CSE 351 Section 3

Overflow and Floating Point

Autumn 2022

Administrivia

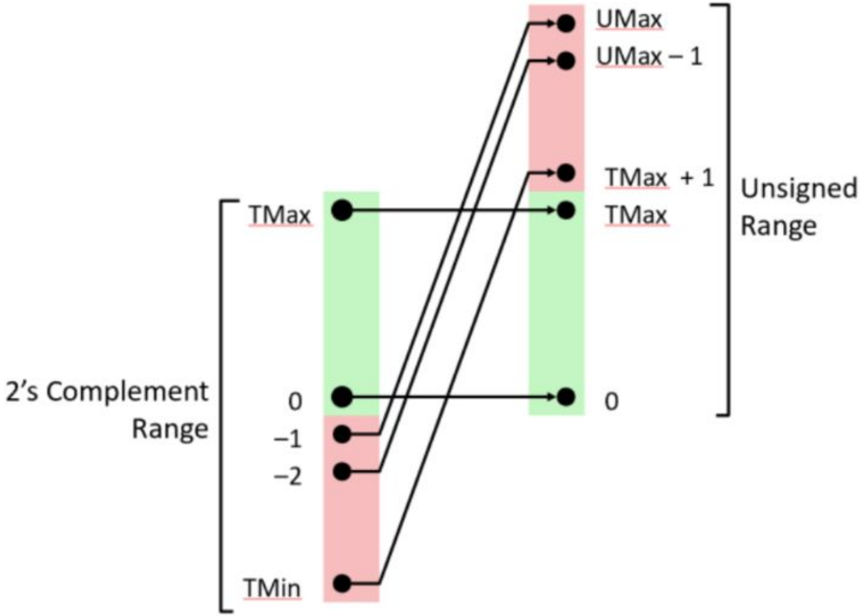
- Lab 1b:
  - Due **Monday, 10/17**
  - Run `make clean` and then `make` to ensure your program compiles!
  - Late days are always an option
- Homeworks on Ed:
  - HW 6 (Floating Point I) due **tomorrow, 10/14**
  - HW 7 (Floating Point II) due **Monday, 10/17**

Integer Overflow

Arithmetic Overflow

Occurs when:

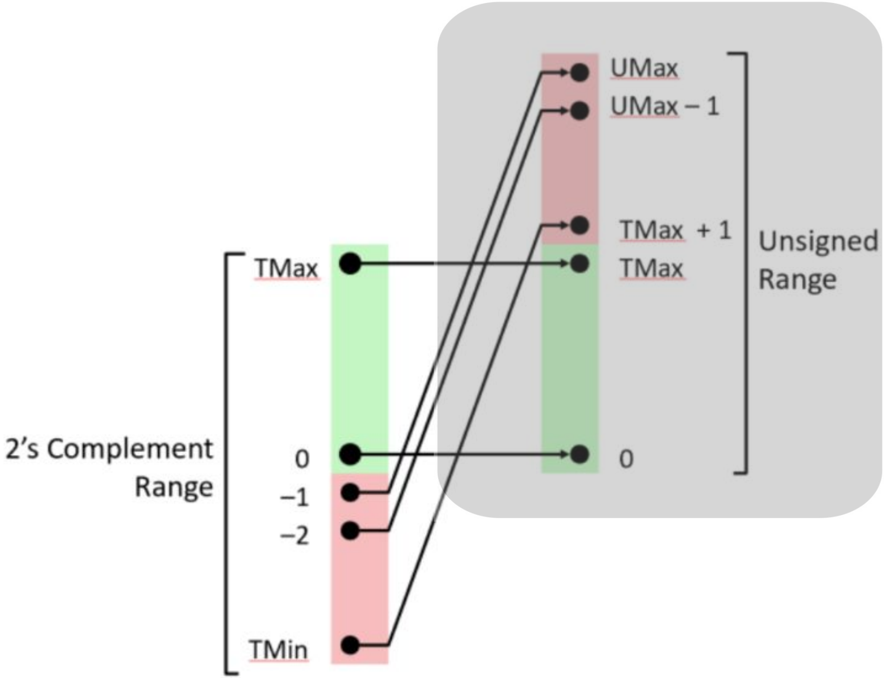
- ➔ The result of a calculation can't be represented in the current encoding scheme
  - ◆ i.e. Lies outside the representable range of values
- ➔ Results in an incorrect result



Unsigned Overflow

Occurs when:

- ➔ The result lies outside [UMin, UMax]
  - ◆ Indicator: Adding two numbers and the result is smaller than either number

