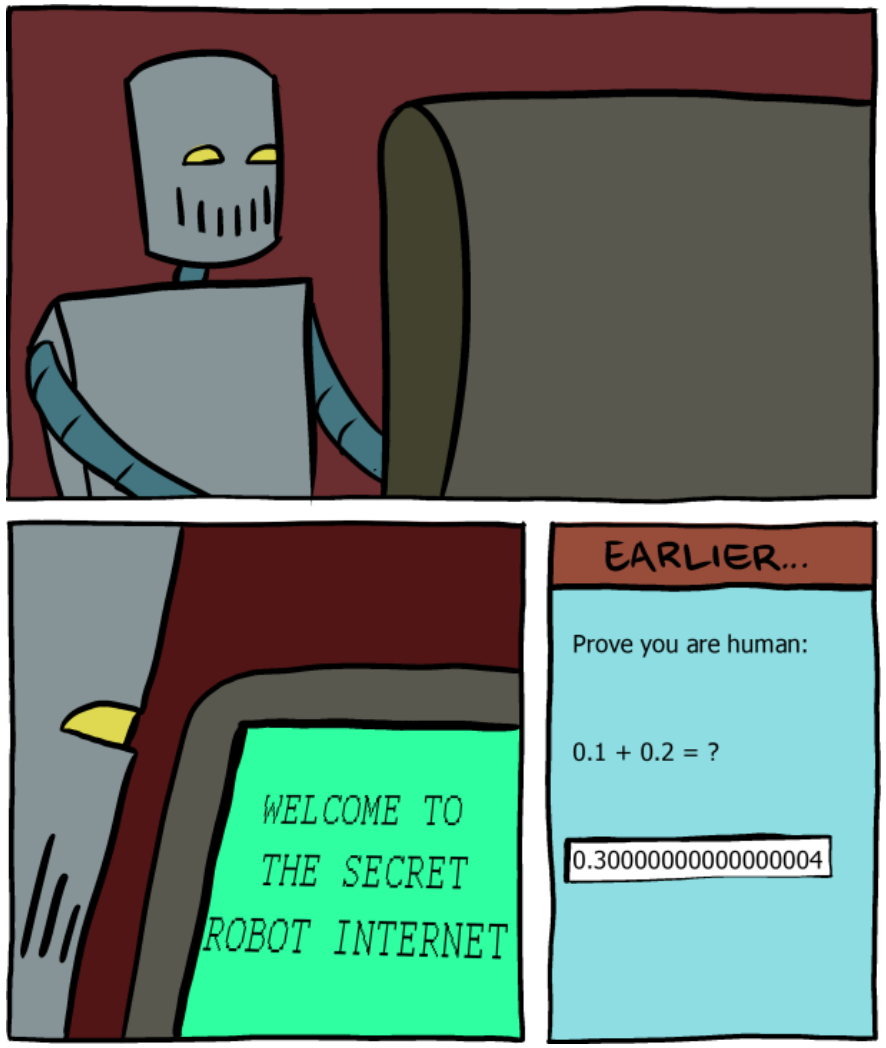
Floating Point

CSE 351 Autumn 2023

Instructor:
Justin Hsia

Teaching Assistants:

Afifah Kashif	Malak Zaki
Bhavik Soni	Naama Amiel
Cassandra Lam	Nayha Auradkar
Connie Chen	Nikolas McNamee
David Dai	Pedro Amarante
Dawit Hailu	Renee Ruan
Ellis Haker	Simran Bagaria
Eyoel Gebre	Will Robertson
Joshua Tan	



