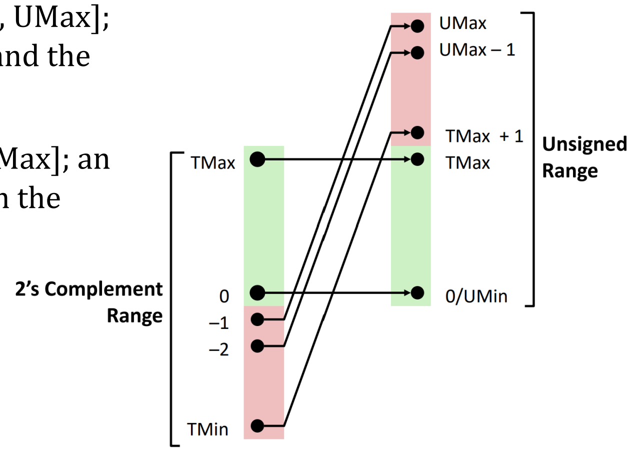
CSE 351 Section 3 – Integers and Floating Point

Welcome back to section, we're happy that you're here ☺

Integers and Arithmetic Overflow

Arithmetic overflow occurs when the result of a calculation can't be represented in the current encoding scheme (*i.e.*, it lies outside of the representable range of values), resulting in an incorrect value.

- Unsigned overflow: the result lies outside of [UMin, UMax]; an indicator of this is when you add two numbers and the result is smaller than either number.
- Signed overflow: the result lies outside of [TMin, TMax]; an indicator of this is when you add two numbers with the same sign and the result has the opposite sign.



Exercises:

1) [Spring 2016 Midterm 1C] Assuming these are all signed two's complement 6-bit integers, compute the result of each of the following additions. For each, indicate if it resulted in overflow.

```
  001001        110001        011001        101111
+ 110110      + 111011      + 001100      + 011111
```

2) [Autumn 2019 Midterm 1C] Find the largest 8-bit unsigned numeral (answer in hex) such that c + 0x80 causes NEITHER signed nor unsigned overflow in 8 bits.

IEEE 754 Floating Point Standard

Goals

- ★ Represent a large range of values (both very small and very large numbers),
- ★ Include a high amount of precision, and
- ★ Allow for real arithmetic results (*e.g.*, ∞ and NaN).

Encoding

The <u>value</u> of a real number can be represented in normalized scientific binary notation as:

$$(-1)^{\text{sign}} \times \text{Mantissa}_2 \times 2^{\text{Exponent}} = (-1)^{S} \times 1.M_2 \times 2^{E\text{-bias}}$$

The <u>binary representation</u> for floating point encodes these three components into separate fields:

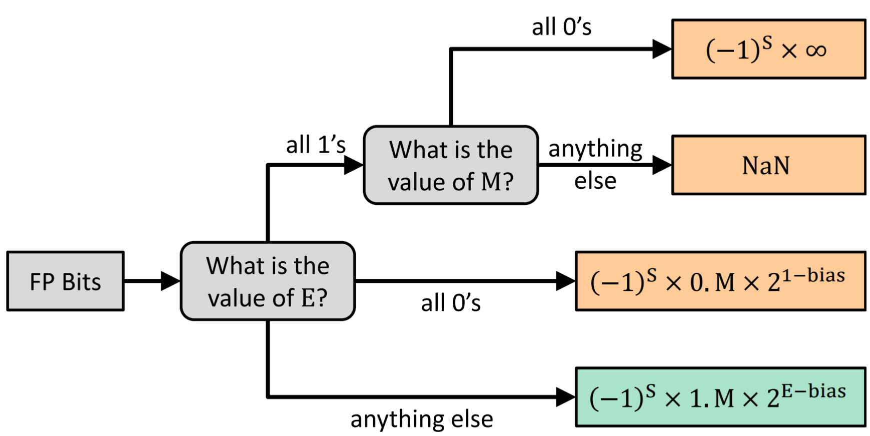
	`S`	`E`	`M`
`float:`	1	8 bits	23 bits
`double:`	1	11 bits	52 bits

- `S`: the sign of the number (0 for positive, 1 for negative)
- `E`: the exponent in biased notation (unsigned with a bias of 2^{w-1}-1)
- `M`: the mantissa (also called the significand or fraction) *without* the implicit leading 1

Special Cases and Interpretations

The interpretation depends on the values in the exponent and mantissa fields:

`E`	`M`	Meaning
0b0…0	anything	denormalized number (denorm)
anything else	anything	normalized number
0b1…1	zero	infinity (∞)
0b1…1	nonzero	not-a-number (NaN)



Mathematical Properties

- Not <u>associative</u>: $(2 + 2^{50}) - 2^{50} \neq 2 + (2^{50} - 2^{50})$
- Not <u>distributive</u>: $100 \times (0.1 + 0.2) \neq 100 \times 0.1 + 100 \times 0.2$
- Not <u>cumulative</u>: $2^{25} + 1 + 1 + 1 + 1 \neq 2^{25} + 4$

Exercises:

3) Let's say that we want to represent the number 3145728.125 (broken down as $2^{21} + 2^{20} + 2^{-3}$)

   a) Convert this number to into single precision floating point representation:

   b) Which limitation of floating point representation does this result highlight?

4) [Summer 2018 Midterm 1E-G] We are working with a new floating point datatype (`flo`) that follows the same conventions as IEEE 754 except using 8 bits split into the following fields:

Sign (1)	Exponent (3)	Mantissa (4)

   a) What is the encoding of the most negative real number that we can represent (∞ is not a real number) in this floating point scheme (binary)?

   b) If we have `signed char x = 0b10101000 = -88`, what will occur if we cast **`flo f = (flo) x`** (*i.e.*, try to represent the value stored in `x` as a `flo`)?

   Rounding   Underflow   Overflow   None of these

5) Based on the floating point representation, explain why each of the three mathematical property examples shown on the previous page occurs.

   a) Not associative:

   b) Not distributive:

   c) Not cumulative:

6) If we have `float x, y;`, give two *different* reasons why **`(x+2*y)-y == x+y`** might evaluate to false.