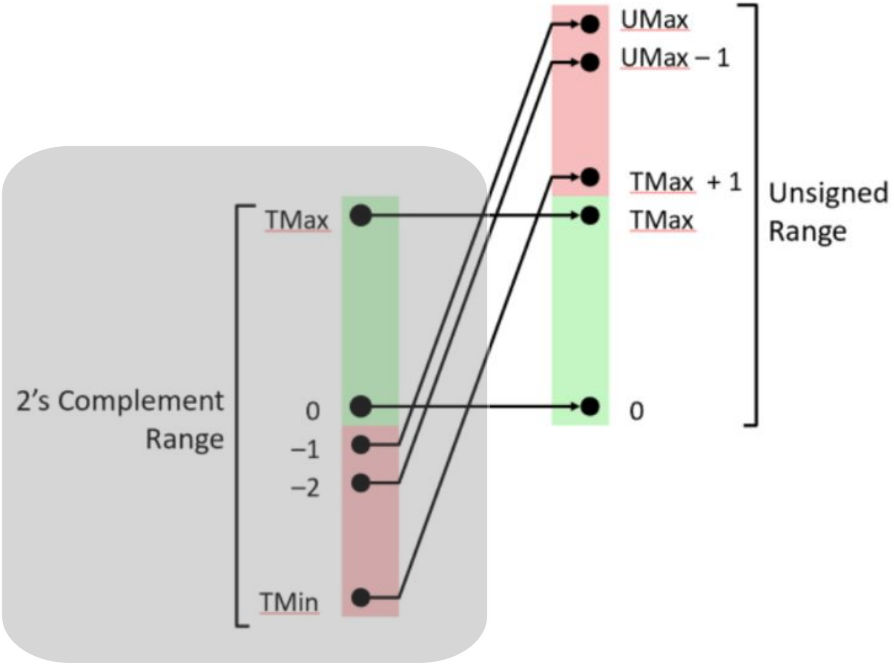


0b 1 1 0 0	12
+ 0b 0 1 1 1	7
1 | 0 0 1 1	3

Signed Overflow

Occurs when:

➔ The result lies outside [TMin, TMax]
- ◆ Indicator: Adding two numbers with the same sign and the result has the opposite sign



0b 0 1 1 0	6
+ 0b 0 0 1 1	3
1 0 0 1	-7

Exercises!

Overflow: Exercise 1

Assuming these are all signed two's complement 6-bit integers, compute the result of each of the following additions. For each, indicate if it resulted in overflow. [Spring 2016 Midterm 1C]

001001	110001	011001	101111
+ 110110	+ 111011	+ 001100	+ 011111

Overflow: Exercise 1

Assuming these are all signed two's complement 6-bit integers, compute the result of each of the following additions. For each, indicate if it resulted in overflow. [Spring 2016 Midterm 1C]

001001	110001	011001	101111
+ 110110	+ 111011	+ 001100	+ 011111
111111	~~1~~101100	100101	~~1~~001110
No	No	Yes	No

Overflow: Exercise 2

Find the largest 8-bit unsigned numeral (answer in hex) such that c + 0x80 causes NEITHER signed nor unsigned overflow in 8 bits. [Autumn 2019 Midterm 1C]

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Unsigned overflow will occur for c >= 0x80. Signed overflow can only happen if c is negative (also >= 0x80). Largest is therefore, 0x7F

Overflow: Exercise 3

Find the smallest 8-bit numeral (answer in hex) such that c + 0x71 causes signed overflow, but NOT unsigned overflow in 8 bits. [Autumn 2018 Midterm 1C]

For signed overflow, need (+) + (+) = (-). For no unsigned overflow, need no carryout from MSB. The first (-) encoding we can reach from 0x71 is 0x80. 0x80 – 0x71 = 0xF.

