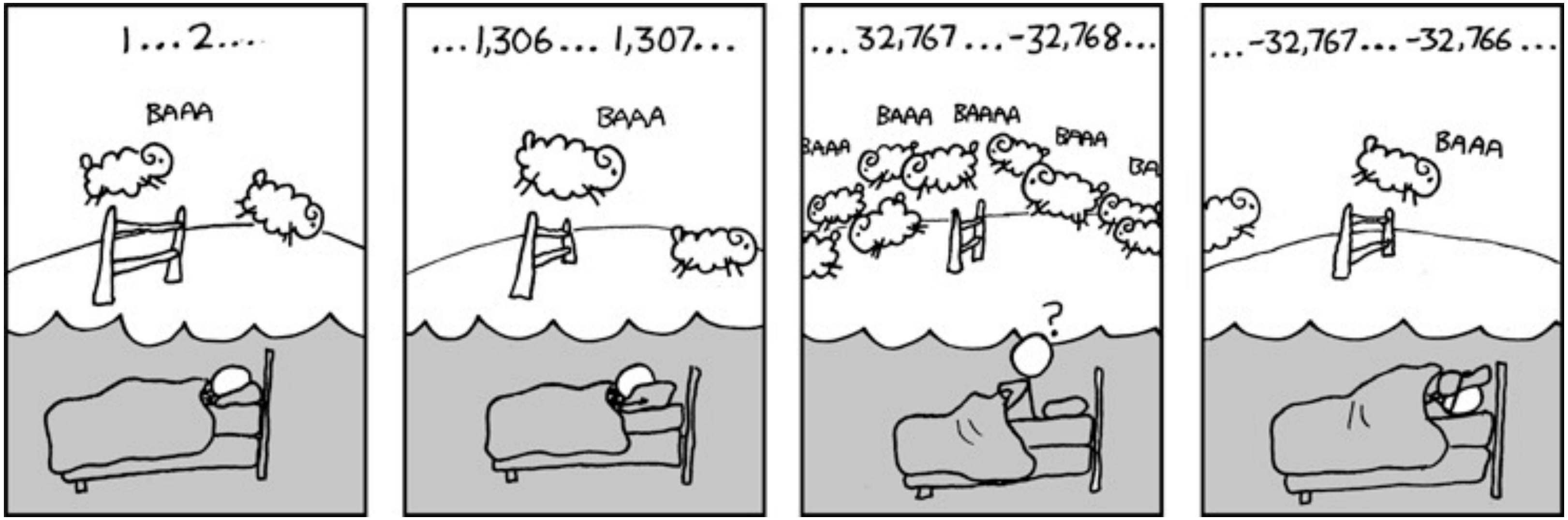
Integers II

CSE 351 Winter 2022

Instructor:
Sam Wolfson

Teaching Assistants:
Angela Xu, Anirudh Kumar, Catherine Guevara
Dara Stotland, Harrison Bay, Ian Hsiao
Kevin Wang, Mara Kirdani-Ryan, Nick Durand
Sanjana Sridhar



http://xkcd.com/571/

Relevant Course Information

- hw4 due tonight, hw5 due Friday (1/14)
- Lab 1a due Wensday (1/19)
  - Use `ptest` and `dlc.py` to check your solution for correctness (on the CSE Linux environment)
  - Submit `pointer.c` and `lab1Asynthesis.txt` to Gradescope
    - Make sure you pass the File and Compilation Check – all the correct files were found and there were no compilation or runtime errors
- Lab 1b released today, due 1/21
  - Bit manipulation on a custom encoding scheme
  - Bonus slides at the end of today's lecture have relevant examples

Reading Review

- Terminology:
  - `UMin`, `UMax`, `TMin`, `TMax`
  - Type casting: implicit vs. explicit
  - Integer extension: zero extension vs. sign extension
  - Modular arithmetic and arithmetic overflow
  - Bit shifting: left shift, logical right shift, arithmetic right shift
- Questions from the Reading?

Review Questions

- What is the value (and encoding) of **`TMin`** for a fictional 6-bit wide integer data type?

0b100000 = $-(2^5)$ = -32

MSB of 1 followed by all zeros.
only the MSB has negative weight

- For `unsigned char uc = 0xA1;`, what are the produced data for the cast **`(unsigned short)uc`**?

Since uc is unsigned, it will be zero-extended to 16 bits (size of a short):
0x00A1

- What is the result of the following expressions?
  - **`(signed char)uc >> 2`**
  - **`(unsigned char)uc >> 3`**

sign extension

zero extension

0xA1 = 0b10100001
shifted off
sign bit copied
0b11101000

0b10100001
shifted off
0b00010100
zero extended

Why Does Two's Complement Work?

