# Integers I

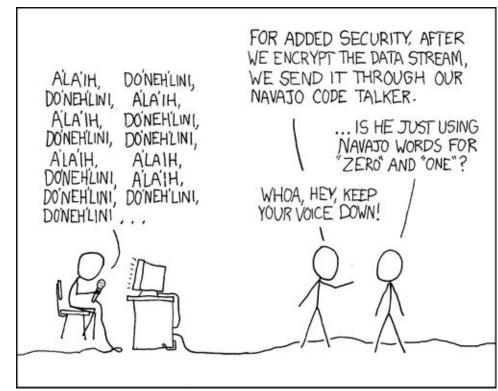
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http://xkcd.com/257/

#### **Gentle and Loving Reminders!**

- No lecture on Monday 7/5 (campus holiday)
- hw3 due Wednesday 6/30 8pm
- Hw4, hw5, lab1a due Friday 7/2 8pm
  - I wanted y'all to have a holiday without deadlines!
  - 1 late day on lab1a would take you to Monday, 8pm
- For the rest of the course:
  - Pre-lecture readings are due at 10am
- Other reminders? Does this feel ok?
  - You all belong in computing!

#### A note on the first survey

- I didn't change everything from last quarter...
  - Boooooooooooooo
- If there's anything that you'd like us to know, difficulties with remote instruction, personal stuff that's coming up, please, please let us know!
  - marakr@cs.washington.edu, if you'd prefer privacy
- This pandemic's been a lot!
  - Last August, I wasn't sure if I could keep living
  - Everyone I know's been through it.
  - Let us know how we can help!

## Classroom norms on language? Reach out with concerns!

# How are y'all feeling today?

## Seattle's hot!

Submit lab1a by Monday@8pm, only using one late day!

#### Memory, Data, and Addressing

- Representing information as bits and bytes
  - Binary, hexadecimal, fixed-widths
- Organizing and addressing data in memory
  - Memory is a byte-addressable array
  - Machine "word" size = address size = register size
  - Endianness ordering bytes in memory
- Manipulating data in memory using C
  - Assignment
  - Pointers, pointer arithmetic, and arrays
- Boolean algebra and bit-level manipulations

#### **Design Tradeoffs**

- When problems don't have a clear answer, you have tradeoffs!
  - Playing card representations
  - Integers (later floats)
- Tradeoffs encode priorities!
  - As well as ideology
- Lots of ideology in 351, emoji to signal ideology?
- When you see ideology (especially when it's unexamined), react with \_\_\_\_

## Breakouts!

Does computing feel like a safe space?
Does it feel oppressive?
...but first...

#### **Discussion Norms**

- Everyone's experience is valid!
- Feelings are valid too!
- Everyone should have space to share!
  - Though, no one's required to share. A "yes" or "no" without explanation is more than enough.
  - Saying "I don't feel like sharing" is good too!
- l've got a whole heap of hurt here!
  - Some of y'all might as well
  - Try to be compassionate to everyone!
- A brief grounding...

#### **Breakouts!**

- What are your experiences bringing other interests into computing?
- Does computing feel like a safe space?
- Have you felt encouraged to pursue non-CS interests?

#### **Learning Objectives**

- Understanding this material means you can...
  - Use bitwise and logical operations to perform bit-level manipulations, &, |, ^, ~, >>, and << (lab1a)</li>
  - Convert between decimal and binary representations for signed and unsigned integers
  - Explain tradeoffs between sign-magnitude and two's complement encodings
  - Begin to recognize CS ideology, especially when considering tradeoffs between different designs
  - Develop a sense of how knowledge insulation affect CS, and more importantly, how knowledge insulation affects you!

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#### **Boolean Algebra**

- Developed by George Boole in 19th Century
  - Algebraic representation of logic (T -> 1, F -> 0)
  - AND: A&B=1 when both A is 1 and B is 1
  - OR: A | B=1 when either A is 1 or B is 1
  - XOR: A^B=1 when either A is 1 or B is 1; not both
  - NOT: ~A=1 when A is 0 and vice versa
  - DeMorgan's law: ~ (A | B) = ~A & ~B

AND			OR			XOR			NOT		
&	0	1	1	0	1	^	0	1		~	
0	0	0	0	0	1	0	0	1		0	1
1	0	1	1	1	1	1	1	0		1	0

#### General Boolean Algebras

- Operate on bit vectors
  - Operations applied bitwise
  - All of the properties of Boolean algebra apply

Examples of useful operations:

$$x \wedge x = 0$$

$$x \mid 1 = 1, \quad x \mid 0 = x$$

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#### **Bit-Level Operations in C**

