Data III & Integers I

CSE 351 Winter 2022

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Relevant Course Information

- Memory & Data I homework due tonight
  Memory & Data II homework due Wednesday
- Lab 1a released
  - Workflow:
    1) Edit `pointer.c`
    2) Run the Makefile (`make clean` followed by `make`) and check for compiler errors & warnings
    3) Run ptest (`./ptest`) and check for correct behavior
    4) Run rule/syntax checker (`python3 dlc.py`) and check output
  - Due Wednesday 1/19, will overlap a bit with Lab 1b
    - We grade just your *last* submission
    - Don't wait until the last minute to submit – the auto-grader may take a few minutes to run, and will identify basic issues

Runnable Code Snippets on Ed

- Ed allows you to embed runnable code snippets (*e.g.*, readings, homework, discussion)
  - These are *editable* and *rerunnable*!
  - Hide compiler warnings, but will show compiler errors and runtime errors

- Suggested use
  - Good for experimental questions about basic behaviors in C
  - *NOT* entirely consistent with the CSE Linux environment, so should not be used for any lab-related work

Lab Synthesis Questions

- All subsequent labs (after Lab 0) have a "synthesis question" portion
  - Can be found on the lab specs and are intended to be done *after* you finish the lab
  - You will type up your responses in a `.txt` file for submission on Gradescope
  - These will be graded "by hand" (read by TAs)

- Intended to check your understanding of what you should have learned from the lab
  - Also great practice for short answer questions on the exams

Reading Review

- Terminology:
  - Bitwise operators (&, |, ^, ~)
  - Logical operators (&&, ||, !)
  - Short-circuit evaluation
  - Unsigned integers
  - Signed integers (Two's Complement)

- Questions from the Reading?

Review Questions

- Compute the result of the following expressions for `char c = 0x81;`
  - `c ^ c` 0b10000001 ^ 0b10000001 ⇒ 0x00 (for any number n, n ^ n = 0)
  - `~c & 0xA9` ~0b10000001 & 0b10101001 = 0b01111110 & 0b10101001 = 0b00101000 ⇒ 0x28
  - `c || 0x80` ⇒ True (True || True)
    - logical op, so output is True (0x1) or false (0x0)
  - `!!c`
    - c is True (non-zero)
    - !c is False
    - !!c is True (0x1)
- Compute the value of `signed char sc = 0xF0;` (Two's Complement)

0xF0 = 0b11110000 (MSB has neg weight)

$-2^7 + 2^6 + 2^5 + 2^4 = -16$

Or...

$-sc = \sim sc + 1$

$-sc = 0b00001111 + 1$

$-sc = 0b00010000$

$-sc = 16$

$\Rightarrow sc = -16$

Bitmasks

- Typically, binary bitwise operators (&, |, ^) are used with one operand being the "input" and other operand being a specially-chosen bitmask (or *mask*) that performs a desired operation

- Operations for a bit b (answer with 0, 1, b, or $\bar{b}$):

$b \;\&\; 0 =$ 0 "set to 0" $\quad b \;\&\; 1 =$ b "keep the same"

$b \mid 0 =$ b "keep the same" $\quad b \mid 1 =$ 1 "set to 1"

$b \;\hat{}\; 0 =$ b "keep the same" $\quad b \;\hat{}\; 1 =$ $\bar{b}$ "flip"

Bitmasks

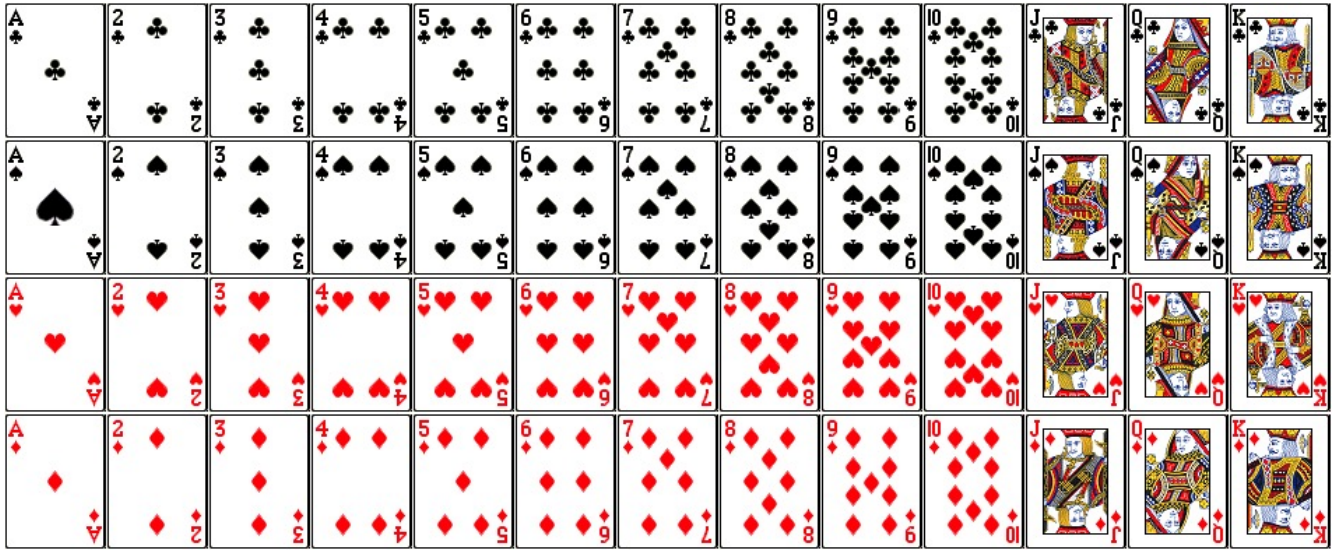
- Typically, binary bitwise operators (&, |, ^) are used with one operand being the "input" and other operand being a specially-chosen bitmask (or *mask*) that performs a desired operation