Floating Point

Floating Point

$$2.75_{10} = 10.11_2 = (+)1.011_2 \times 2^1$$

sign

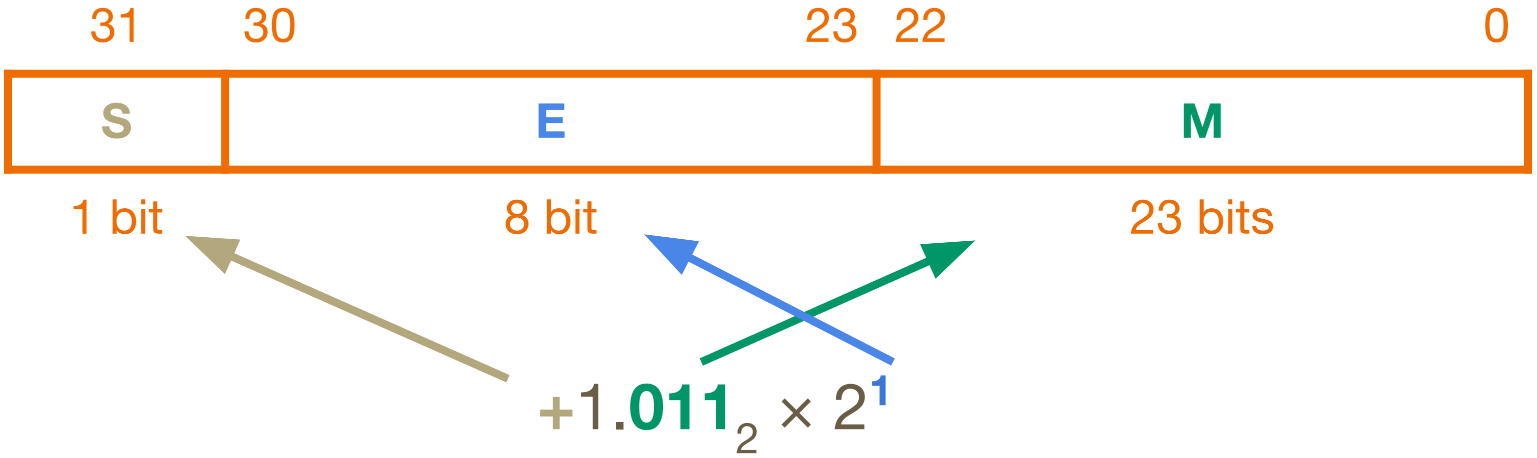
mantissa
(or significand)

exponent

How can we build a representation that has a large range of values, high precision, and handle real arithmetic results (including special values like infinity and NaN)?

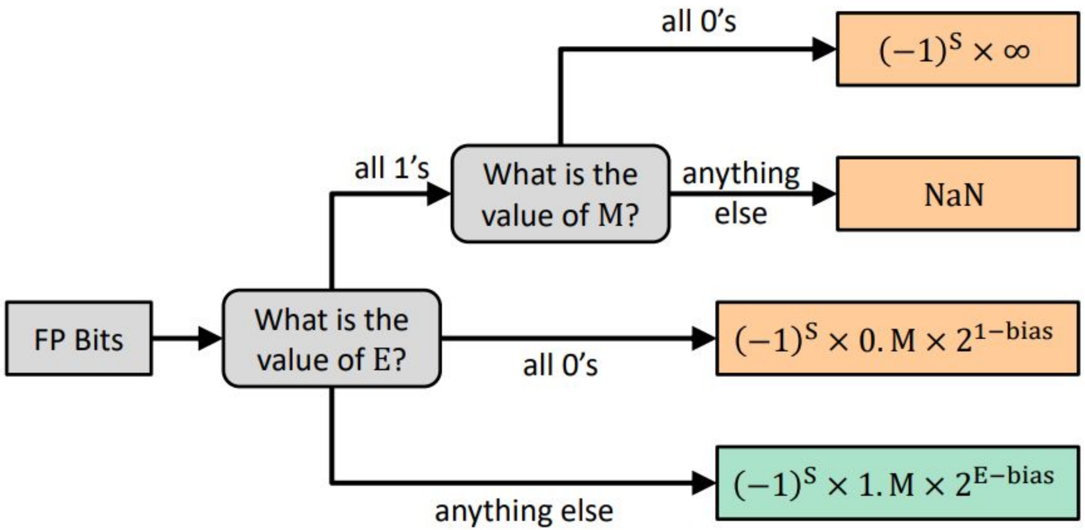
Floating Point Representation

- ➔ Encode the sign, mantissa, and exponent in three “fields” (S, E, M)
- ➔ For a single-precision floating point number (float), we have the following field widths:



Special Cases in Floating Point

E	M	Meaning
0b0...0	0b0...0	+/- 0
0b0...0	non-zero	denormalized number
everything else	anything	normalized number
0b1...1	0b0...0	+/- ∞
0b1...1	non-zero	Not-a-Number (NaN)



Exponent & E-Field

$+1.011_2 \times 2^{1}$

- ➔ We only care about the exponent value, not the base
- ➔ *Biased notation* to represent + and - values (unsigned with a *bias*/offset)
- ➔ The bias is $\mathbf{2^{w-1}-1}$ where the width of the E field is **w**

Ex) $1.011_2 \times 2^1$ stored as a 32-bit float

- *Exp* = 1
- *Bias* = $2^{8-1}-1 = 127$
- **E** = Exp + Bias = 1 + 127 = 128

Note: **Overflow/underflow** occurs when we try to calculate numbers outside of the representable range.

