Data III & Integers I

CSE 351 Spring 2022

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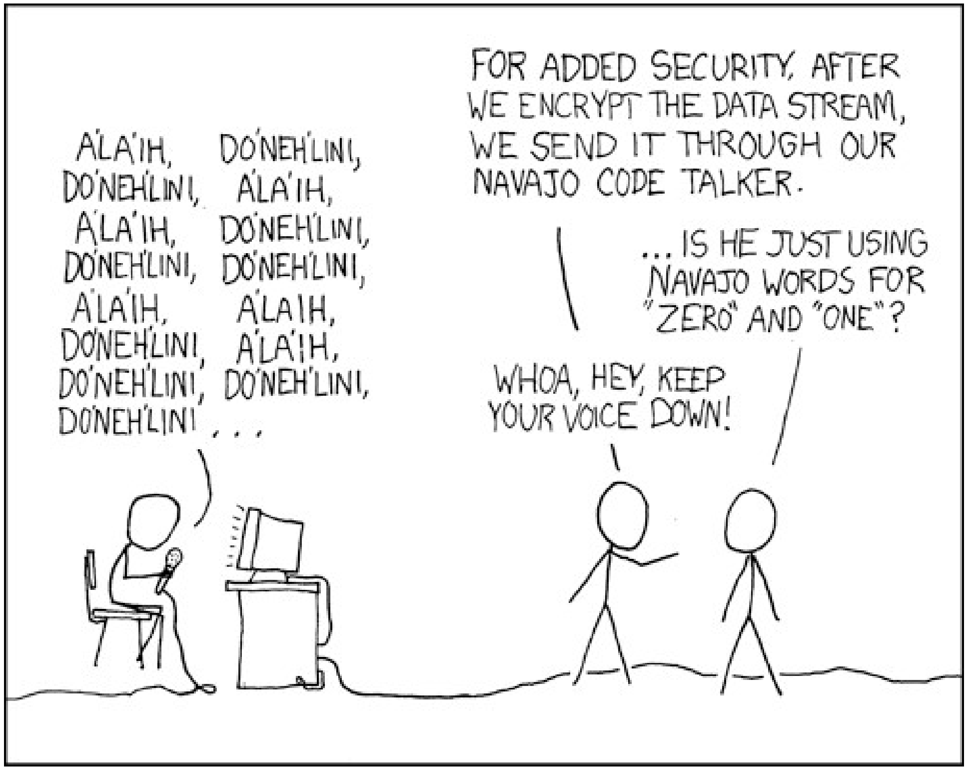


http://xkcd.com/257/

Relevant Course Information

- Lab 0 and hw2 due tonight (4/04) @ 11:59 pm
- hw3 due Wednesday (4/06) @ 11:59 pm
- hw4 due Friday (4/08) @ 11:59 pm
- From here on out, at 11am on day of lecture:
  - Reading for that lecture is DUE at 11am
  - Lecture activities from the previous lecture are DUE at 11am

Lab 1a released!

- Labs can be found linked on our course home page:
  - https://courses.cs.washington.edu/courses/cse351/21sp/labs/lab1a.php
- Workflow:
  1) Edit `pointer.c`
  2) Run the Makefile (`make clean` followed by `make`) and check for compiler errors & warnings
  3) Run ptest (`./ptest`) and check for correct behavior
  4) Run rule/syntax checker (`python3 dlc.py`) and check output
- Due Monday 4/11, will overlap a bit with Lab 1b
  - Submit in Gradescope - we grade just your *last* submission
  - Don't wait until the last minute to submit! Check autograder output!

Lab Synthesis Questions

- All subsequent labs (after Lab 0) have a "synthesis question" portion
  - Can be found on the lab specs and are intended to be done *after* you finish the lab
  - You will type up your responses in a `.txt` file for submission on Gradescope
  - These will be graded "by hand" (read by TAs)

- Intended to check your understanding of what you should have learned from the lab
  - Also great practice for short answer questions on the exams

Memory, Data, and Addressing

- Representing information as bits and bytes
  - Binary, hexadecimal, fixed-widths
- Organizing and addressing data in memory
  - Memory is a byte-addressable array
  - Machine "word" size = address size = register size
  - Endianness – ordering bytes in memory
- Manipulating data in memory using C
  - Assignment
  - Pointers, pointer arithmetic, and arrays
- **Boolean algebra and bit-level manipulations**

Reading Review

- Terminology:
  - Bitwise operators (&, |, ^, ~)
  - Logical operators (&&, ||, !)
  - Short-circuit evaluation
  - Unsigned integers
  - Signed integers (Two's Complement)

Review Questions

- Compute the result of the following expressions for `char c = 0x81;`
  - `c ^ c`
  - `~c & 0xA9`
  - `c || 0x80`
  - `!!c`
- Compute the value of `signed char sc = 0xF0;` (Two's Complement)

Bitmasks

(Input) op (Bitmask) → Result

- Typically binary bitwise operators (&, |, ^) are used with one operand being the "input" and other operand being a specially-chosen bitmask (or *mask*) that performs a desired operation

- Operations for a bit b (answer with 0, 1, b, or $\bar{b}$):

$b \,\&\, 0 =$ 0 "set to zero" $\qquad b \,\&\, 1 =$ b "keep as it is"