- Example: $b|0 = b,\ b|1 = 1$

```
  01010101   ← input
| 11110000   ← bitmask
----------
  11110101
```

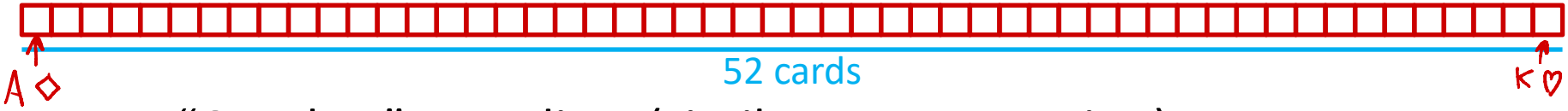
Numerical Encoding Design Example

- Encode a standard deck of playing cards
  - 52 cards in 4 suits
- Operations to implement:
  - Which is the higher value card?
  - Are they the same suit?



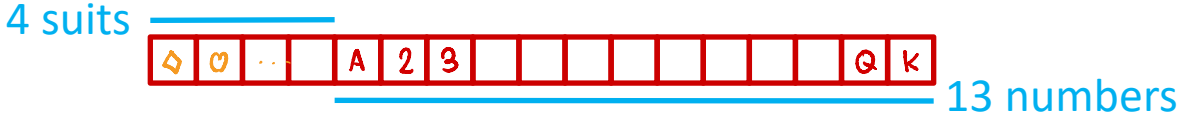
Representations and Fields

1) 1 bit per card (52): bit corresponding to card set to 1



- "One-hot" encoding (similar to set notation)
- Drawbacks:
  - Hard to compare values and suits
  - Large number of bits required

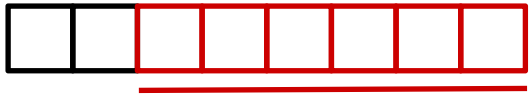
2) 1 bit per suit (4), 1 bit per number (13): 2 bits set



- Pair of one-hot encoded values
- Easier to compare suits and values, but still lots of bits used

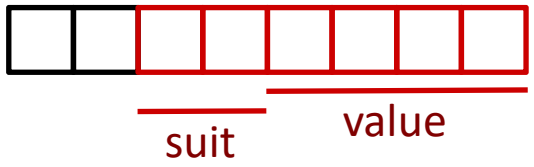
Representations and Fields

3) Binary encoding of all 52 cards – only 6 bits needed
   - $2^6 = 64 \geq 52$



   - Fits in one byte (smaller than one-hot encodings)
   - How can we make value and suit comparisons easier?

4) Separate binary encodings of suit (2 bits) and value (4 bits)



   - Also fits in one byte, and easy to do comparisons

K	Q	J	. . .	3	2	A
1101	1100	1011	. . .	0011	0010	0001

♣	00
♦	01
♥	10
♠	11

Compare Card Suits

mask: a bit vector designed to achieve a desired behavior when used with a bitwise operator on another bit vector *v*.
Here we turn all *but* the bits of interest in *v* to 0.

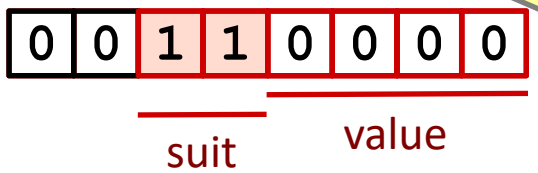
```
char hand[5];          // represents a 5-card hand
char card1, card2;     // two cards to compare
card1 = hand[0];
card2 = hand[1];
...
if ( sameSuitP(card1, card2) ) { ... }
```

```
#define SUIT_MASK   0x30

int sameSuitP(char card1, char card2) {
  return (!((card1 & SUIT_MASK) ^ (card2 & SUIT_MASK)));
  //return (card1 & SUIT_MASK) == (card2 & SUIT_MASK);
}
```