- For all representable positive integers x, we want:

$$\begin{array}{r} \textit{bit representation of } \; x \\ + \; \textit{bit representation of} -x \\ \hline 0 \end{array}$$ (ignoring the carry-out bit)

  - What are the 8-bit negative encodings for the following?

$$\begin{array}{r} 00000001 \\ + \;\; ???????? \\ \hline 00000000 \end{array} \qquad \begin{array}{r} 00000010 \\ + \;\; ???????? \\ \hline 00000000 \end{array} \qquad \begin{array}{r} 11000011 \\ + \;\; ???????? \\ \hline 00000000 \end{array}$$

for any x: $x + \sim x = 0b111\ldots1$ ← in 2's complement, all 1s represents -1

$x + \sim x = -1$

$\sim x = -1 - x$

$\sim x + 1 = -x$

wow!

Why Does Two's Complement Work?

- For all representable positive integers x, we want:

$$\begin{array}{r} \textit{bit representation of } x \\ +\ \textit{bit representation of } -x \\ \hline 0 \end{array}$$ (ignoring the carry-out bit)

  - What are the 8-bit negative encodings for the following?

$$\begin{array}{r} 00000001 \\ +\ 11111111 \\ \hline 100000000 \end{array} \qquad \begin{array}{r} 00000010 \\ +\ 11111110 \\ \hline 100000000 \end{array} \qquad \begin{array}{r} 11000011 \\ +\ 00111101 \\ \hline 100000000 \end{array}$$

These are the bitwise complement plus 1!