$b \mid 0 =$ b "keep as is" $\qquad b \mid 1 =$ 1 "set as one"

$b \wedge 0 =$ b $\qquad b \wedge 1 =$ $\bar{b}$ "flip"

Bitmasks

- Typically binary bitwise operators (&, |, ^) are used with one operand being the "input" and other operand being a specially-chosen bitmask (or *mask*) that performs a desired operation

- Example: $b|0 = b,\ b|1 = 1$

```
   01010101  ← input
 | 11110000  ← bitmask
   11110101  ← output
```

"set to one" "keep as is"

Short-Circuit Evaluation

- If the result of a binary logical operator (&&, ||) can be determined by its first operand, then the second operand is never evaluated
  - Also known as *early termination*
- Example: (p && *p) for a pointer p to "protect" the dereference
  - Dereferencing NULL (0) results in a segfault

Roadmap

C:

```
car *c = malloc(sizeof(car));
c->miles = 100;
c->gals = 17;
float mpg = get_mpg(c);
free(c);
```

Java:

```
Car c = new Car();
c.setMiles(100);
c.setGals(17);
float mpg =
    c.getMPG();
```

Memory & data
Integers & floats
x86 assembly
Procedures & stacks
Executables
Arrays & structs
Memory & caches
Processes
Virtual memory
Memory allocation
Java vs. C

Assembly language:

```
get_mpg:
    pushq   %rbp
    movq    %rsp, %rbp
    ...
    popq    %rbp
    ret
```

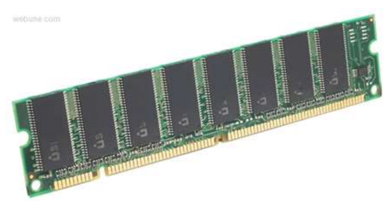
Machine code:

```
0111010000011000
100011010000010000000010
1000100111000010
110000011111101000011111
```

OS:

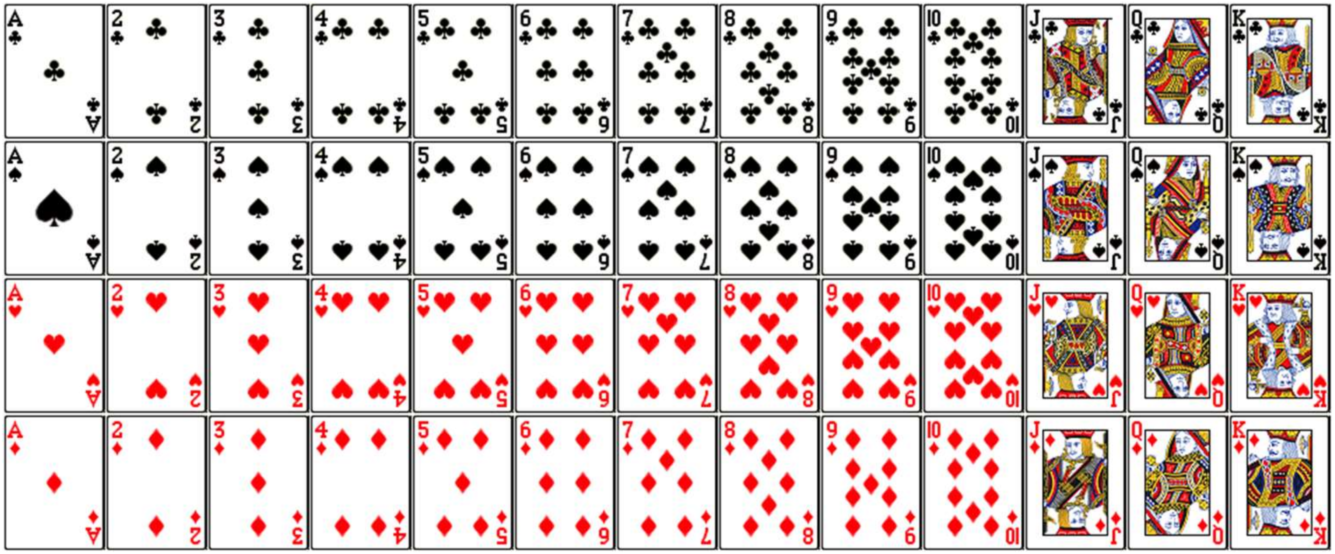
Windows 10
OS X Yosemite

Computer system:



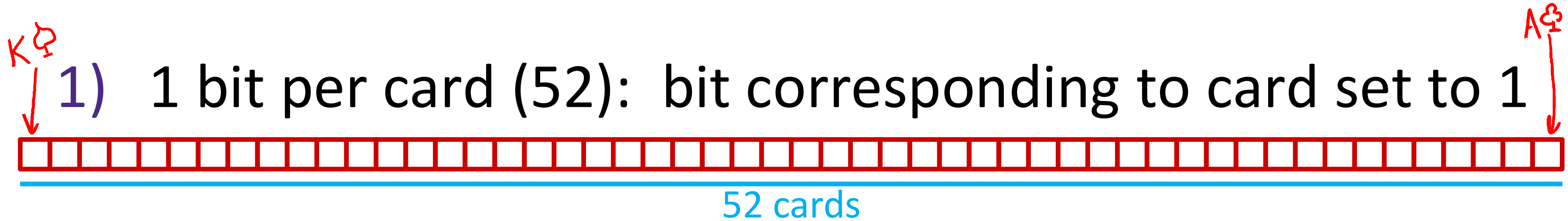
Numerical Encoding Design Example

- Encode a standard deck of playing cards
- 52 cards in 4 suits
  - How do we encode suits, face cards?
- What operations do we want to make easy to implement?
  - Which is the higher value card?
  - Are they the same suit?