returns **int**

SUIT_MASK = 0x30 =



equivalent

Compare Card Suits

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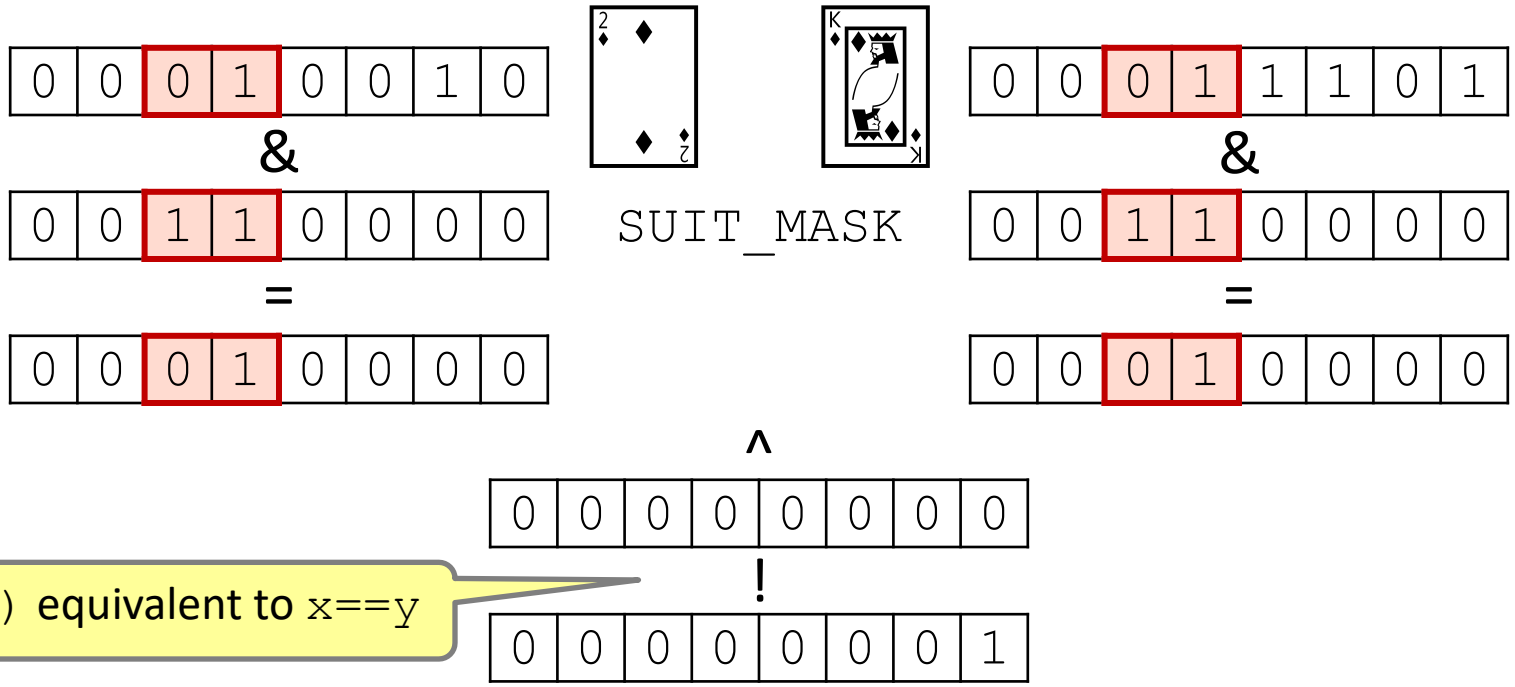
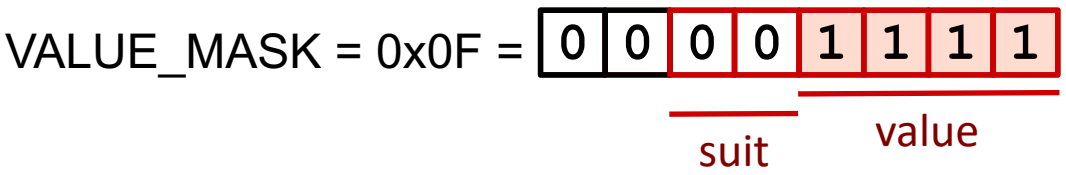


Compare Card Values

```
char hand[5];          // represents a 5-card hand
char card1, card2;     // two cards to compare
card1 = hand[0];
card2 = hand[1];
...
if ( greaterValue(card1, card2) ) { ... }
```

```
#define VALUE_MASK   0x0F

int greaterValue(char card1, char card2) {
  return ((unsigned int)(card1 & VALUE_MASK) >
          (unsigned int)(card2 & VALUE_MASK));
}
```