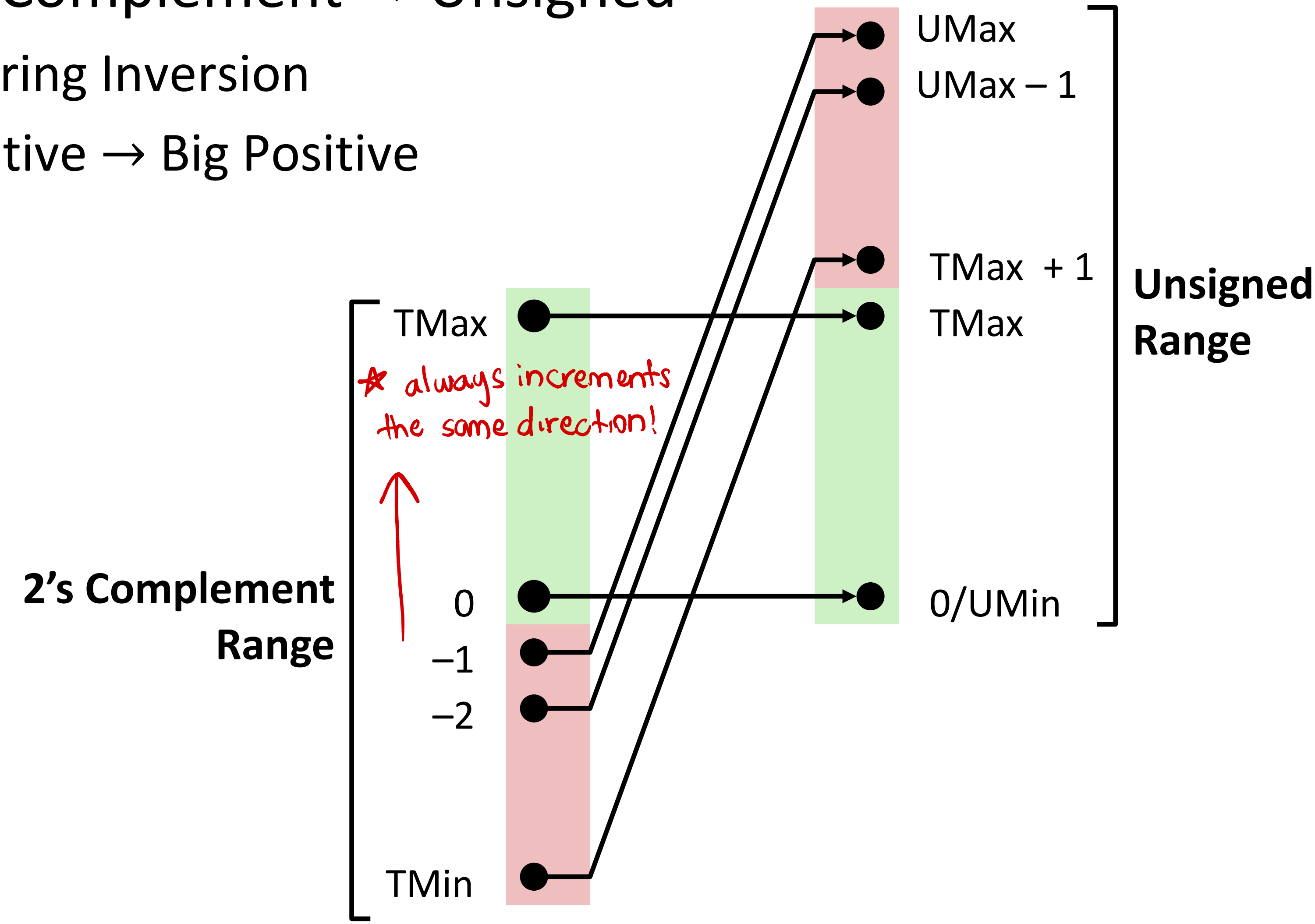
`-x == ~x + 1`

Integers

- **Binary representation of integers**
  - **Unsigned and signed**
  - **Casting in C**
- Consequences of finite width representations
  - Sign extension, overflow
- Shifting and arithmetic operations

Signed/Unsigned Conversion Visualized

- Two's Complement → Unsigned
  - Ordering Inversion
  - Negative → Big Positive



Values To Remember (Review)

- Unsigned Values
  - UMin = 0b00…0 = 0
  - UMax = 0b11…1 = $2^w - 1$
- Two's Complement Values
  - TMin = 0b10…0 = -2^{w-1}
  - TMax = 0b01…1 = $2^{w-1} - 1$
  - -1 = 0b11…1
- **Example:** Values for $w = 64$

	Decimal	Hex
UMax	18,446,744,073,709,551,615	FF FF FF FF FF FF FF FF
TMax	9,223,372,036,854,775,807	7F FF FF FF FF FF FF FF
TMin	-9,223,372,036,854,775,808	80 00 00 00 00 00 00 00
-1	-1	FF FF FF FF FF FF FF FF
0	0	00 00 00 00 00 00 00 00

In C: Signed vs. Unsigned (Review)

- Casting
  - Bits are unchanged, just interpreted differently!
    - **int** tx, ty;
    - **unsigned int** ux, uy;
  - *Explicit* casting
    - tx = (**int**) ux;
    - uy = (**unsigned int**) ty;
  - *Implicit* casting can occur during assignments or function calls
    - tx = ux;
    - uy = ty;

example:

```
char c = -20;              // 0b11101100
unsigned char uc = c;      // same bits, but now
                           // interpreted as 236
```

Casting Surprises (Review)

!!!

- Integer literals (constants)
  - By default, integer constants are considered *signed* integers
    - Hex constants already have an explicit binary representation
  - Use "U" (or "u") suffix to explicitly force *unsigned*
    - **Examples:** 0U, 4294967259u

- Expression Evaluation
  - When you mixed unsigned and signed in a single expression, then **signed values are implicitly cast to <u>unsigned</u>**
  - Including comparison operators <, >, ==, <=, >=

Integers

- Binary representation of integers
  - Unsigned and signed
  - Casting in C
- **Consequences of finite width representations**
  - **Sign extension, overflow**
- Shifting and arithmetic operations

Sign Extension (Review)

- **Task:** Given a w-bit signed integer X, convert it to w+k-bit signed integer X' *with the same value*
- **Rule: Add k copies of sign bit**
  - Let x_i be the i-th digit of X in binary
  - $\mathrm{X}' = \underbrace{x_{w-1}, \ldots, x_{w-1}}_{k \text{ copies of MSB}}, \underbrace{x_{w-1}, x_{w-2}, \ldots, x_1, x_0}_{\text{original X}}$

