Integers II
CSE 351 Autumn 2022

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http://xkcd.com/571/
Relevant Course Information

- hw4 due Monday, hw5 due Wednesday

- Lab 1a due Monday (10/10)
  - Use ptest and dlc.py to check your solution for correctness (on the CSE Linux environment)
  - Submit pointer.c and lab1Asynthesis.txt to Gradescope
    - Make sure you pass the File and Compilation Check – all the correct files were found and there were no compilation or runtime errors

- Lab 1b released today, due 10/17
  - Bit manipulation on a custom encoding scheme
  - Bonus slides at the end of today’s lecture have relevant examples
Runnable Code Snippets on Ed

- Ed allows you to embed runnable code snippets (e.g., readings, homework, discussion)
  - These are editable and rerunnable!
  - Hides compiler warnings, but will show compiler errors and runtime errors

- Suggested use
  - Good for experimental questions about basic behaviors in C
  - NOT entirely consistent with the CSE Linux environment, so should not be used for any lab-related work
Reading Review

❖ Terminology:
  ▪ UMin, UMax, TMin, Tmax
  ▪ Type casting: implicit vs. explicit
  ▪ Integer extension: zero extension vs. sign extension
  ▪ Modular arithmetic and arithmetic overflow
  ▪ Bit shifting: left shift, logical right shift, arithmetic right shift

❖ Questions from the Reading?
Review Questions

❖ What is the value (and encoding) of $\text{TMin}$ for a fictional 6-bit wide integer data type?

❖ For unsigned char uc = 0xA1;, what are the produced data for the cast (unsigned short)uc?

❖ What is the result of the following expressions?
  ▪ (signed char)uc >> 2
  ▪ (unsigned char)uc >> 3
Integers

- Binary representation of integers
  - Unsigned and signed
  - Casting in C

- Consequences of finite width representations
  - Sign extension, overflow

- Shifting and arithmetic operations
Signed/Unsigned Conversion Visualized

- Two’s Complement → Unsigned
  - Ordering Inversion
  - Negative → Big Positive
Values To Remember (Review)

❖ Unsigned Values
  ▪ UMin = 0b00...0
         = 0
  ▪ UMax = 0b11...1
         = $2^w - 1$

❖ Two’s Complement Values
  ▪ Tmin = 0b10...0
         = $-2^{w-1}$
  ▪ Tmax = 0b01...1
         = $2^{w-1} - 1$
  ▪ $-1$ = 0b11...1

❖ Example: Values for $w = 64$

<table>
<thead>
<tr>
<th></th>
<th>Decimal</th>
<th>Hex</th>
</tr>
</thead>
<tbody>
<tr>
<td>UMax</td>
<td>18,446,744,073,709,551,615</td>
<td>FF FF FF FF FF FF FF FF FF FF</td>
</tr>
<tr>
<td>Tmax</td>
<td>9,223,372,036,854,775,807</td>
<td>7F FF FF FF FF FF FF FF FF FF</td>
</tr>
<tr>
<td>Tmin</td>
<td>-9,223,372,036,854,775,808</td>
<td>80 00 00 00 00 00 00 00 00 00</td>
</tr>
<tr>
<td>-1</td>
<td>-1</td>
<td>FF FF FF FF FF FF FF FF FF FF</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>00 00 00 00 00 00 00 00 00 00</td>
</tr>
</tbody>
</table>

Decimal values are shown in base 10, and hex values are shown in base 16.
In C: Signed vs. Unsigned (Review)

❖ Casting
  ▪ Bits are unchanged, just interpreted differently!
    • `int  tx, ty;`
    • `unsigned int   ux, uy;`
  ▪ Explicit casting
    • `tx = (int)  ux;`
    • `uy = (unsigned int)  ty;`
  ▪ Implicit casting can occur during assignments or function calls
    • `tx = ux;`
    • `uy = ty;`
Casting Surprises (Review)

- Integer literals (constants)
  - By default, integer constants are considered *signed* integers
    - Hex constants already have an explicit binary representation
  - Use “U” (or “u”) suffix to explicitly force *unsigned*
    - Examples: 0U, 4294967259u

- Expression Evaluation
  - When you mixed unsigned and signed in a single expression, then *signed values are implicitly cast to unsigned*
  - Including comparison operators <, >, ==, <=, >=
Expression Evaluation Examples

- Assuming 8-bit data *(i.e., bit position 7 is the MSB)*, what will the following expression evaluate to?

