Floating Point I
CSE 351 Autumn 2022

Instructor: Justin Hsia

Teaching Assistants:
Angela Xu  Arjun Narendra  Armin Magness
Assaf Vayner  Carrie Hu  Clare Edmonds
David Dai  Dominick Ta  Effie Zheng
James Froelich  Jenny Peng  Kristina Lansang
Paul Stevans  Renee Ruan  Vincent Xiao

http://xkcd.com/899/
Relevant Course Information

- hw5 due Wednesday, hw6 due Friday
- Don’t change your poll answers after-the-fact!
  - Graded on completion; misrepresents your understanding
- Lab 1a due tonight at 11:59 pm
  - Submit pointer.c and lab1Asynthesis.txt
    - Make sure there are no lingering printf statements in your code!
  - Make sure you submit something to Gradescope before the deadline and that the file names are correct
  - Can use late days to submit up until Wed 11:59 pm
- Lab 1b due next Monday (10/17)
  - Submit aisle_manager.c, store_client.c, and lab1Bsynthesis.txt
Lab 1b Aside: C Macros

❖ C macros basics:

▪ Basic syntax is of the form: `#define NAME expression`
▪ Allows you to use “NAME” instead of “expression” in code
  • Does naïve copy and replace *before* compilation – everywhere the characters “NAME” appear in the code, the characters “expression” will now appear instead
  • NOT the same as a Java constant
▪ Useful to help with readability/factoring in code

❖ You’ll use C macros in Lab 1b for defining bit masks
▪ See Lab 1b starter code and Lecture 4 slides (card operations) for examples
Reading Review

❖ Terminology:
  ▪ normalized scientific binary notation
  ▪ trailing zeros
  ▪ sign, mantissa, exponent ↔ bit fields S, M, and E
  ▪ float, double
  ▪ biased notation (exponent), implicit leading one (mantissa)
  ▪ rounding errors

❖ Questions from the Reading?
Review Questions

❖ Convert $11.375_{10}$ to normalized binary scientific notation

❖ What is the value encoded by the following floating point number?

0b 0 | 1000 0000 | 110 0000 0000 0000 0000 0000 0000

- bias = $2^{w-1}$-1
- exponent = $E - bias$
- mantissa = 1.M
Number Representation Revisited

❖ What can we represent in one word?
  ▪ Signed and Unsigned Integers
  ▪ Characters (ASCII)
  ▪ Addresses

❖ How do we encode the following:
  ▪ Real numbers (e.g., 3.14159)
  ▪ Very large numbers (e.g., $6.02 \times 10^{23}$)
  ▪ Very small numbers (e.g., $6.626 \times 10^{-34}$)
  ▪ Special numbers (e.g., $\infty$, NaN)
Floating Point Topics

- IEEE floating-point standard
- Floating-point operations and rounding
- Floating-point in C

There are many more details that we won’t cover
  - It’s a 58-page standard...
Binary Scientific Notation (Review)

- **Normalized form**: exactly one digit (non-zero) to left of binary point

- Computer arithmetic that supports this called floating point due to the “floating” of the binary point
  - Declare such variable in C as float (or double)
IEEE Floating Point

- IEEE 754 (established in 1985)
  - Standard to make numerically-sensitive programs portable
  - Specifies two things: *representation scheme* and result of *floating point operations*
  - Supported by all major CPUs

- Driven by numerical concerns
  - **Scientists**/numerical analysts want them to be as **real** as possible
  - **Engineers** want them to be **easy to implement** and **fast**
  - Scientists mostly won out:
    - Nice standards for rounding, overflow, underflow, but...
    - Hard to make fast in hardware
    - **Float operations can be an order of magnitude slower than integer ops**
Floating Point Encoding (Review)

❖ Use normalized, base 2 scientific notation:
  ▪ Value: \( \pm 1 \times \text{Mantissa} \times 2^{\text{Exponent}} \)
  ▪ Bit Fields: \((-1)^S \times 1.M \times 2^{(E-\text{bias})}\)

