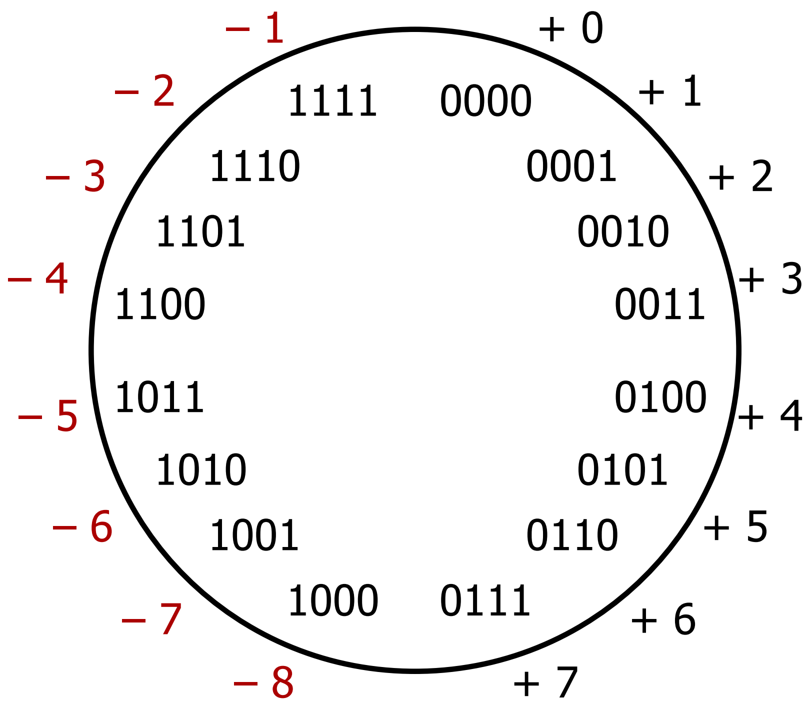
- Let’s fix these problems:
  1) “Flip” negative encodings so incrementing works
  2) “Shift” negative numbers to eliminate –0



Two's Complement Development (2/2)

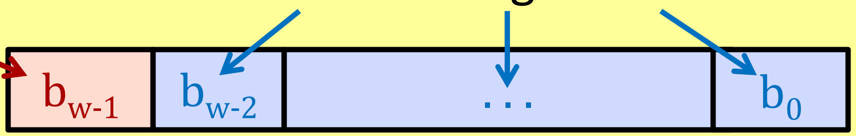
- Let's fix these problems:
  1) "Flip" negative encodings so incrementing works
  2) "Shift" negative numbers to eliminate –0

- MSB *still* indicates sign!
  - This is why we represent one more negative than positive number (-2^{N-1} to $2^{N-1} - 1$)

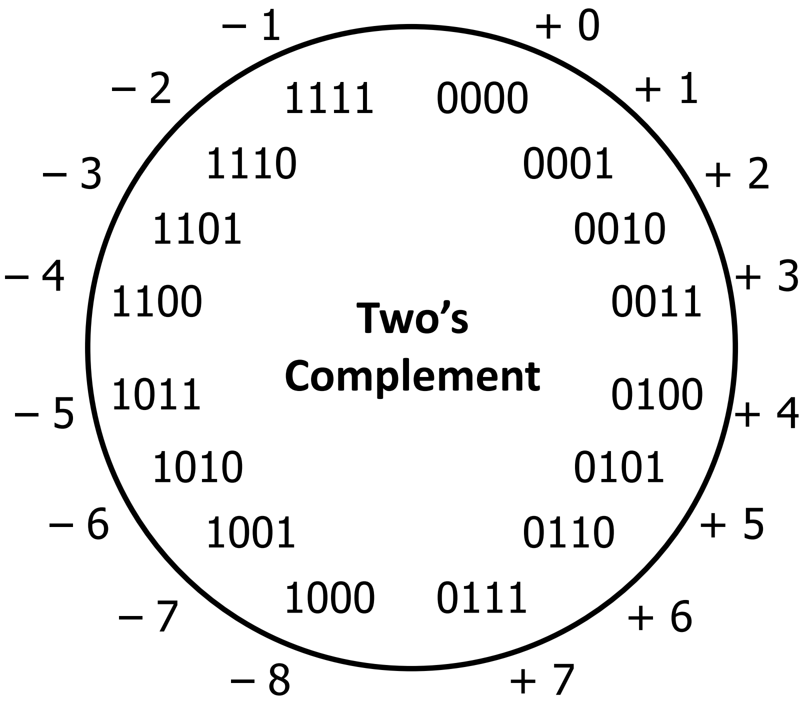


Two's Complement Negatives (Review)

- Accomplished with one neat mathematical trick!