http://www.smbc-comics.com/?id=2999

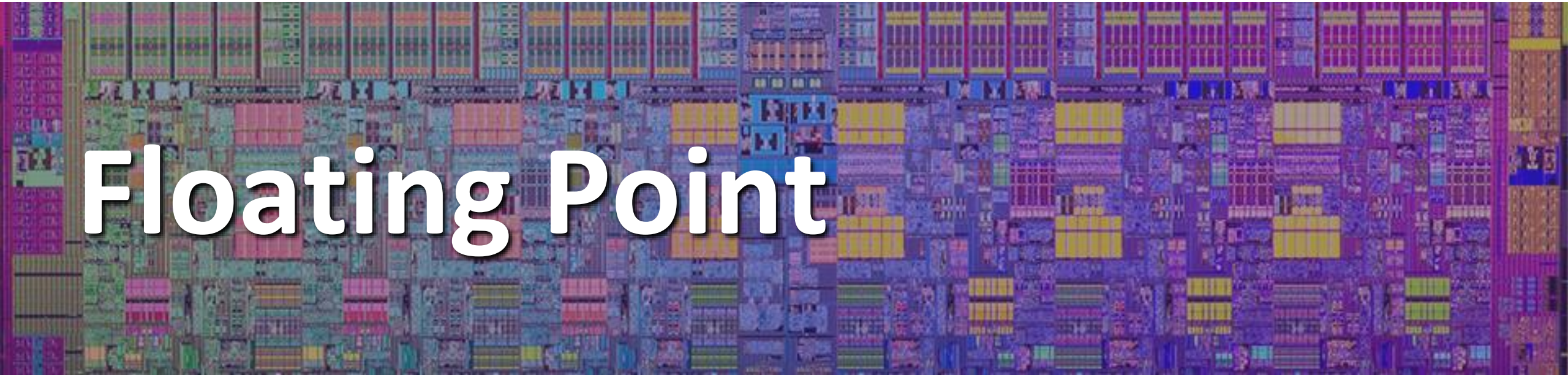
Relevant Course Information

- hw5 due Wednesday, hw6 due Friday
- Lesson questions are graded on *completion*
  - Don't change your answer afterward; misrepresents your understanding
- Lab 1a due tonight at 11:59 pm
  - Submit `pointer.c` and `lab1Asynthesis.txt`
    - Make sure there are no lingering `printf` statements in your code!
  - Make sure you submit *something* to Gradescope before the deadline and that the file names are correct
  - Can use late days to submit up until Wed 11:59 pm
- Lab 1b due next Monday (10/16)
  - Submit `aisle_manager.c`, `store_client.c`, and `lab1Bsynthesis.txt`

Lab 1b Aside: C Macros

- C macros basics:
  - Basic syntax is of the form: `#define NAME expression`
  - Allows you to use "`NAME`" instead of "`expression`" in code
    - Does naïve copy and replace *before* compilation – everywhere the characters "NAME" appear in the code, the characters "expression" will now appear instead
    - NOT the same as a Java constant
  - Useful to help with readability/factoring in code

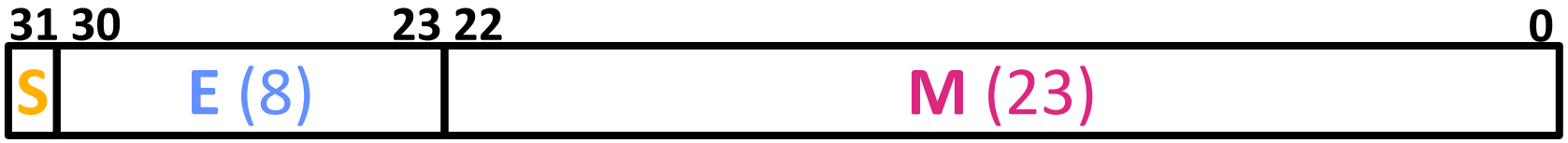
- You'll use C macros in Lab 1b for defining bit masks
  - See Lab 1b starter code and Lesson 4 (card operations) for examples



Floating Point

Lesson Summary (1/3)

- Floating point approximates real numbers:



  - Handles large numbers, small numbers, special numbers
  - Exponent in biased notation (bias = $2^{w-1} - 1$)
    - Size of exponent field determines our representable *range*
    - Outside of representable exponents is *overflow* and *underflow*
  - Mantissa approximates fractional portion
    - Size of mantissa field determines our representable *precision*
    - Exceeding length causes *rounding*

E	M	Meaning
0b0…0	anything	± denorm num (including 0)
anything else	anything	± norm num
0b1…1	0	± ∞
0b1…1	non-zero	NaN

Lesson Summary (2/3)