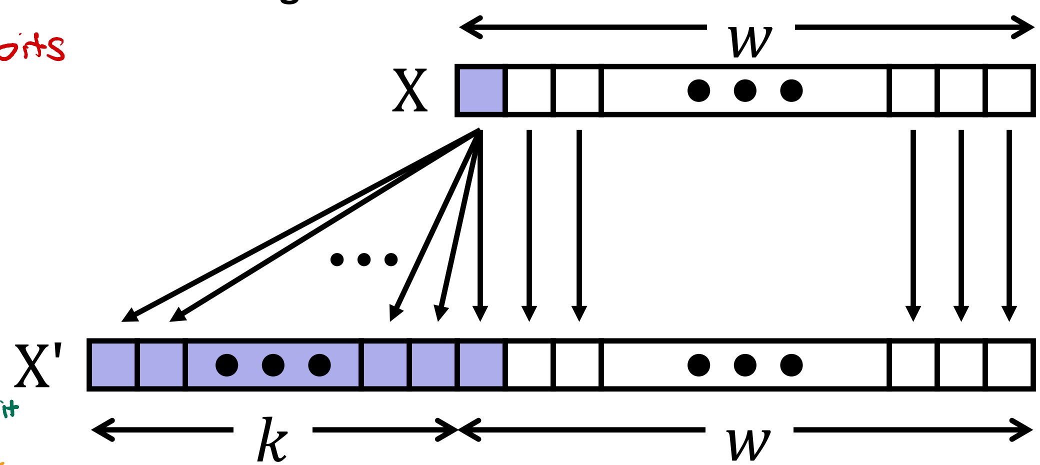
example: sign-extend 4 bits to 5 bits

$0b1100 = -(2^3) + 2^2 = -4$

$0b11100 = -(2^4) + 2^3 + 2^2 = -4$

new sign bit is worth double the old sign bit, but old sign bit is now positive. so they cancel:

before extension $[-(2^n)] = [\underbrace{-(2^{n+1})}_{\text{new sign bit}} + \underbrace{2^n}_{\text{old sign bit}}]$ after extension



Two's Complement Arithmetic

- The same addition procedure works for both unsigned and two's complement integers
  - **Simplifies hardware:** only one algorithm for addition
  - **Algorithm:** simple addition, discard the highest carry bit
    - Called modular addition: result is sum *modulo* 2^w

Arithmetic Overflow (Review)

Bits	Unsigned	Signed
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	-8
1001	9	-7
1010	10	-6
1011	11	-5
1100	12	-4
1101	13	-3
1110	14	-2
1111	15	-1

- When a calculation produces a result that can't be represented in the current encoding scheme
  - Integer range limited by fixed width
  - Can occur in both the positive and negative directions
- C and Java ignore overflow exceptions
  - You end up with a bad value in your program and no warning/indication… oops!

Overflow: Unsigned

- **Addition:** drop carry bit (-2^N)

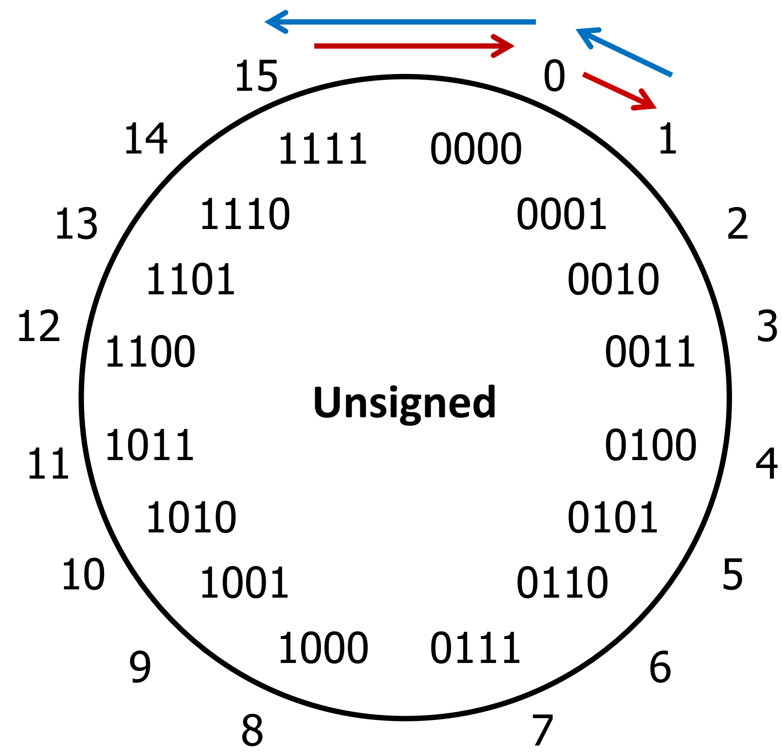
$$\begin{array}{r} 15 \\ +\ \ 2 \\ \hline \cancel{17} \end{array} \qquad \begin{array}{r} 1111 \\ +\ \ 0010 \\ \hline \cancel{1}0001 \end{array}$$

17 (mod 16) = 1

- **Subtraction:** borrow ($+2^N$)

$$\begin{array}{r} 1 \\ -\ \ 2 \\ \hline \cancel{-1} \end{array} \qquad \begin{array}{r} 10001 \\ -\ \ 0010 \\ \hline 1111 \end{array}$$