- - View arguments as bit vectors, apply operations bitwise
  - Apply to any "integral" data type
    - · long, int, short, char, unsigned
- Examples with char a, b, c;

```
a = (char) 0x41; // 0x41->0b 0100 0001
b = ~a; // 0b 1011 1110 ->0xBE
a = (char) 0x69; // 0x69->0b 0110 1001
b = (char) 0x55; // 0x55->0b 0101 0101
c = a & b; // 0b 0100 0001 ->0x41
a = (char) 0x41; // 0x41->0b 0100 0001
b = a; // 0b 0100 0001
c = a ^ b; // 0b 0000 0000 ->0x0
```

#### **Contrast: Logic Operations**

- Logical ops in C: & & (AND), | | (OR), ! (NOT)
  - **0** is False, **anything nonzero** is True
  - Always return 0 or 1
  - Early termination (short-circuit evaluation) of & &, | |

■ 0xCC && 0x33 -> 0x01

-0x00 | 10x33 -> 0x01

Examples (char data type)

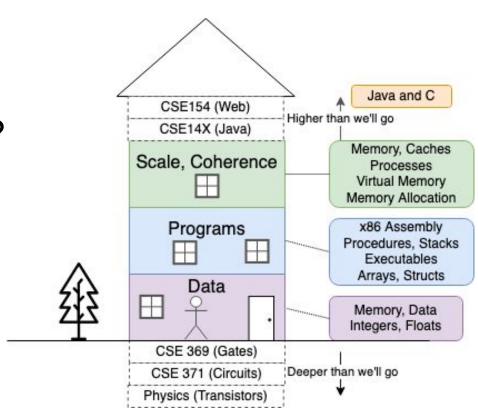
```
• !0x41 -> 0x00
```

• 
$$!!0x41 \rightarrow 0x01$$

- p && \*p
  - If p is the null pointer (0x0), then p is never dereferenced!

#### First Floor: Data

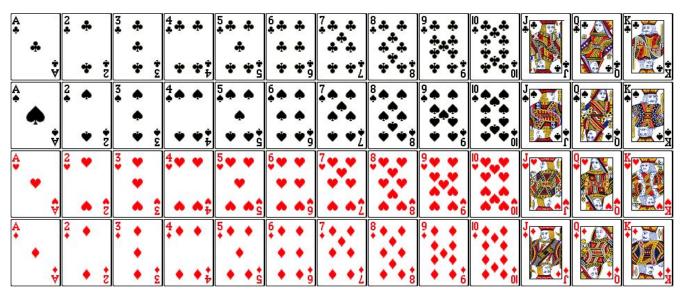
- How do we represent data (strings, numbers) computationally?
- What limits exist? Why?
- What values were encoded into data representations?
- What was prioritized? Why?



Today: Integers!

#### But before we get to integers....

- Encode a standard deck of playing cards
- 52 cards in 4 suits
  - How do we encode suits, face cards?
- What operations do we want to make easy to implement?
  - Which is the higher value card?
  - Are they the same suit?



#### Two possible representations

1. 1 bit/card (52): bit that refers to card set to 1

low-order 52 bits of 64-bit word

- "One-hot" encoding (similar to set notation)
- Drawbacks:
  - Hard to compare values and suits
  - Large number of bits required

4 suits

13 numbers

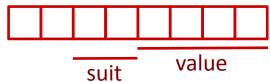
- 2. 1 bit/suit (4), 1 bit/number (13): 2 bits set
  - Pair of one-hot encoded values
  - Easier to compare suits/values; but lots of bits used

#### Two better representations

- 3) Binary encoding of all 52 cards 6 bits needed
  - $2^6 = 64 \ge 52$

low-order 6 bits of a byte

- Fits in one byte (smaller than one-hot encodings)
- How can we make value and suit comparisons easier?
- 4) Separate encodings of suit (2 bits), value (4 bits)



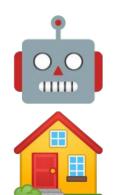
Also fits in one byte; easy to do comparisons

K	Q	J		3	2	Α	
1101	1100	1011	• • •	0011	0010	0001	

<b>%</b>	00
	01
	10
	11

#### **Design Tradeoffs**

- We went from 52 bits to 6 bits
  - Are 6 bit representations "better" than 52 bit ones?
  - It depends!!! What are we prioritizing?
- In general, folks prioritize along their ideology
  - CS: Efficiency, minimalism
  - Frequently choice between space/speed, without considering socio-technical factors
- There usually isn't a "perfect" decision!