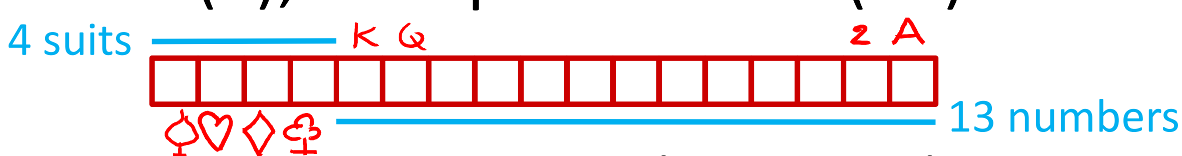
Two possible representations

1) 1 bit per card (52): bit corresponding to card set to 1



- "One-hot" encoding (similar to set notation)
- Drawbacks:
  - Hard to compare values and suits
  - Large number of bits required 52 bits fits in → 7 bytes (56 bits)

2) 1 bit per suit (4), 1 bit per number (13): 2 bits set



- **Pair** of one-hot encoded values (two fields)
- Easier to compare suits and values, but still lots of bits used

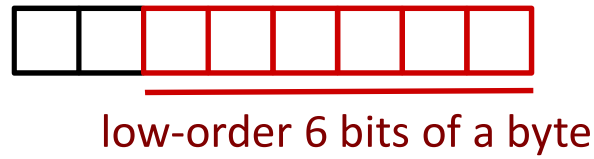
17 bits → 3 bytes

Two better representations

3) Binary encoding of all 52 cards – only 6 bits needed

- $2^6 = 64 \geq 52$

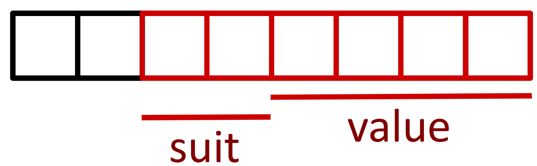
  $2^5 = 32 < 52$



- Fits in one byte (smaller than one-hot encodings)
- How can we make value and suit comparisons easier?

4) Separate binary encodings of suit (2 bits) and value (4 bits)

4 suits — 13 values



- Also fits in one byte, and easy to do comparisons

C	♣	00
D	♦	01
H	♥	10
S	♠	11

K	Q	J	. . .	3	2	A
1101	1100	1011	. . .	0011	0010	0001
13			. . .			1

Compare Card **Suits**

mask: a bit vector designed to achieve a desired behavior when used with a bitwise operator on another bit vector *v*.
Here we turn all *but* the bits of interest in *v* to 0.

```
char hand[5];         // represents a 5-card hand
char card1, card2;    // two cards to compare
card1 = hand[0];
card2 = hand[1];
...
if ( sameSuitP(card1, card2) ) { ... }
```

text substitution

```
#define SUIT_MASK  0x30

int sameSuitP(char card1, char card2) {
  return (!((card1 & SUIT_MASK) ^ (card2 & SUIT_MASK)));
  //return (card1 & SUIT_MASK) == (card2 & SUIT_MASK);
}
```

returns **int**

equivalent

SUIT_MASK = 0x30 =

0	0	1	1	0	0	0	0

suit (keep)   value (discard)

x & 0 = 0

x & 1 = x

Compare Card Suits

```
#define SUIT_MASK  0x30

int sameSuitP(char card1, char card2) {
  return (!((card1 & SUIT_MASK) ^ (card2 & SUIT_MASK)));
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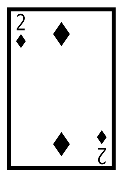
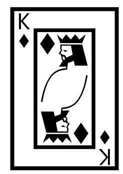
}
```





card1

0	0	0	1	0	0	1	0

&

0	0	1	1	0	0	0	0

=

0	0	0	1	0	0	0	0

card2

0	0	0	1	1	1	0	1

&

0	0	1	1	0	0	0	0

=

0	0	0	1	0	0	0	0

SUIT_MASK

x & 0 = 0

x & 1 = x

^

0	0	0	0	0	0	0	0

! ← logical

0	0	0	0	0	0	0	1

`!(x^y)` equivalent to `x==y`

Compare Card Values

```
char hand[5];         // represents a 5-card hand
char card1, card2;    // two cards to compare
card1 = hand[0];
card2 = hand[1];
...
if ( greaterValue(card1, card2) ) { ... }
```

```
#define VALUE_MASK  0x0F

int greaterValue(char card1, char card2) {
  return ((unsigned int)(card1 & VALUE_MASK) >
          (unsigned int)(card2 & VALUE_MASK));
}
```

VALUE_MASK = 0x0F =

0	0	0	0	1	1	1	1

suit (discard) value (keep)

Compare Card Values

```
#define VALUE_MASK  0x0F

int greaterValue(char card1, char card2) {
  return ((unsigned int)(card1 & VALUE_MASK) >
          (unsigned int)(card2 & VALUE_MASK));
}
```