-1 (mod 16) = 15



$\pm 2^N$ because of modular arithmetic

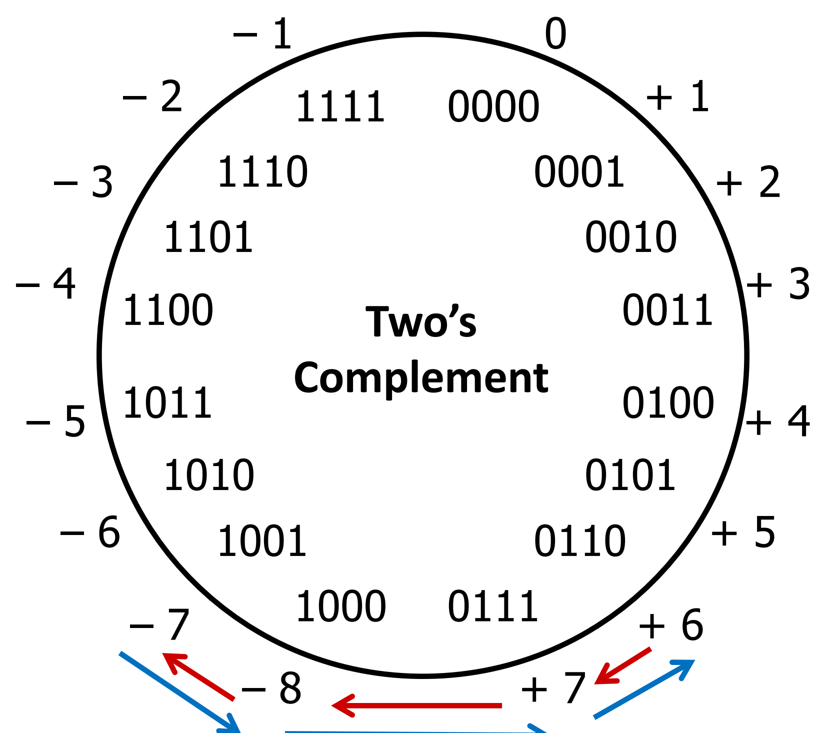
Overflow: Two's Complement

- **Addition:** (+) + (+) = (−) result?

$$\begin{array}{r} 6 \\ +\quad 3 \\ \hline \cancel{9} \\ -7 \end{array} \qquad \begin{array}{r} 0110 \\ +\quad 0011 \\ \hline 1001 \end{array}$$

- **Subtraction:** (−) + (−) = (+)?

$$\begin{array}{r} -7 \\ -\quad 3 \\ \hline \cancel{-10} \\ 6 \end{array} \qquad \begin{array}{r} 1001 \\ -\quad 0011 \\ \hline 0110 \end{array}$$



For signed: overflow if operands have same sign and result's sign is different

Integers

- Binary representation of integers
  - Unsigned and signed
  - Casting in C
- Consequences of finite width representations
  - Sign extension, overflow
- **Shifting and arithmetic operations**

Shift Operations (Review)

- Throw away (drop) extra bits that "fall off" the end
- Left shift (x<<n) bit vector x by n positions
  - Fill with 0's on right
- Right shift (x>>n) bit-vector x by n positions
  - Logical shift (for unsigned values)
    - Fill with 0's on left
  - Arithmetic shift (for signed values)
    - Replicate most significant bit on left (maintains sign of x)

	x	0010 0010
	x<<3	0001 0000
logical:	x>>2	0000 1000
arithmetic:	x>>2	0000 1000

	x	1010 0010
	x<<3	0001 0000
logical:	x>>2	0010 1000
arithmetic:	x>>2	1110 1000

Shift Operations (Review)

- Arithmetic:
  - Left shift (`x<<n`) is equivalent to <u>multiply</u> by 2^n
  - Right shift (`x>>n`) is equivalent to <u>divide</u> by 2^n
  - Shifting is faster than general multiply and divide operations!

- Notes:
  - Shifts by `n<0` or `n`≥`w` (`w` is bit width of `x`) are *undefined*
  - **In C:** behavior of `>>` is determined by the compiler
    - In gcc / C lang, depends on data type of `x` (signed/unsigned)
  - **In Java:** logical shift is `>>>` and arithmetic shift is `>>`

Left Shifting Arithmetic 8-bit Example

- No difference in left shift operation for unsigned and signed numbers (just manipulates bits)
  - Difference comes during interpretation: `x*2`^{n}?