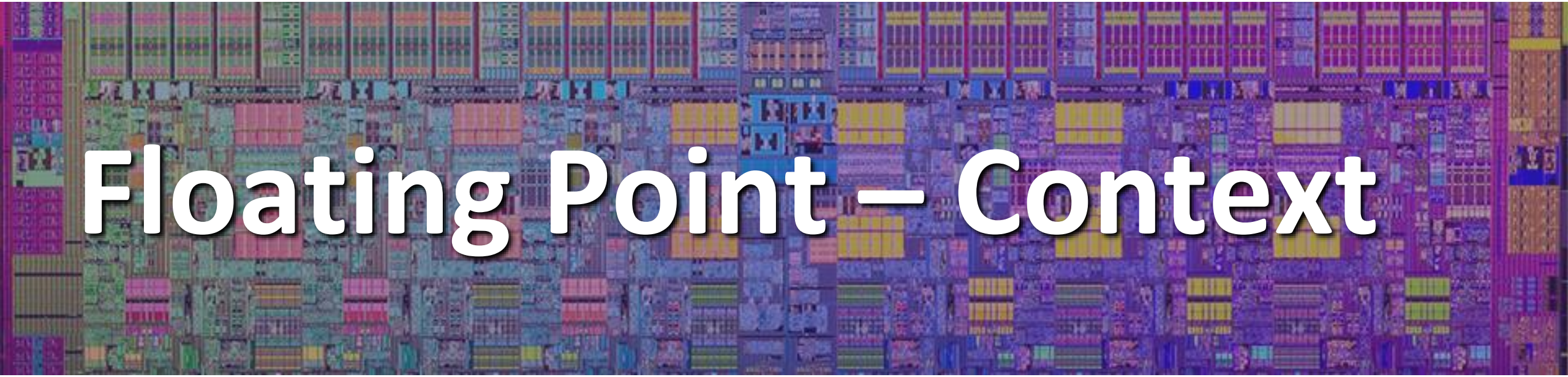
- Limitations of FP affect programmers all the time
  - Overflow, underflow, rounding
    - Rounding is a HUGE issue due to limited mantissa bits and gaps that are scaled by the value of the exponent
  - Floating point arithmetic is NOT associative or distributive
    - ∞ and NaN are valid operands, but can produce unintuitive results
  - Do NOT use equality (==) with floating point numbers
  - Converting between integral and floating point data types *does* change the bits

Lesson Summary (3/3)

- Terminology:
  - `float`, `double`
  - sign (S), exponent (E), mantissa (M), biased notation, implicit leading one
  - denormalized numbers, $\pm\infty$, Not-a-Number (NaN), overflow, underflow, rounding

- Learning Objectives:
  - Describe how the bits in floating point are organized and how they represent real numbers (and special cases).
  - Describe the distribution of representable values in floating point.
  - Explain the limitations of floating point and write C code that accounts for them.

- What lingering questions do you have from the lesson?



Floating Point – Context

Floating Point Issues in Real Life

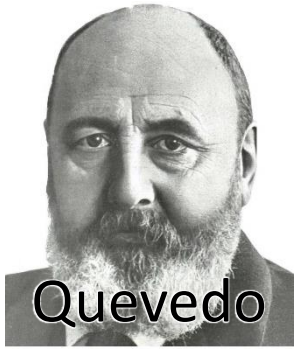
- **1991:** Patriot missile targeting error
  - Time in system stored in integer (tenths of a second since boot)
  - Converted to seconds by multiplying by $0.1 = 0.0\ \overline{0011}_2$ leading to erroneous time (error grows the longer system has been on)

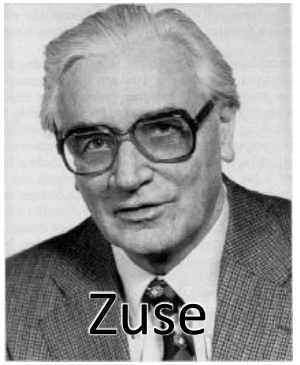
- **1996:** V88 Ariane 501 rocket exploded 37 seconds after launch
  - Reused code from Ariane 4 inertial reference platform
  - Overflow when converting a 64-bit floating point number to a 16-bit integer (not protected by extra lines of code)

- **Other related bugs:**
  - 1982: Vancouver Stock Exchange 50% error in less than 2 years due to truncation
  - 1994: Intel Pentium FDIV (floating point division) hardware bug costs company $475 million in recall

More on Floating Point History

- Early days
  - First design with floating-point arithmetic in 1914 by Leonardo Torres y Quevedo
  - Implementations started in 1940 by Konrad Zuse, but with differing field lengths (usually not summing to 32 bits) and different subsets of the special cases
- IEEE 754 standard created in 1985
  - Primary architect was William Kahan, who won a Turing Award for this work
  - Standardized bit encoding, well-defined behavior for *all* arithmetic operations





Floating Point in the "Wild"