Mantissa & M-Field

$+1.011_2 \times 2^1$

- ➔ Significand is stored with the leading 1 implied
- ➔ Numbers of this form are *normalized*

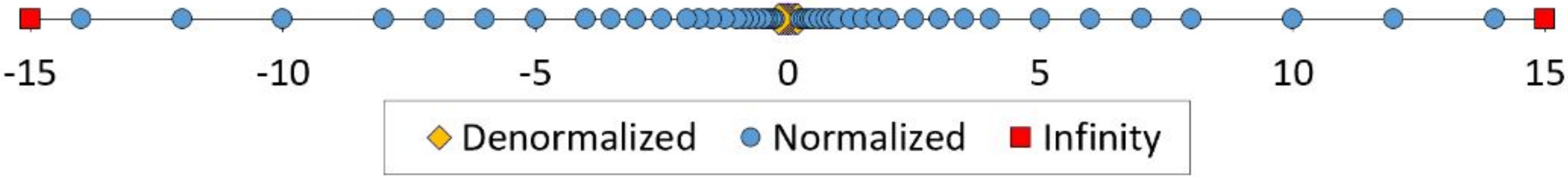
Ex) $1.011_2 \times 2^1$, 32-bit float:

- *Mantissa* = 1.**011**
- **E** is not 0, so the leading 1 is implied (normalized)
- **M** = 0b01100 … 0

Note: **Rounding errors** may occur to to limitations of the precision we have (i.e., the field width).

Limitations of Floating Point

- ➔ **Not associative!** $(2 + 2^{50}) - 2^{50} \neq 2 + (2^{50} - 2^{50})$
- ➔ **Not distributive!** $100 \times (0.1 + 0.2) \neq 100 \times 0.1 + 100 \times 0.2$
- ➔ **Not cumulative!** $2^{25} + 1 + 1 + 1 + 1 \neq 2^{25} + 4$



Exercises!

E = Exp + Bias

Exercises 1 & 2

Bias (4 bits) = ?

Exponent	E (4 bits)	E (8 bits)
1		
0		
-1		

E = Exp + Bias

Exercises 1 & 2

Bias (4 bits) = 2^(4-1) - 1 = 7

Exponent	E (4 bits)				E (8 bits)							
1	1	0	0	0	1	0	0	0	0	0	0	0
0	0	1	1	1	0	1	1	1	1	1	1	1
-1	0	1	1	0	0	1	1	1	1	1	1	0

Exercise 3

Represent 3145728.125_{10} as a single precision floating point ($2^{21}+2^{20}+2^{-3}$)

a) Convert to single precision floating point

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01001010010000000000000000000000

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b) How does this number highlight a limitation of floating point representation?

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b) How does this number highlight a limitation of floating point representation?

Could only represent 2^21 + 2^20. Not enough bits in the mantissa to hold 2^-3, which caused *rounding*

Exercise 4

0x80000000	-0
0xFF94BEEF	NaN
0x41180000	+9.5

Exercise 5

Based on the floating point representation, explain why each of the three mathematical property examples shown on the previous page occurs.

a) Not Associative

b) Not Distributive

c) Not Cumulative

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Based on the floating point representation, explain why each of the three mathematical property examples shown on the previous page occurs.

a) Not Associative

Only 23 bits of mantissa, so 2 + 2^50 = 2^50 (2 gets rounded off).
So LHS = 0, RHS = 2.

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b) Not Distributive

0.1 and 0.2 have infinite representations in binary point (0.2 = 0b 0.00110011...), so the LHS and RHS suffer from different amounts of rounding (try it!).

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c) Not Cumulative

1 = 2^0 is 25 powers of 2 away from 2^25, so 2^25 + 1 = 2^25, but 4 = 2^2 is 23 powers of 2 away from 2^25, so it doesn't get rounded off.

Exercise 6

If x and y are variable type float, give two different reasons why (x+2*y)-y == x+y might evaluate to false.

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1. Rounding Error
2. Overflow - if x and y are large enough, then x+2y may result in infinity while x+y does not

DECODING FLOWCHART

It can seem a bit confusing to interpret a floating point from a bit-level representation.

Thankfully, the process is very methodical, so we can illustrate it using a diagram like so →

From Exercise 4
0xFF94BEEF
0b1111 1111 1001 …
NaN