			Signed	Unsigned
`x = 25;`	`00011001`	=	25	25
`L1=x<<2;`	`0001100100`	=	100	100
`L2=x<<3;`	`00011001000`	=	-56	200
`L3=x<<4;`	`000110010000`	=	-112	144

shifting causes sign bit to change (signed overflow)

signed overflow

shifting causes bits to "fall off" the end (unsigned overflow)

unsigned overflow

Right Shifting Arithmetic 8-bit Examples

- **Reminder:** C operator >> does *logical* shift on unsigned values and *arithmetic* shift on signed values
  - Logical Shift: `x/2`^{n}?

```
xu = 240u;    11110000          = 240
R1u=xu>>3;    00011110000       =  30
R2u=xu>>5;    0000011110000     =   7
```

round down b/c
LSB is lost

rounding (down)

Right Shifting Arithmetic 8-bit Examples

- **Reminder:** C operator >> does *logical* shift on unsigned values and *arithmetic* shift on signed values
  - Arithmetic Shift: `x/2`^{n}?

```
xs  = -16;   11110000          = -16
R1s=xu>>3;   11111110000       =  -2
R2s=xu>>5;   1111111110000     =  -1
```

rounding (down)

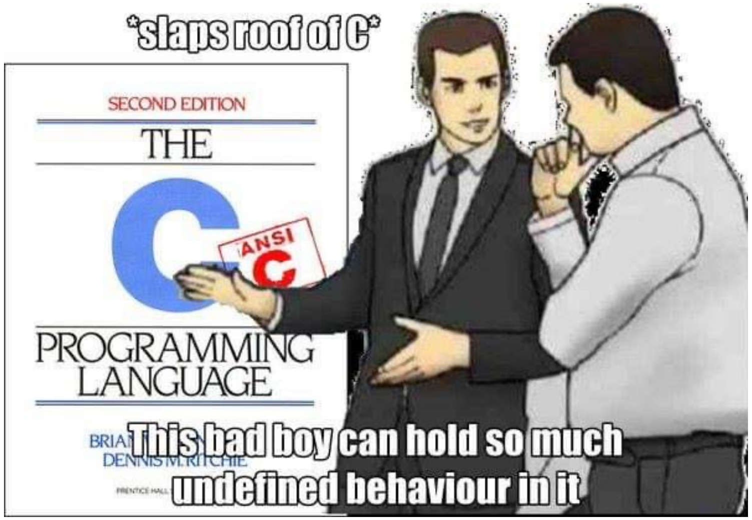
NB: rounding down means toward the negative, not the same as rounding toward 0

Summary

- Sign and unsigned variables in C
  - Bit pattern remains the same, just *interpreted* differently
  - Strange things can happen with our arithmetic when we convert/cast between sign and unsigned numbers
    - Type of variables affects behavior of operators (shifting, comparison)
- We can only represent so many numbers in *w* bits
  - When we exceed the limits, *arithmetic overflow* occurs
  - *Sign extension* tries to preserve value when expanding
- Shifting is a useful bitwise operator
  - Right shifting can be arithmetic (sign) or logical (0)
  - Can be used in multiplication with constant or bit masking

Undefined Behavior in C

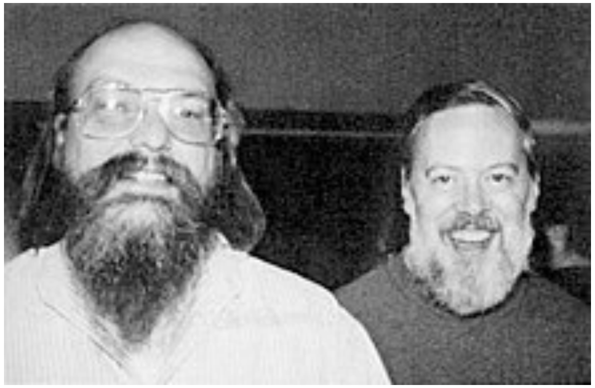
- How much **undefined behavior** have we talked about in just the past 5 lectures?
  - Integer overflow
  - Pointers, pointers, pointers
  - Mystery data in unassigned variables
  - …and there will be more (I promise)

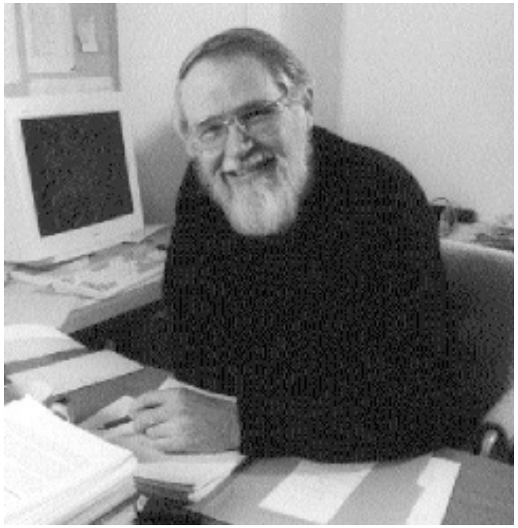


What does this tell us about the values that were embedded in C?

C language (1978)

- Created in 1972, "standardized" in 1978
  - Goal of writing Unix (precursor to Linux, macOS and others)
  - Different time, greater performance and resource limits
- Explicit Goals:
  - Portability, performance (better than B, it's C!)