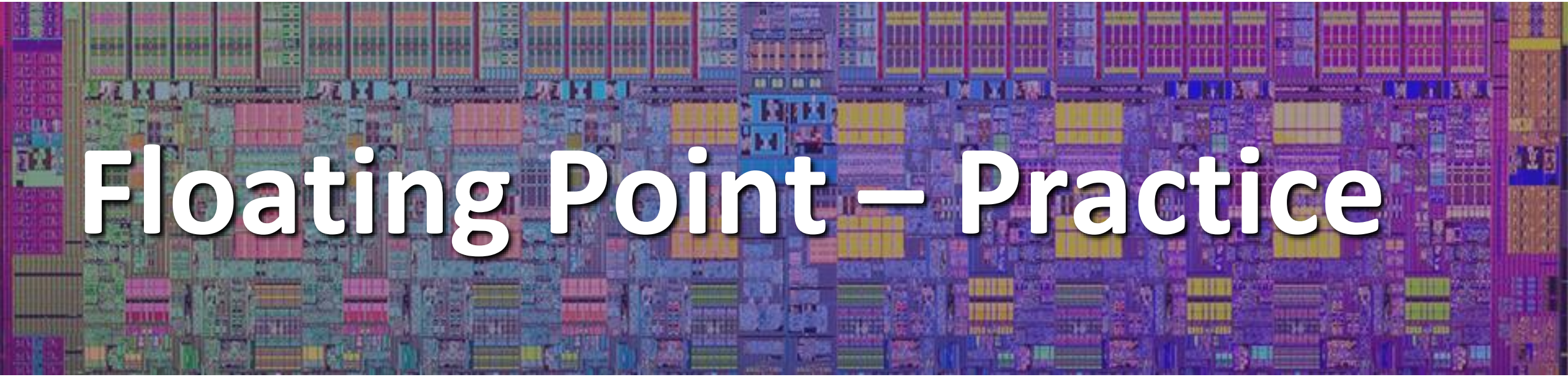
- 3 formats from IEEE 754 standard widely used in computer hardware and languages
  - In C, called `float`, `double`, `long double`
- Common applications:
  - 3D graphics: textures, rendering, rotation, translation
  - "Big Data": scientific computing at scale, machine learning
- Non-standard formats in domain-specific areas:
  - **Bfloat16:** training ML models; range more valuable than precision
  - **TensorFloat-32:** Nvidia-specific hardware for Tensor Core GPUs

Type	S bits	E bits	M bits	Total bits
Half-precision	1	5	10	16
Bfloat16	1	8	7	16
TensorFloat-32	1	8	10	19
Single-precision	1	8	23	32

Discussion Question

- Discuss the following question(s) in groups of 3-4 students
  - I will call on a few groups afterwards so please be prepared to share out
  - Be respectful of others' opinions and experiences

- How do you feel about floating point?
  - Do you feel like the limitations are acceptable?
  - Does this affect the way you'll think about non-integer arithmetic in the future?
  - Are there any changes or different encoding schemes that you think would be an improvement?



Floating Point – Practice

Group Work Time

- During this time, you are encouraged to work on the following:
  1) If desired, continue your discussion
  2) Work on the lesson problems (solutions at the end of class)
  3) Work on the homework problems

- Resources:
  - You can revisit the lesson material
  - Work together in groups and help each other out
  - Course staff will circle around to provide support

$2^{-1} = 0.5$
$2^{-2} = 0.25$
$2^{-3} = 0.125$
$2^{-4} = 0.0625$

Practice Questions (1/2)

- What is the value encoded by the following floating point number?

  0b 0 | 1000 0000 | 110 0000 0000 0000 0000 0000

  - bias = $2^{w-1}-1$ $= 2^7 - 1 = 127$
  - exponent = E – bias $= 2^7 - 127 = 128 - 127 = 1$
  - mantissa = 1.M $= 1.110\ldots0_2$

  $(-1)^0 \times 1.11_2 \times 2^1 = 11.1_2 =$ **+3.5**

- Convert the decimal number **-7.375 = -1.11011 x 2^2** into floating point representation.

  $S = 1$, $E = 2 + 127 = 129 =$ 0b 1000 0001, $M =$ 0b 11011 0…0

  0b 1 100 0000 1 110 1100 0…0 = **0x C0EC 0000**

Practice Questions (2/2)

- What is the value of the following floats?
  - 0x00000000 ⇒ S=0, E=0, M=0 ⇒ +0
  - 0xFF800000 ⇒ S=1, E= all 1's, M=0 ⇒ −∞

0b 1 1111 1111 1000 0…0
S E M

- For the following code, what is the smallest value of n that will encounter a limit of representation?

```
float f = 1.0;  // 2^0
for (int i = 0; i < n; ++i)
    f *= 1024;  // 1024 = 2^10
printf("f = %f\n", f);
```

E_{max} = 0xFE, Exp_{max} = 254 − 127 = 127

n	f
1	2^{10}
2	2^{20}
3	2^{30}
4	2^{40}
⋮	⋮

always M = 0

for n = 13, we hit 2^{130}, which causes overflow