Compare Card Values

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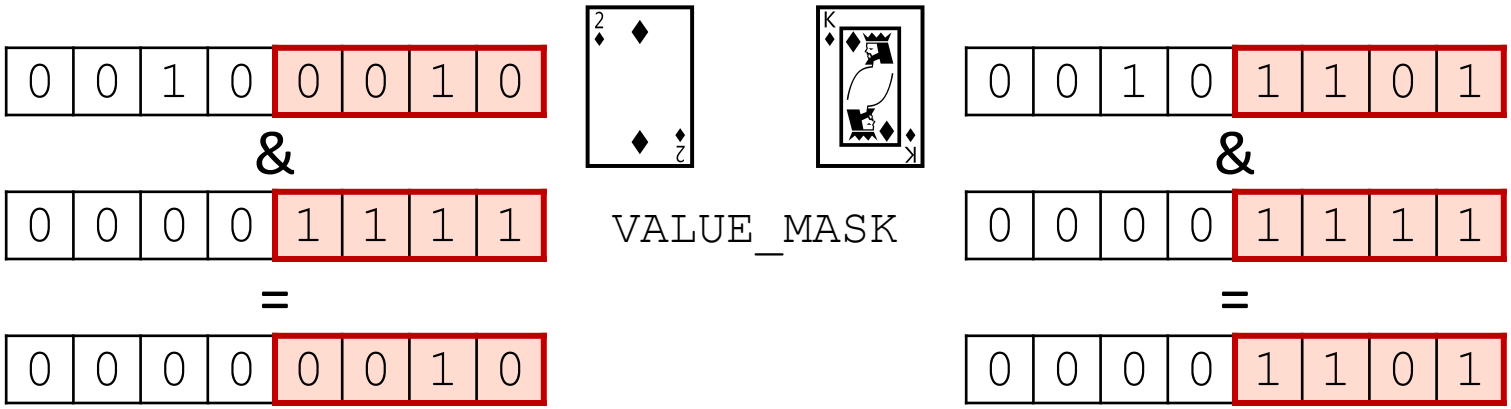
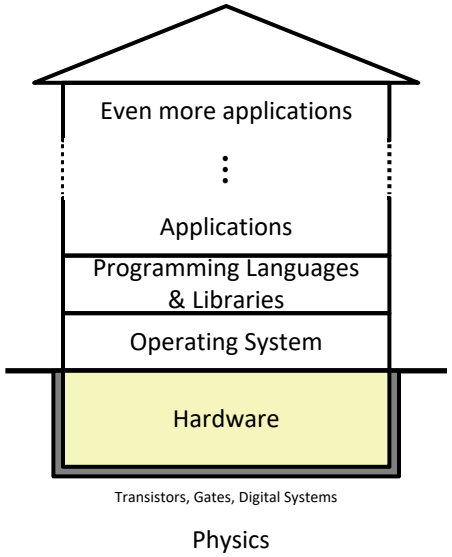


$2_{10} > 13_{10}$

0 (false)

The Hardware/Software Interface

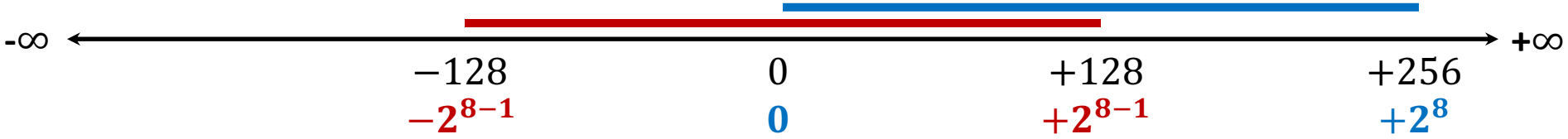
- Topic Group 1: **Data**
  - Memory, Data, **Integers**, Floating Point, Arrays, Structs



- How do we store information for other parts of the house of computing to access?
  - How do we represent data and what limitations exist?
  - What design decisions and priorities went into these encodings?

Encoding Integers

- The hardware (and C) supports two flavors of integers
  - *unsigned* – only the non-negatives
  - *signed* – both negatives and non-negatives
- Cannot represent all integers with w bits
  - Only 2^w distinct bit patterns
  - Unsigned values: $0 \ldots 2^w - 1$
  - Signed values: $-2^{w-1} \ldots 2^{w-1} - 1$
- **Example:** 8-bit integers (*e.g.*, `char`)



Unsigned Integers (Review)

- Unsigned values follow the standard base 2 system
  - $b_7b_6b_5b_4b_3b_2b_1b_0 = b_7 2^7 + b_6 2^6 + \cdots + b_1 2^1 + b_0 2^0$

- Useful formula: $2^{N-1} + 2^{N-2} + \ldots + 2 + 1 = 2^N - 1$
  - *i.e.*, N ones in a row = $2^N - 1$
  - *e.g.*, 0b00111111 = 63

why does this work?

adding 1 will cause cascading carry up to the next power of 2:

0b00111111 = 0b01000000

= 64

Sign and Magnitude

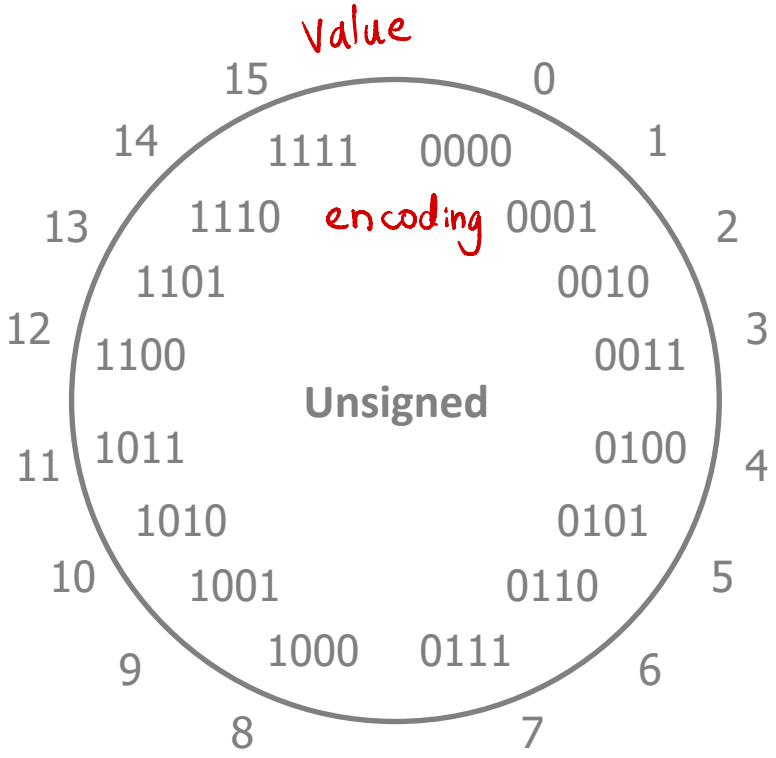
Not used in practice for integers!

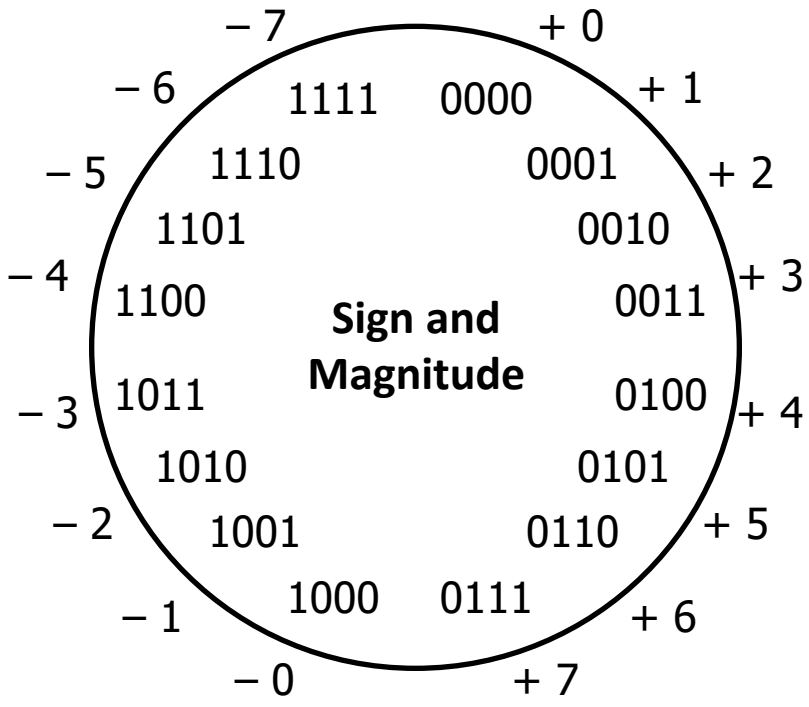
- Designate the high-order bit (MSB) as the "sign bit"
  - `sign=0`: positive numbers; `sign=1`: negative numbers
- Benefits:
  - Using MSB as sign bit matches positive numbers with unsigned
  - All zeros encoding is still = 0
- Examples (8 bits):
  - 0x00 = 00000000_2 is non-negative, because the sign bit is 0
  - 0x7F = 01111111_2 is non-negative ($+127_{10}$)
  - 0x85 = 10000101_2 is negative (-5_{10})
  - 0x80 = 10000000_2 is negative… zero???

Sign and Magnitude

Not used in practice for integers!

- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks?