Principles of C, viewed today

- *“Since C is relatively small, it can be described in small space, and learned quickly.”*
- "Only the bare essentials”
- “Close to the *hardware*”
- “Shows what’s *really happening*”
- “No one to help you”
- “You’re on your own”
- “I know what I’m doing, get out of my way”

Principles of C, viewed today

- **Minimalist**:
  - *"Since C is relatively small, it can be described in small space, and learned quickly."*
  - "Only the bare essentials"
- **Rugged**:
  - "Close to the *hardware*"
  - "Shows what's *really happening*"
- **Individualistic**
  - "No one to help you"
  - "You're on your own"
  - "I know what I'm doing, get out of my way"

Minimalism, Rugged, Individualistic... Pioneers!

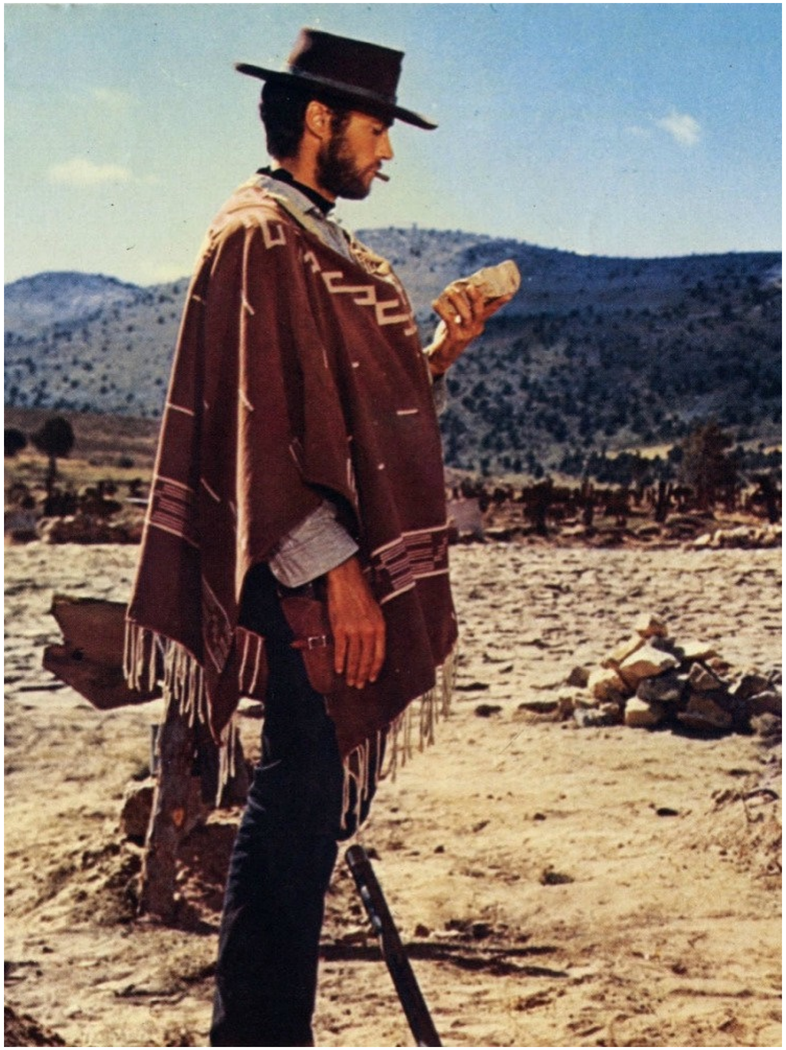
Pioneers in the Wild West

- American Frontierism (~1800 – 1890)
  - Vast expansion westward, from original 13 colonies to Pacific Ocean
- Manifest Destiny
  - Burgeoning theory that White Americans were "destined" to connect from coast to coast
  - Cultural phenomenon, Indigenous genocide

Manifest Destiny

John Gast, *American Progress*, 1872

Immortalized in Popular Culture



Replicated in Computing Culture

Takeaways

- C: **Minimalistic, Rugged, Individualistic**
  - Embodied what was culturally valued at the time!
  - Frontierism! Moon landing was 1969!
- Explore the digital frontier!
  - Only carry the essentials!
  - American frontierism!
  - Manifest destiny (1800 – 1890), genocide
  - Glorified in popular culture: westerns, video games
- K&R didn't mean to do harm!
  - But they didn't question the values glorified by society
  - And C's laissez-faire attitude **has** caused harm…

"C is good for two things: being beautiful and creating catastrophic 0days in memory management."

https://medium.com/message/everything-is-broken-81e5f33a24e1

Ideology: You don’t even need to ask

Contrast with: “best”, “better”, “more important”

"We shape our tools, and thereafter, our tools shape us"

1967

"Reification", if you want a single word. To make the abstract concrete.