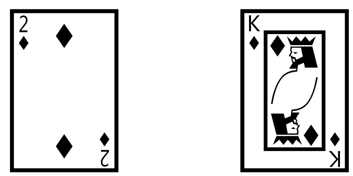
0	0	1	0	0	0	1	0

&

0	0	0	0	1	1	1	1

=

0	0	0	0	0	0	1	0



VALUE_MASK

0	0	1	0	1	1	0	1

&

0	0	0	0	1	1	1	1

=

0	0	0	0	1	1	0	1

$2_{10} > 13_{10}$

0 (false)

Roadmap

C:

```
car *c = malloc(sizeof(car));
c->miles = 100;
c->gals = 17;
float mpg = get_mpg(c);
free(c);
```

Java:

```
Car c = new Car();
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float mpg =
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Assembly language:

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get_mpg:
    pushq   %rbp
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Machine code:

```
0111010000011000
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110000011111101000011111
```

OS:

Windows 10

OS X Yosemite

Computer system:

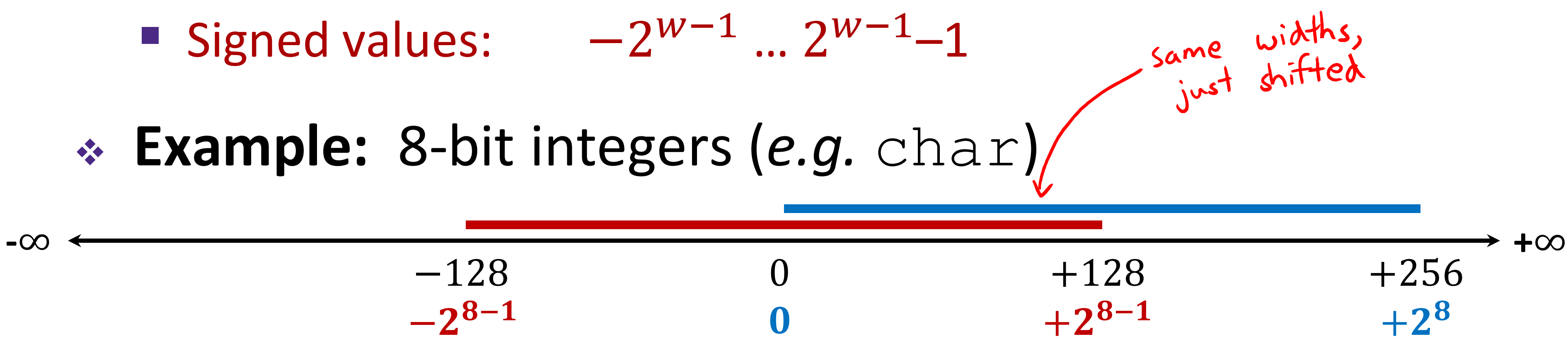
- Memory & data
- **Integers & floats**
- x86 assembly
- Procedures & stacks
- Executables
- Arrays & structs
- Memory & caches
- Processes
- Virtual memory
- Memory allocation
- Java vs. C

Integers

- **Binary representation of integers**
  - **Unsigned and signed**
- Shifting and arithmetic operations