  - 127 < 128u

  - 127 < (signed char) 128u
Integers

❖ Binary representation of integers
  ▪ Unsigned and signed
  ▪ Casting in C

❖ Consequences of finite width representations
  ▪ Sign extension, overflow

❖ Shifting and arithmetic operations
Sign Extension (Review)

- **Task:** Given a $w$-bit signed integer $X$, convert it to $w+k$-bit signed integer $X'$ *with the same value*

- **Rule:** Add $k$ copies of sign bit
  - Let $x_i$ be the $i$-th digit of $X$ in binary
  - $X' = x_{w-1}, \ldots, x_{w-1}, x_{w-1}, x_{w-2}, \ldots, x_1, x_0$

![Diagram of sign extension](diagram.png)
Two’s Complement Arithmetic

- The same addition procedure works for both unsigned and two’s complement integers
  - **Simplifies hardware**: only one algorithm for addition
  - **Algorithm**: simple addition, **discard the highest carry bit**
    - Called modular addition: result is sum $modulo \ 2^w$
Arithmetic Overflow (Review)

- When a calculation produces a result that can’t be represented in the current encoding scheme
  - Integer range limited by fixed width
  - Can occur in both the positive and negative directions

- C and Java ignore overflow exceptions
  - You end up with a bad value in your program and no warning/indication... oops!

<table>
<thead>
<tr>
<th>Bits</th>
<th>Unsigned</th>
<th>Signed</th>
</tr>
</thead>
<tbody>
<tr>
<td>0000</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0001</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0010</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>0011</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>0100</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>0101</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>0110</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>0111</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>1000</td>
<td>8</td>
<td>-8</td>
</tr>
<tr>
<td>1001</td>
<td>9</td>
<td>-7</td>
</tr>
<tr>
<td>1010</td>
<td>10</td>
<td>-6</td>
</tr>
<tr>
<td>1011</td>
<td>11</td>
<td>-5</td>
</tr>
<tr>
<td>1100</td>
<td>12</td>
<td>-4</td>
</tr>
<tr>
<td>1101</td>
<td>13</td>
<td>-3</td>
</tr>
<tr>
<td>1110</td>
<td>14</td>
<td>-2</td>
</tr>
<tr>
<td>1111</td>
<td>15</td>
<td>-1</td>
</tr>
</tbody>
</table>
Overflow: Unsigned

- **Addition**: drop carry bit ($-2^N$)
  
  \[
  \begin{array}{c@{}c@{}c@{}c@{}c@{}c@{}c}
  & & & & & & \\
  & 1 & 5 & & & & \\
  + & & 2 & & & & \\
  \hline
  & & 1 & 7 & & & \\
  \end{array}
  \quad \quad \quad
  \begin{array}{c@{}c@{}c@{}c@{}c@{}c@{}c}
  & & & & & & \\
  & 1 & 1 & 0 & 1 & 1 & \\
  + & 0 & 0 & 0 & 1 & 0 & \\
  \hline
  & 1 & 0 & 0 & 0 & 1 & \\
  \end{array}
  
  \]

- **Subtraction**: borrow ($+2^N$)
  
  \[
  \begin{array}{c@{}c@{}c@{}c@{}c@{}c@{}c}
  & & & & & & \\
  & 1 & & & & & \\
  - & & 2 & & & & \\
  \hline
  & & & & & & \\
  & 0 & 0 & 0 & 1 & 0 & \\
  - & & 0 & 0 & 1 & 0 & \\
  \hline
  & & & & & & \\
  & 1 & 1 & 1 & 1 & & \\
  \end{array}
  \]

$\pm 2^N$ because of modular arithmetic
Overflow: Two’s Complement

- **Addition:** $(+)+(+)=(-)$ result?

  \[
  \begin{array}{ccc}
  6 &  & 0110 \\
  +3 & + & 0011 \\
  \hline
  9 & & 1001 \\
  \end{array}
  \]

  \(-7\)

- **Subtraction:** $(-)+(-)=(+)$?

  \[
  \begin{array}{ccc}
  -7 &  & 1001 \\
  +3 & - & 0011 \\
  \hline
  -10 & & 0110 \\
  \end{array}
  \]

  \(6\)

For signed: overflow if operands have same sign and result’s sign is different
Practice Questions