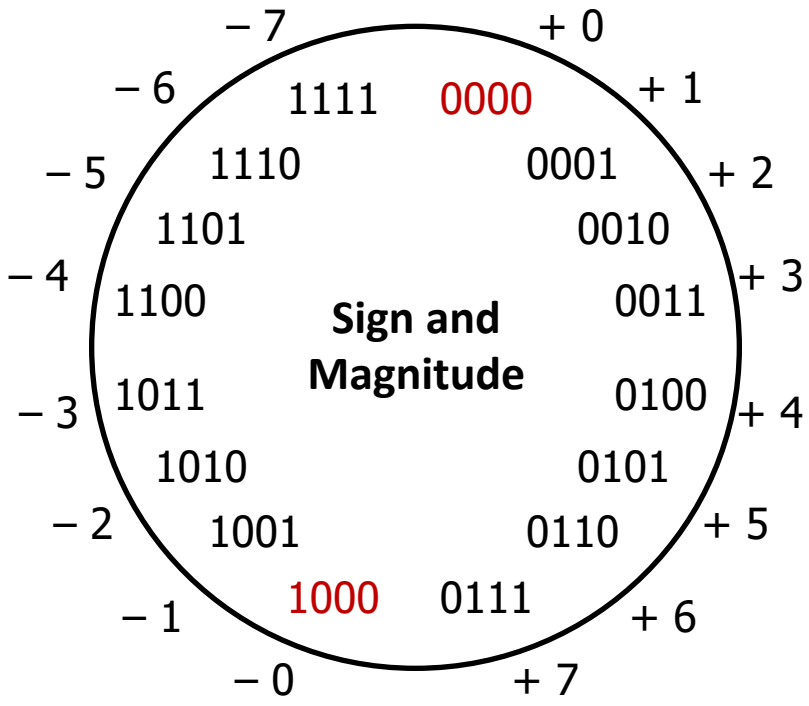




Sign and Magnitude

Not used in practice for integers!

- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks:
  - Two representations of 0 (bad for checking equality)



Sign and Magnitude

Not used in practice for integers!

- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks:
  - Two representations of 0 (bad for checking equality)
  - Arithmetic is cumbersome
    - Example: `4-3 != 4+(-3)`