- In C: Signed, Unsigned and Casting
- Consequences of finite width representations
  - Overflow, sign extension

Encoding Integers

- The hardware (and C) supports two flavors of integers
  - *unsigned* – only the non-negatives
  - *signed* – both negatives and non-negatives
- Cannot represent all integers with w bits
  - Only 2^w distinct bit patterns
  - Unsigned values: $0 \ldots 2^w-1$
  - Signed values: $-2^{w-1} \ldots 2^{w-1}-1$
- **Example:** 8-bit integers (*e.g.* `char`)



Unsigned Integers (Review)

- Unsigned values follow the standard base 2 system
  - $b_7 b_6 b_5 b_4 b_3 b_2 b_1 b_0 = b_7 2^7 + b_6 2^6 + \cdots + b_1 2^1 + b_0 2^0$
- Useful formula: $2^{N-1} + 2^{N-2} + \ldots + 2 + 1 = 2^N - 1$
  - *i.e.*, N ones in a row = $2^N - 1$
  - *e.g.*, 0b111111 = 63 ← x, 6 1's in a row

x+1 = 0b1 000 000
= 2^6

$x = 2^6 - 1$

Sign and Magnitude

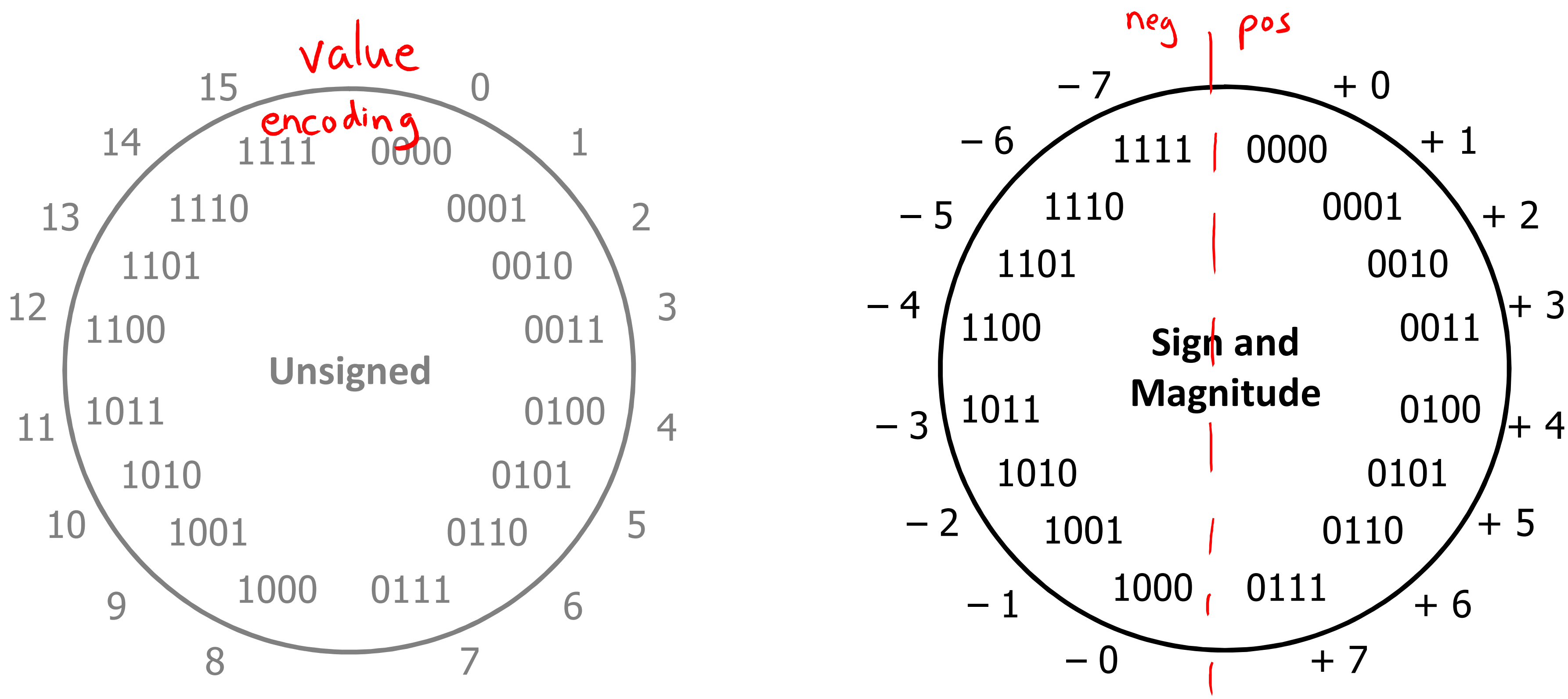
Not used in practice for integers!

- Designate the high-order bit (MSB) as the "sign bit"
  - `sign=0`: positive numbers; `sign=1`: negative numbers
- Benefits:
  - Using MSB as sign bit matches positive numbers with unsigned unsigned: 0b 0010 = 2^1 = 2 ; sign + mag: 0b 0010 = $+2^1$ = 2 ✓
  - All zeros encoding is still = 0
- Examples (8 bits):
  - 0x00 = 00000000_2 is non-negative, because the sign bit is 0
  - 0x7F = 01111111_2 is non-negative ($+127_{10}$) 2^7-1
  - 0x85 = 10000101_2 is negative (-5_{10})
  - 0x80 = 10000000_2 is negative… zero???

Sign and Magnitude

Not used in practice for integers!

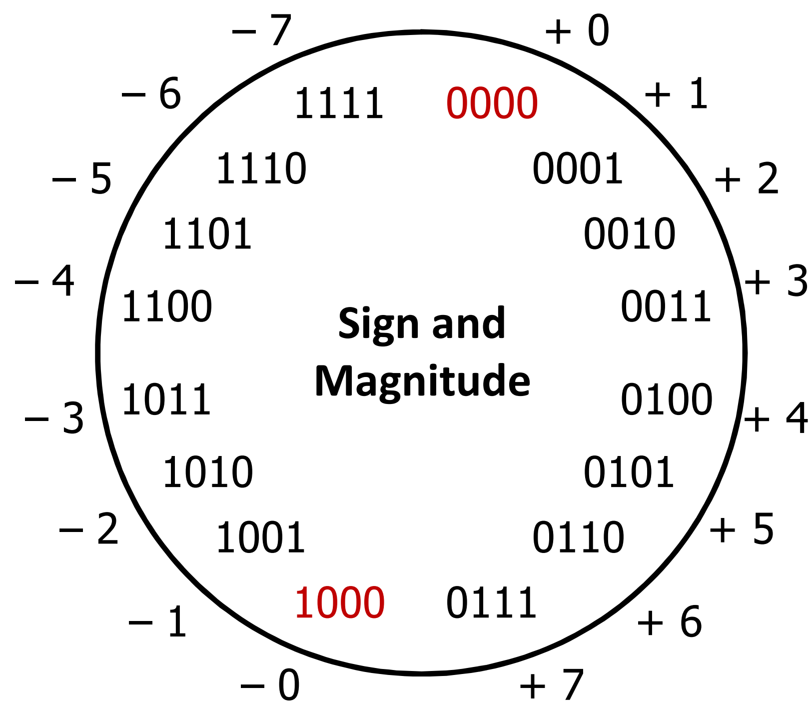
- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks?



Sign and Magnitude

Not used in practice for integers!

- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks:
  - Two representations of 0 (bad for checking equality)



Sign and Magnitude

Not used in practice for integers!

- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks:
  - Two representations of 0 (bad for checking equality)
  - Arithmetic is cumbersome
    - Example: `4-3 != 4+(-3)`

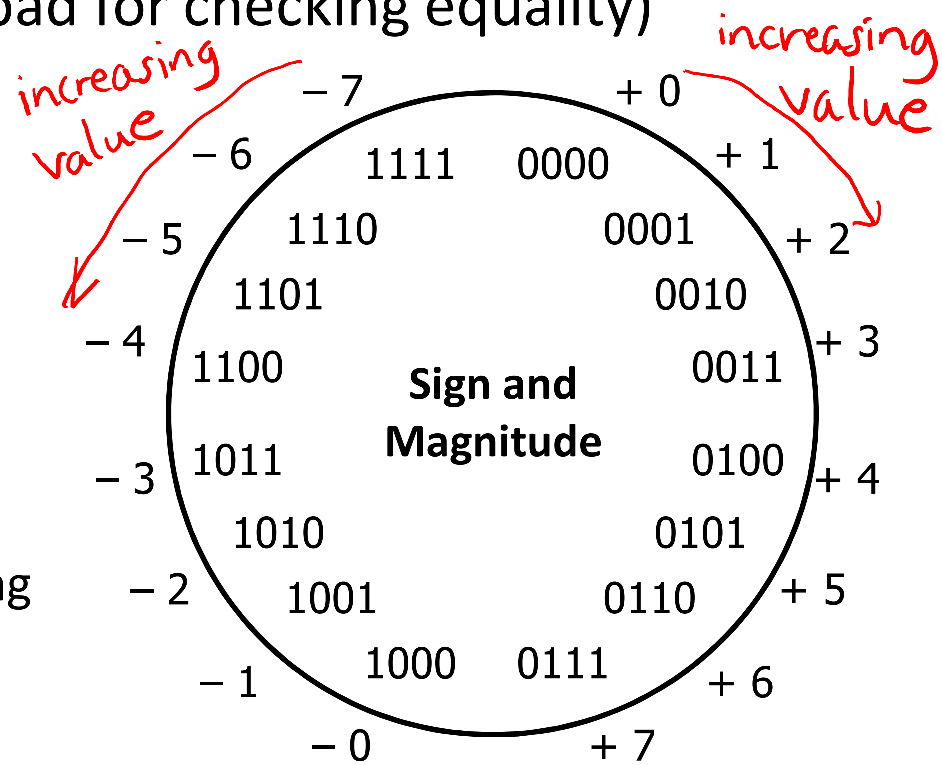
4	0100
– 3	– 0011
1	0001

✓

4	0100
+ -3	+ 1011
-7	1111

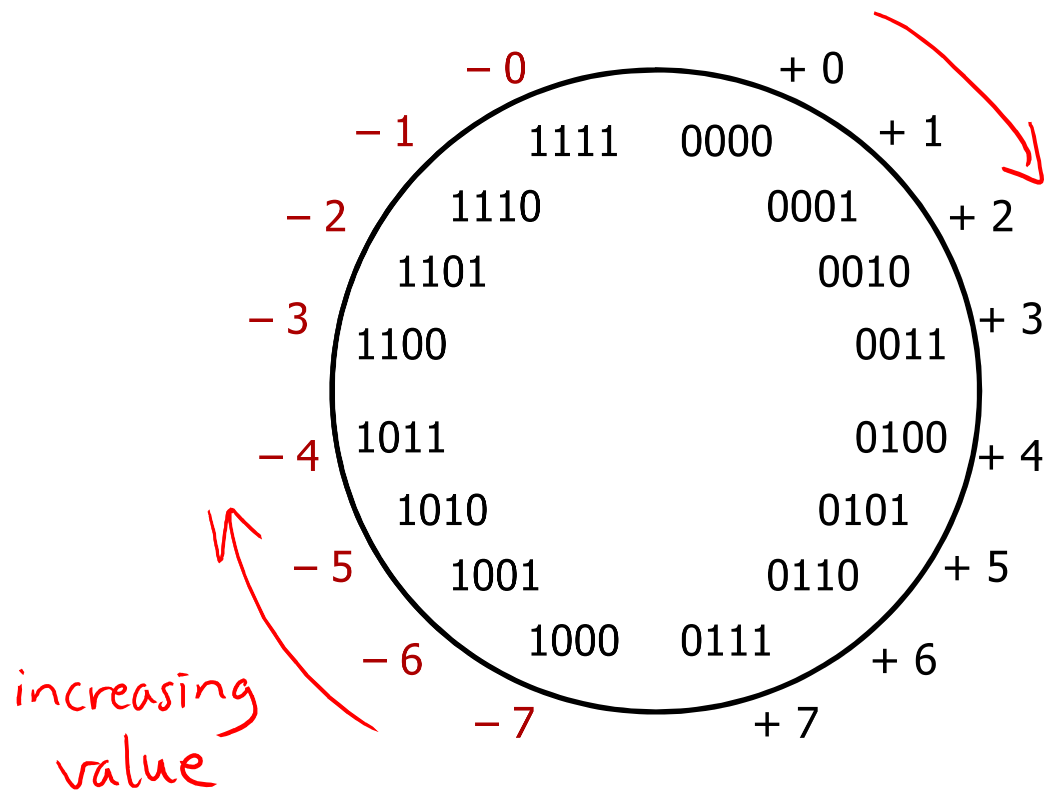
✗

    - Negatives "increment" in wrong direction!