❖ Assuming 8-bit integers:
  ▪ 0x27 = 39 (signed) = 39 (unsigned)
  ▪ 0xD9 = -39 (signed) = 217 (unsigned)
  ▪ 0x7F = 127 (signed) = 127 (unsigned)
  ▪ 0x81 = -127 (signed) = 129 (unsigned)

❖ For the following additions, did signed and/or unsigned overflow occur?
  ▪ 0x27 + 0x81
  ▪ 0x7F + 0xD9
Integers

- Binary representation of integers
  - Unsigned and signed
  - Casting in C
- Consequences of finite width representations
  - Sign extension, overflow
- Shifting and arithmetic operations
Shift Operations (Review)

- Throw away (drop) extra bits that “fall off” the end
- Left shift ($x << n$) bit vector $x$ by $n$ positions
  - Fill with 0’s on right
- Right shift ($x >> n$) bit-vector $x$ by $n$ positions
  - Logical shift (for unsigned values)
    - Fill with 0’s on left
  - Arithmetic shift (for signed values)
    - Replicate most significant bit on left (maintains sign of $x$)

<table>
<thead>
<tr>
<th>$x$</th>
<th>0010 0010</th>
<th>$x$</th>
<th>1010 0010</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x &lt;&lt; 3$</td>
<td>0001 0000</td>
<td>$x &lt;&lt; 3$</td>
<td>0001 0000</td>
</tr>
<tr>
<td>logical: $x &gt;&gt; 2$</td>
<td>0000 1000</td>
<td>logical: $x &gt;&gt; 2$</td>
<td>0010 1000</td>
</tr>
<tr>
<td>arithmetic: $x &gt;&gt; 2$</td>
<td>0000 1000</td>
<td>arithmetic: $x &gt;&gt; 2$</td>
<td>1110 1000</td>
</tr>
</tbody>
</table>
Shift Operations (Review)

❖ Arithmetic:
  ▪ Left shift \( x << n \) is equivalent to multiply by \( 2^n \)
  ▪ Right shift \( x >> n \) is equivalent to divide by \( 2^n \)
  ▪ Shifting is faster than general multiply and divide operations!

❖ Notes:
  ▪ Shifts by \( n < 0 \) or \( n \geq w \) (\( w \) is bit width of \( x \)) are undefined
  ▪ **In C:** behavior of \( >> \) is determined by the compiler
    • In gcc / C lang, depends on data type of \( x \) (signed/unsigned)
  ▪ **In Java:** logical shift is \( >>> \) and arithmetic shift is \( >> \)
Left Shifting Arithmetic 8-bit Example

- No difference in left shift operation for unsigned and signed numbers (just manipulates bits)
  - Difference comes during interpretation: \( x \times 2^n \)?

\[
x = 25; \quad 00011001 = 25 \\
L1=x<<2; \quad 0001100100 = 100 \\
L2=x<<3; \quad 00011001000 = -56 \\
L3=x<<4; \quad 000110010000 = -112 \\
\]

- Signed overflow
- Unsigned overflow
Right Shifting Arithmetic 8-bit Examples

❖ Reminder: C operator `>>` does *logical* shift on unsigned values and *arithmetic* shift on signed values
  - Logical Shift: $x/2^n$?

$$x_u = 240u; \quad 11110000 \quad = \quad 240$$

$$R1u=x_u>>3; \quad 00011110000 \quad = \quad 30$$

$$R2u=x_u>>5; \quad 0000011110000 \quad = \quad 7$$

(rounding (down))
Right Shifting Arithmetic 8-bit Examples

- **Reminder:** C operator `>>` does *logical* shift on unsigned values and *arithmetic* shift on signed values
  - **Arithmetic Shift:** $x/2^n$?

```plaintext
xs = -16; 11110000 = -16
R1s=xu>>3; 111111100000 = -2
R2s=xu>>5; 11111111100000 = -1 (rounding (down))
```
Exploration Questions

For the following expressions, find a value of signed char \( x \), if there exists one, that makes the expression True.

- Assume we are using 8-bit arithmetic:
  - \( x == (\text{unsigned char}) \ x \)  
  - \( x >= 128U \)  
  - \( x != (x>>2) << 2 \)  
  - \( x == -x \)  
    - Hint: there are two solutions  
  - \( (x < 128U) && (x > 0x3F) \)
Summary

❖ Sign and unsigned variables in C
  ▪ Bit pattern remains the same, just interpreted differently
  ▪ Strange things can happen with our arithmetic when we convert/cast between sign and unsigned numbers
    • Type of variables affects behavior of operators (shifting, comparison)

❖ We can only represent so many numbers in $w$ bits
  ▪ When we exceed the limits, arithmetic overflow occurs
  ▪ Sign extension tries to preserve value when expanding

❖ Shifting is a useful bitwise operator
  ▪ Right shifting can be arithmetic (sign) or logical (0)
  ▪ Can be used in multiplication with constant or bit masking
Some examples of using shift operators in combination with bitmasks, which you may find helpful for Lab 1b.