❖ Representation Scheme:
  ▪ Sign bit (0 is positive, 1 is negative)
  ▪ Mantissa (a.k.a. significand) is the fractional part of the number in normalized form and encoded in bit vector \(M\)
  ▪ Exponent weights the value by a (possibly negative) power of 2 and encoded in the bit vector \(E\)
The Exponent Field (Review)

❖ Use biased notation

- Read exponent as unsigned, but with *bias of* \(2^{w-1}-1 = 127\)
- Representable exponents roughly ½ positive and ½ negative
- \(\text{Exp} = E - \text{bias} \leftrightarrow E = \text{Exp} + \text{bias}\)
  - Exponent 0 (\(\text{Exp} = 0\)) is represented as \(E = 0b 0111 1111\)

❖ Why biased?

- Now it’s a sign-and-magnitude representation!
- Makes floating point arithmetic easier (somewhat compatible with two’s complement hardware)
The Mantissa (Fraction) Field (Review)

\[ (-1)^S \times (1 . M) \times 2^{(E - \text{bias})} \]

- Note the implicit leading 1 in front of the M bit vector
  - Example: 0b 0011 1111 1100 0000 0000 0000 0000 0000 is read as \( 1.1_{12} = 1.5_{10} \), not \( 0.1_{12} = 0.5_{10} \)
  - Gives us an extra bit of precision

- Mantissa “limits”
  - Low values near \( M = 0b0...0 \) are close to \( 2^{\text{Exp}} \)
  - High values near \( M = 0b1...1 \) are close to \( 2^{\text{Exp}+1} \)
Normalized Floating Point Conversions

❖ FP → Decimal
1. Append the bits of M to implicit leading 1 to form the mantissa.
2. Multiply the mantissa by $2^{E - \text{bias}}$.
3. Multiply the sign $(-1)^S$.
4. Multiply out the exponent by shifting the binary point.
5. Convert from binary to decimal.

❖ Decimal → FP
1. Convert decimal to binary.
2. Convert binary to normalized scientific notation.
3. Encode sign as $S (0/1)$.
4. Add the bias to exponent and encode E as unsigned.
5. The first bits after the leading 1 that fit are encoded into M.
Practice Question

❖ Convert the decimal number $-7.375 = -1.11011 \times 2^2$ into floating point representation.
Precision and Accuracy

❖ **Accuracy** is a measure of the difference between the *actual value of a number* and its computer representation

❖ **Precision** is a count of the number of bits in a computer word used to represent a value
- Capacity for accuracy

❖ *High precision permits high accuracy but doesn’t guarantee it*
- **Example:** `float pi = 3.14;` will be represented using all 24 bits of the mantissa (highly precise), but is only an approximation (not accurate)
Need Greater Precision?

- **Double Precision** (vs. Single Precision) in 64 bits

- C variable declared as double
- Exponent bias is now $2^{10} - 1 = 1023$
- **Advantages:** greater precision (larger mantissa), greater range (larger exponent)
- **Disadvantages:** more bits used, slower to manipulate
Current Limitations

❖ Largest magnitude we can represent?
❖ Smallest magnitude we can represent?
  ▪ Limited range due to width of E field

❖ What happens if we try to represent $2^0 + 2^{-30}$?
  ▪ Rounding due to limited precision: stores $2^0$

❖ There is a need for special cases
  ▪ How do we represent the value zero?
  ▪ What about $\infty$ and NaN?
Summary

❖ Floating point approximates real numbers:

- Handles large numbers, small numbers, special numbers
- Exponent in biased notation (bias = $2^{w-1} - 1$)
  - Size of exponent field determines our representable range
  - Outside of representable exponents is overflow and underflow
- Mantissa approximates fractional portion of binary point
  - Size of mantissa field determines our representable precision
  - Implicit leading 1 (normalized) except in special cases
  - Exceeding length causes rounding
Preview Question

❖ Find the sum of the following binary numbers in normalized scientific binary notation:

\[1.01_2 \times 2^0 + 1.11_2 \times 2^2\]