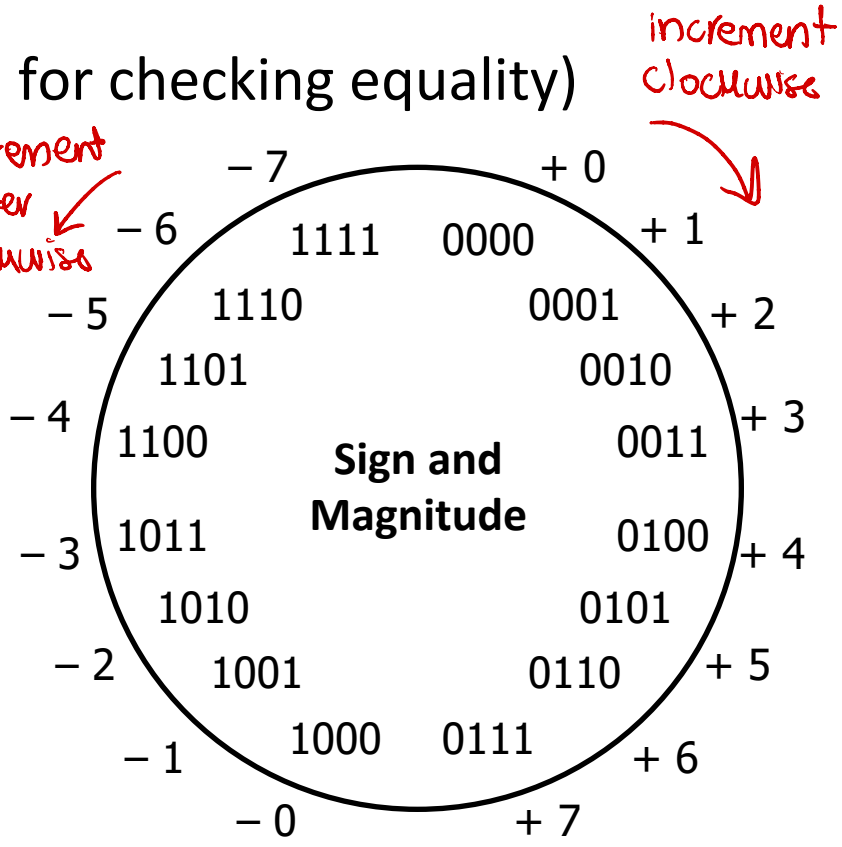
4	0100
− 3	− 0011
1	0001

✓

4	0100
+ −3	+ 1011
−7	1111

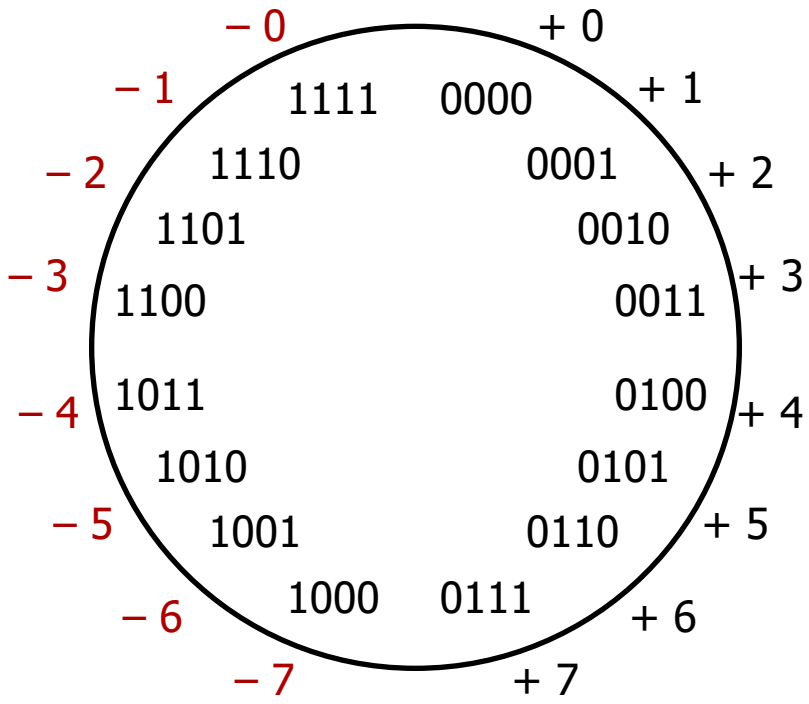
✗

- Negatives "increment" in wrong direction!



Two's Complement

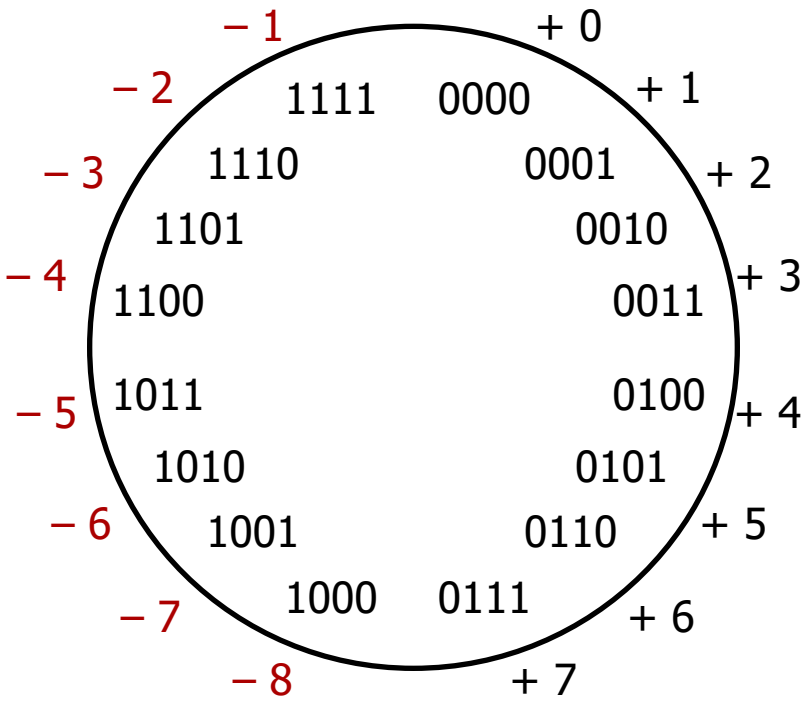
- Let's fix these problems:
  1) "Flip" negative encodings so incrementing works



Two's Complement

- Let's fix these problems:
  1) "Flip" negative encodings so incrementing works
  2) "Shift" negative numbers to eliminate –0

- MSB *still* indicates sign!
  - This is why we can represent one more negative than positive number (-2^{N-1} to $2^{N-1}-1$)



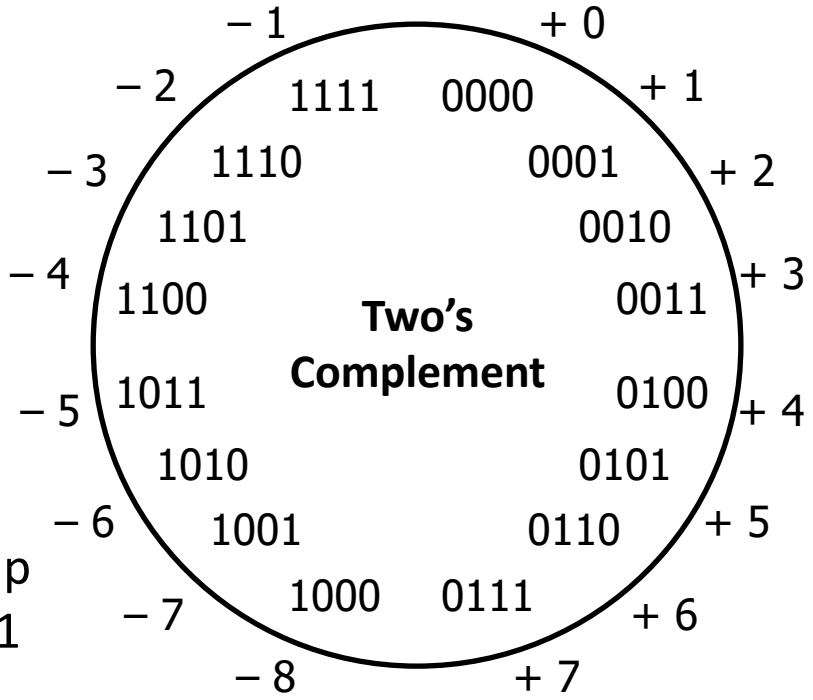
Two's Complement Negatives (Review)