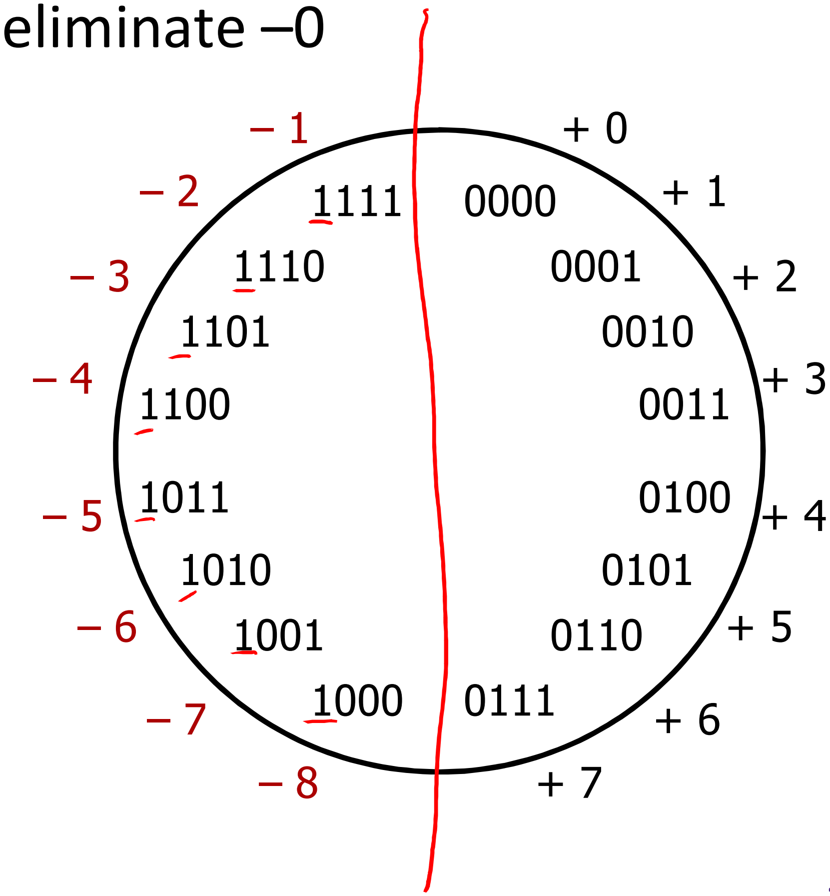
Two's Complement

- Let's fix these problems:
  1) "Flip" negative encodings so incrementing works



Two's Complement

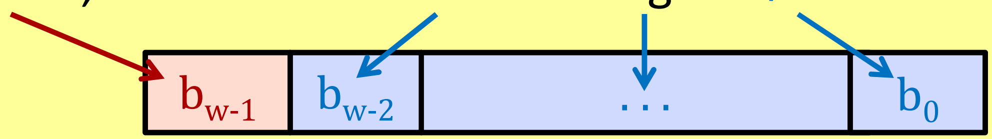
- Let's fix these problems:
  1) "Flip" negative encodings so incrementing works
  2) "Shift" negative numbers to eliminate –0
- MSB *still* indicates sign!
  - This is why we represent one more negative than positive number (-2^{N-1} to 2^{N-1} –1)



Two's Complement Negatives (Review)

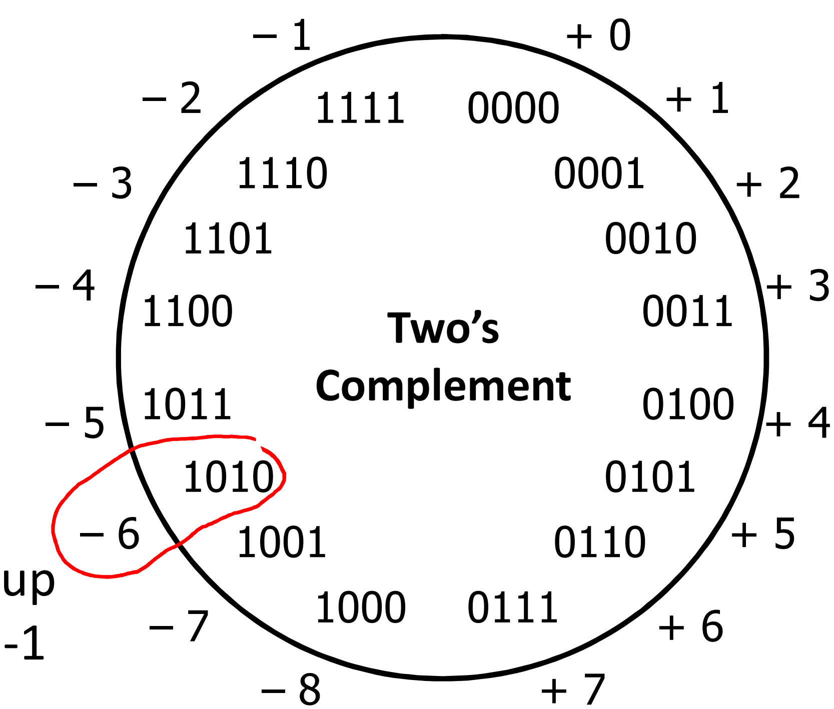
- Accomplished with one neat mathematical trick!

b_{w-1} has weight -2^{w-1}, other bits have usual weights $+2^{i}$



- 4-bit Examples:
  - 1010_2 unsigned:
    $1*2^3+0*2^2+1*2^1+0*2^0$ = **10**
  - 1010_2 two's complement:
    $-1*2^3+0*2^2+1*2^1+0*2^0$ = **–6**

- -1 represented as:
  $1111_2 = -2^3+(2^3 - 1)$
  3 one's in a row
  - MSB makes it super negative, add up all the other bits to get back up to -1



Polling Question

- Take the 4-bit number encoding **x = 0b1011** (MSB)
- Which of the following numbers is NOT a valid interpretation of x using any of the number representation schemes discussed today?