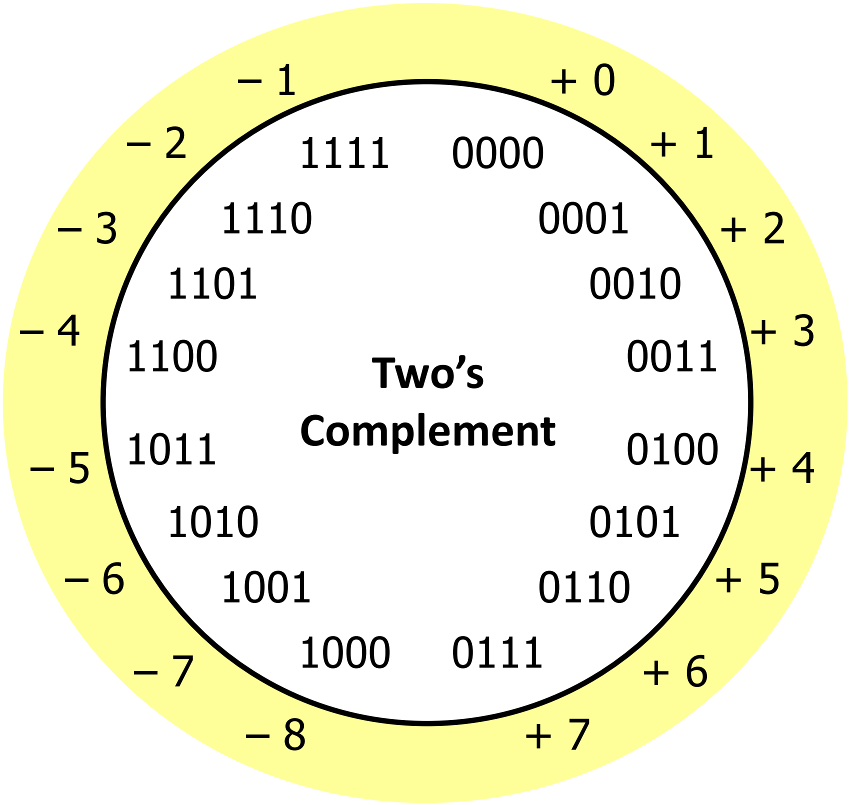
- 4-bit Example:
  - 1010_2 unsigned:
    $1*2^3+0*2^2+1*2^1+0*2^0$ = **10**
  - 1010_2 two's complement:
    $-1*2^3+0*2^2+1*2^1+0*2^0$ = **–6**



Two's Complement is Great (Review)

- Roughly same number of (+) and (–) numbers
- Positive number encodings match unsigned
- Single zero (with all `0`'s encoding)

- Simple negation procedure:
  - Get negative representation of any integer by taking bitwise complement and then adding one!
    `( ~x + 1 == -x )`



Polling Questions (2/2)

- Take the 4-bit number encoding `x = 0b1011`
- Which of the following numbers is NOT a valid interpretation of x using any of the number representation schemes discussed today?
  - Unsigned, Sign and Magnitude, Two's Complement

A. -4

B. -5

C. 11

D. -3

E. We're lost…

Integer Hardware

- In practice, all modern system use **unsigned** and **two's complement** encoding schemes for integers
  - Sign and magnitude for integers is a historical artifact, but useful context for design decision and for floating point (next unit)
  - Much of the same hardware can be used for both encoding schemes (*e.g.*, $+$, $-$)

- Fun fact: Java was designed to only support <u>signed</u> data types
  - *Assumed easier for beginners to understand* than having unsigned as well (*i.e.*, eliminate potential sources of error)
  - Unsigned operation support provided with Unsigned Integer API (starting with Java SE 8 in 2014)

Summary (1/2)

- Bit-level operators allow for fine-grained manipulation
  - Bitwise AND (&), OR (|), XOR (^) and NOT (~) operate on the individual bits of the data
  - Especially useful with bitmasks, chosen bit vectors used with &, |, or ^
    - `b & 0 = 0, b & 1 = b` (set to zero or keep as-is)
    - `b | 0 = b, b | 1 = 1` (keep as-is or set to one)
    - `b ^ 0 = b, b & 1 = ~b` (keep as-is or flip the bit)

AND

Outputs 1 only when both input bits are 1:

&	0	1
0	**0**	**0**
1	**0**	**1**

OR

Outputs 1 when either input bit is 1:

|	0	1
0	**0**	**1**
1	**1**	**1**

XOR

Outputs 1 when either input is *exclusively* 1:

^	0	1
0	**0**	**1**
1	**1**	**0**

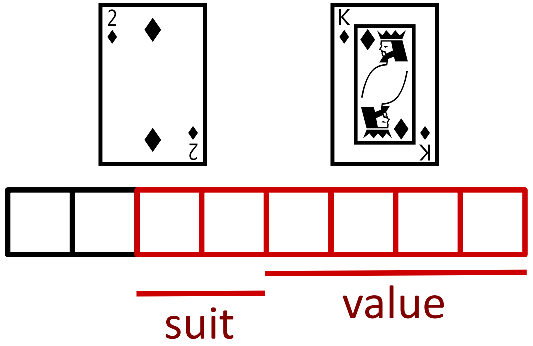
NOT

Outputs the opposite of its input:

~	
0	**1**
1	**0**

- Logical operators work on "truthiness" of data
  - 0 = False, anything else = True
  - Logical AND (&&), OR (||), and NOT (!) → always evaluate to 1 for True

Summary (2/2)



- Choice of *encoding scheme* is important
  - Tradeoffs based on size requirements and desired operations
- Integers represented using unsigned and two's complement representations (sign and magnitude not used in practice)
  - Limited by fixed bit width, satisfy desirable arithmetic properties