- Accomplished with one neat mathematical trick!

b_{w-1} has weight -2^{w-1}, other bits have usual weights $+2^i$

- 4-bit Examples:
  - 1010_2 unsigned:
    $1*2^3+0*2^2+1*2^1+0*2^0$ = **10**
  - 1010_2 two's complement:
    $-1*2^3+0*2^2+1*2^1+0*2^0$ = **–6**

- -1 represented as:
  $1111_2 = -2^3+(2^3-1)$
  - MSB makes it super negative, add up all the other bits to get back up to -1



Polling Question

- Take the 4-bit number encoding **`x = 0b1011`**
- Which of the following numbers is NOT a valid interpretation of x using any of the number representation schemes discussed today?
  - Unsigned, Sign and Magnitude, Two's Complement
  - Vote at: PollEv.com/wolfson

A. -4

B. -5 2's complement: $-(2^3) + 2^1 + 2^0 = -5$

C. 11 unsigned: $2^3 + 2^1 + 2^0 = 11$

D. -3 Sign + mag: $-(2^1 + 2^0) = -3$

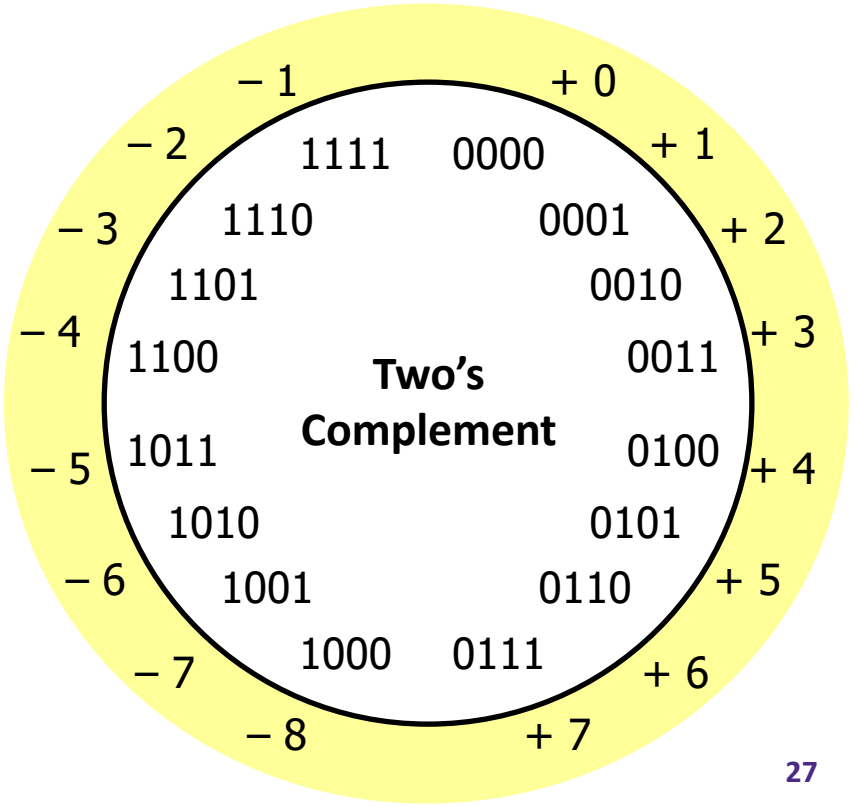
E. We're lost…

Two's Complement is Great (Review)

- ✓ Roughly same number of (+) and (–) numbers
- ✓ Positive number encodings match unsigned
- ✓ Single zero
- ✓ All zeros encoding = 0

- Simple negation procedure:
  - Get negative representation of any integer by taking bitwise complement and then adding one!

```
( ~x + 1 == -x )
```



Integer Representation Design Decisions

- Some explicitly mentioned and some implicit:
  - Represent consecutive integers
    - What would happen if we had gaps?
  - Represent the same number of positives and negatives
    - Could we choose a different shift/bias?
  - Positive number encodings match unsigned
    - What's the advantage here?

Summary

- Bit-level operators allow for fine-grained manipulations of data
  - Bitwise AND (&), OR (|), and NOT (~) different than logical AND (&&), OR (||), and NOT (!)
  - Especially useful with bit masks
- Choice of *encoding scheme* is important
  - Tradeoffs based on size requirements and desired operations
- Integers represented using unsigned and two's complement representations
  - Limited by fixed bit width
  - We'll examine arithmetic operations next lecture