- Extract the 2\textsuperscript{nd} most significant byte of an \texttt{int}
- Extract the sign bit of a signed \texttt{int}
- Conditionals as Boolean expressions
Using Shifts and Masks

❖ Extract the 2\textsuperscript{nd} most significant byte of an int:

- First shift, then mask: \((x\gg16) \& 0xFF\)

<table>
<thead>
<tr>
<th>(x)</th>
<th>00000001 00000010 00000011 00000100</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x\gg16)</td>
<td>00000000 00000000 00000001 00000010</td>
</tr>
<tr>
<td>0xFF</td>
<td>00000000 00000000 00000000 11111111</td>
</tr>
<tr>
<td>((x\gg16) &amp; 0xFF)</td>
<td>00000000 00000000 00000000 00000010</td>
</tr>
</tbody>
</table>

- Or first mask, then shift: \((x \& 0xFFF0000) \gg16\)

<table>
<thead>
<tr>
<th>(x)</th>
<th>00000001 00000010 00000011 00000100</th>
</tr>
</thead>
<tbody>
<tr>
<td>0xFFF0000</td>
<td>00000000 11111111 00000000 00000000</td>
</tr>
<tr>
<td>(x &amp; 0xFFF0000)</td>
<td>00000000 00000010 00000000 00000000</td>
</tr>
<tr>
<td>((x&amp;0xFFF0000) \gg16)</td>
<td>00000000 00000000 00000000 00000010</td>
</tr>
</tbody>
</table>
Using Shifts and Masks

❖ Extract the **sign bit** of a signed `int`:

- **First shift, then mask:** `(x>>31) & 0x1`
  - Assuming arithmetic shift here, but this works in either case
  - Need mask to clear 1s possibly shifted in

<table>
<thead>
<tr>
<th>x</th>
<th>00000001 00000010 00000011 00000100</th>
</tr>
</thead>
<tbody>
<tr>
<td>x&gt;&gt;31</td>
<td>00000000 00000000 00000000 00000000 0</td>
</tr>
<tr>
<td>0x1</td>
<td>00000000 00000000 00000000 00000001</td>
</tr>
<tr>
<td>(x&gt;&gt;31) &amp; 0x1</td>
<td>00000000 00000000 00000000 00000000 0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>x</th>
<th>10000001 00000010 00000011 00000100</th>
</tr>
</thead>
<tbody>
<tr>
<td>x&gt;&gt;31</td>
<td>11111111 11111111 11111111 11111111 1</td>
</tr>
<tr>
<td>0x1</td>
<td>00000000 00000000 00000000 00000001</td>
</tr>
<tr>
<td>(x&gt;&gt;31) &amp; 0x1</td>
<td>00000000 00000000 00000000 00000001</td>
</tr>
</tbody>
</table>
Using Shifts and Masks

- Conditionals as Boolean expressions
  - For `int x`, what does `(x<<31)>>31` do?

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>x=!!123</td>
<td>00000000 00000000 00000000 00000000 00000001</td>
</tr>
<tr>
<td>x&lt;&lt;31</td>
<td>10000000 00000000 00000000 00000000 00000000</td>
</tr>
<tr>
<td>(x&lt;&lt;31)&gt;&gt;31</td>
<td>11111111 11111111 11111111 11111111</td>
</tr>
<tr>
<td>!x</td>
<td>00000000 00000000 00000000 00000000 00000000</td>
</tr>
<tr>
<td>!x&lt;&lt;31</td>
<td>00000000 00000000 00000000 00000000 00000000</td>
</tr>
<tr>
<td>(!x&lt;&lt;31)&gt;&gt;31</td>
<td>00000000 00000000 00000000 00000000</td>
</tr>
</tbody>
</table>

- Can use in place of conditional:
  - In C: `if(x) {a=y;} else {a=z;}` equivalent to `a=x?y:z;`
  - `a=(((!!x<<31)>>31)&y) | (((!x<<31)>>31)&z);`