Computing is a tool, but a tool built by a distinctly non-neutral society!
We've always had values!

C is like camping!

BONUS SLIDES

Some examples of using shift operators in combination with bitmasks, which you may find helpful for Lab 1b.

- Extract the 2nd most significant byte of an `int`
- Extract the sign bit of a signed `int`
- Conditionals as Boolean expressions

Practice Question 1

- Assuming 8-bit data (*i.e.*, bit position 7 is the MSB), what will the following expression evaluate to?
  - UMin = 0, UMax = 255, TMin = -128, TMax = 127

signed

signed

- `127 < (signed char) 128u`

← both sides are signed, so signed comparison

0b0111 1111

0b1000 0000

signed comparison:

0b0111 1111 0b1000 0000

127 < -128

$\boxed{\text{False}}$

unsigned comparison:

127 < 128

True

(e.g., if LHS was 127u)

Practice Questions 2

[Umin, Umax] = [0, 255]
[Tmin, Tmax] = [-128, 127]

- Assuming 8-bit integers:
  - `0x27` = 39 (signed) = 39 (unsigned)
  - `0xD9` = -39 (signed) = 217 (unsigned)
  - `0x7F` = 127 (signed) = 127 (unsigned)
  - `0x81` = -127 (signed) = 129 (unsigned)

- For the following additions, did signed and/or unsigned overflow occur?
  - **`0x27 + 0x81`**
    signed: 39 + -127 = -88 ✓ no overflow
    unsigned: 39 + 129 = 168 ✓ no overflow
  - **`0x7F + 0xD9`**
    signed: 127 + -39 = 88 ✓ no overflow
    unsigned: 127 + 217 = 344 × overflow (> Umax)

Exploration Questions

UMin = 0 , UMax = 255
8-bits, so TMin = -128, TMax = 127

For the following expressions, find a value of `signed char x`, if there exists one, that makes the expression True.

- Assume we are using 8-bit arithmetic:

Expression	Example:	All solutions:
`x == (unsigned char) x` (unsigned)	x = 0	works for all x
`x >= 128U` (unsigned) 0b1000 0000	x = -1	any x < 0
`x != (x>>2)<<2`	x = 3	any x where lowest two bits are not 0b00
`x == -x` • Hint: there are two solutions	x = 0	① x = 0b0...0 = 0 ② x = 0b10...0 = -128
`(x < 128U) && (x > 0x3F)`		any x where upper two bits are exactly 0b01

Using Shifts and Masks

- Extract the 2nd most significant *byte* of an `int`:
  - First shift, then mask: `(x>>16) & 0xFF`

x	00000001 00000010 00000011 00000100
x>>16	00000000 00000000 00000001 00000010
0xFF	00000000 00000000 00000000 11111111
(x>>16) & 0xFF	00000000 00000000 00000000 00000010

  - Or first mask, then shift: `(x & 0xFF0000)>>16`

x	00000001 00000010 00000011 00000100
0xFF0000	00000000 11111111 00000000 00000000
x & 0xFF0000	00000000 00000010 00000000 00000000
(x&0xFF0000)>>16	00000000 00000000 00000000 00000010

Using Shifts and Masks

- Extract the *sign bit* of a signed `int`:
  - First shift, then mask: `(x>>31) & 0x1`
    - Assuming arithmetic shift here, but this works in either case
    - Need mask to clear `1`s possibly shifted in

x	**0**0000001 00000010 00000011 00000100
x>>31	00000000 00000000 00000000 0000000**0**
0x1	00000000 00000000 00000000 00000001
(x>>31) & 0x1	00000000 00000000 00000000 00000000

x	**1**0000001 00000010 00000011 00000100
x>>31	11111111 11111111 11111111 1111111**1**
0x1	00000000 00000000 00000000 00000001
(x>>31) & 0x1	00000000 00000000 00000000 00000001

Using Shifts and Masks

- Conditionals as Boolean expressions
  - For **`int`** `x`, what does `(x<<31)>>31` do?

`x=!!123`	`00000000 00000000 00000000 00000001`
`x<<31`	`10000000 00000000 00000000 00000000`
`(x<<31)>>31`	`11111111 11111111 11111111 11111111`
`!x`	`00000000 00000000 00000000 00000000`
`!x<<31`	`00000000 00000000 00000000 00000000`
`(!x<<31)>>31`	`00000000 00000000 00000000 00000000`

  - Can use in place of conditional:
    - In C: `if(x) {a=y;} else {a=z;}` equivalent to `a=x?y:z;`
    - `a=(((!!x<<31)>>31)&y) | (((!x<<31)>>31)&z);`