  - Unsigned, Sign and Magnitude, Two's Complement
  - Vote in Ed Lessons

A. -4

B. ~~-5~~

C. ~~11~~

D. ~~-3~~

E. We're lost…

unsigned: $8+2+1=11$

sign + mag: $1011 \rightarrow -(2+1)=-3$

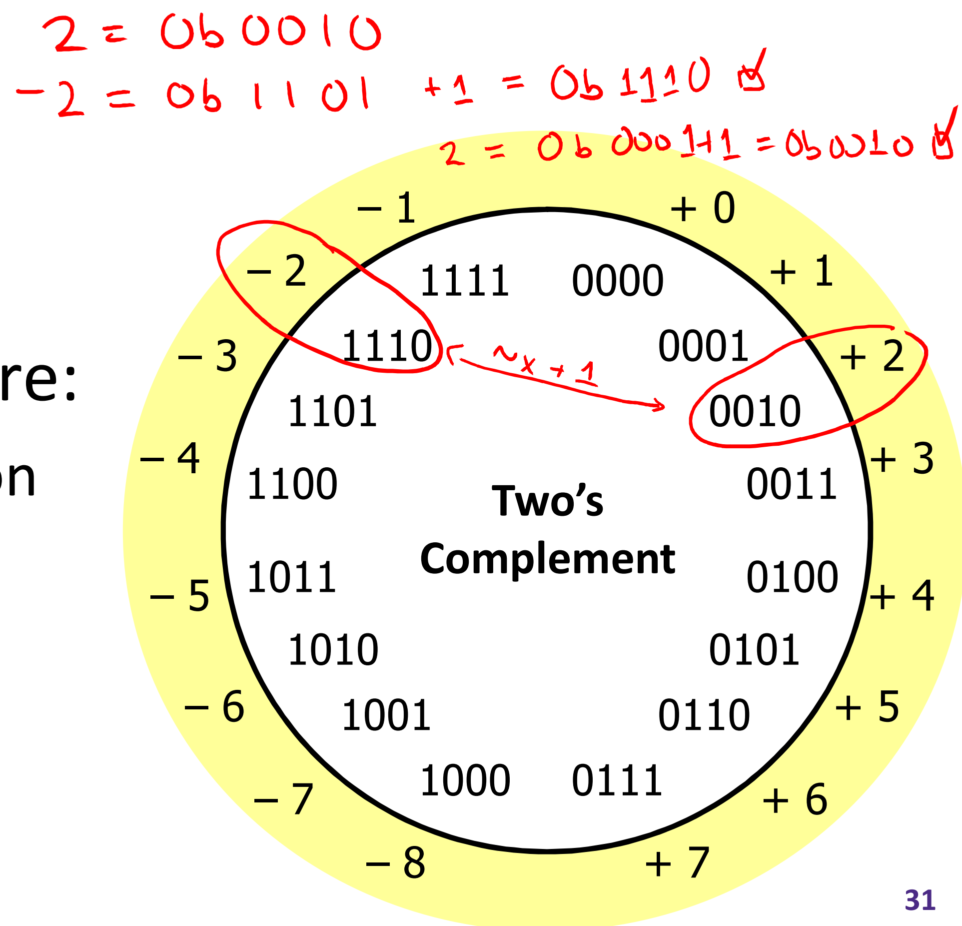
two's: $-8+2+1=-5$

$-x = 0b\ 0100+1 = 5 \rightarrow x=-5$

Two's Complement is Great (Review)

- Roughly same number of (+) and (–) numbers
- Positive number encodings match unsigned
- Single zero
- All zeros encoding = 0

- Simple negation procedure:
  - Get negative representation of any integer by taking bitwise complement and then adding one!
    `( ~x + 1 == -x )`



Summary

- Bit-level operators allow for fine-grained manipulations of data
  - Bitwise AND (`&`), OR (`|`), and NOT (`~`) different than logical AND (`&&`), OR (`||`), and NOT (`!`)
  - Especially useful with bit masks
- Choice of *encoding scheme* is important
  - Tradeoffs based on size requirements and desired operations
- Integers represented using unsigned and two's complement representations
  - Limited by fixed bit width
  - We'll examine arithmetic operations next lecture