#### **Compare Card Suits**

```
#define SUIT_MASK 0x30

int sameSuitP(char card1, char card2) {
   return (!((card1 & SUIT_MASK) ^ (card2 & SUIT_MASK)));
   //return (card1 & SUIT_MASK) == (card2 & SUIT_MASK);
}
```

```
SUIT_MASK = 0x30 = 0 0 1 1 0 0 0 0 

suit value
```

#### Compare Card Sui another bit vector v.

mask: a bit vector designed to achieve a desired behavior when used with a bitwise operator on

Here we turn all but the bits of interest in v to 0.

```
char card1, card2; // two/ cards to compare
 card1 = hand[0];
 card2 = hand[1];
 if ( sameSuitP(card1, card2) ) { ... }
#define SUIT MASK
                 0x30 // 0011 0000
int sameSuitP(char card1, char card2) {
 return (!((card1 & SUIT MASK) ^ (card2 & SUIT MASK)));
   return (card1 & SUIT MASK) == (card2 & SUIT MASK);
                                        equivalent
 returns int SUIT MASK = 0x30 = 0 0
                                   value
                              suit
                                                   23
```

#### Compare Card Sui behavior when used another bit vector v.

**mask:** a bit vector designed to achieve a desired behavior when used with a bitwise operator on another bit vector *v*.

Here we turn all but the bits of interest in v to 0.

```
#define SUIT MASK
                     0 \times 30
int sameSuitP(char card1, char card2) {
  return (!((card1 & SUIT MASK) ^ (card2 & SUIT MASK)));
  //return (card1 & SUIT MASK) == (card2 & SUIT MASK);
                          SUIT MASK
! (x^y) equivalent to x==y
```

#### Compare Card Values bitwise operator on another bit vector v.

mask: a bit vector designed to achieve a desired behavior when used with a

```
char card1, card2; // two cards to compare
card1 = hand[0];
card2 = hand[1];
if ( greaterValue(card1, card2) ) { ... }
```

```
#define VALUE MASK 0x0F // 0000 1111
int greaterValue(char card1, char card2) {
  return ((unsigned int)(card1 & VALUE MASK) >
          (unsigned int) (card2 & VALUE MASK));
```

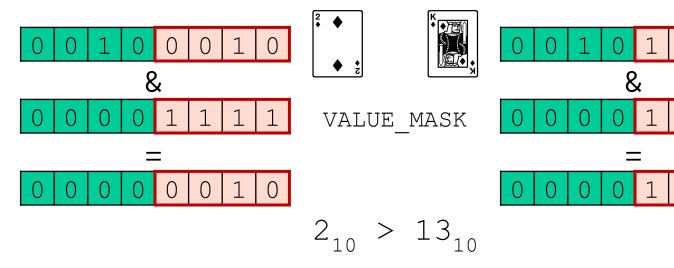
```
VALUE MASK = 0x0F = 0
                                   value
                            suit
```

#### Compare Card Values bitwise operator on another bit vector v.

mask: a bit vector designed to achieve a desired behavior when used with a

```
#define VALUE MASK
                    0x0F
int greaterValue(char card1, char card2) {
  return ((unsigned int)(card1 & VALUE MASK) >
          (unsigned int) (card2 & VALUE MASK));
```

(false)



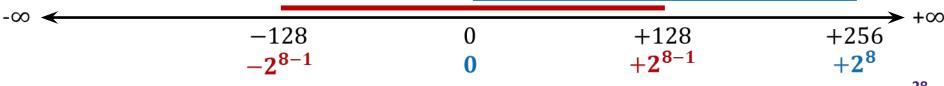
#### Integers

- Binary representation of integers
  - Unsigned and signed
- Shifting and arithmetic operations for Lab 1a
- In C: Signed, Unsigned and Casting
- Consequences of finite width representations
  - Overflow, sign extension
- CS sprung out of mathematics
  - Creators wanted to match existing axioms

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#### **Encoding Integers**

- The hardware (and C) supports two flavors of integers
  - unsigned only the non-negatives
  - signed both negatives and non-negatives
- Cannot represent all integers with w bits
  - Only 2<sup>w</sup> distinct bit patterns
  - Unsigned values:  $0 \dots 2^w 1$
  - Signed values:  $-2^{w-1} \dots 2^{w-1} 1$
- \* Example: 8-bit integers (e.g. char)



#### Unsigned Integers; "standard" binary

- Unsigned values follow the standard base 2 system
  - $b_7b_6b_5b_4b_3b_2b_1b_0 = b_72^7 + b_62^6 + \dots + b_12^1 + b_02^0$
- Add and subtract using the normal "carry" and "borrow" rules, just in binary

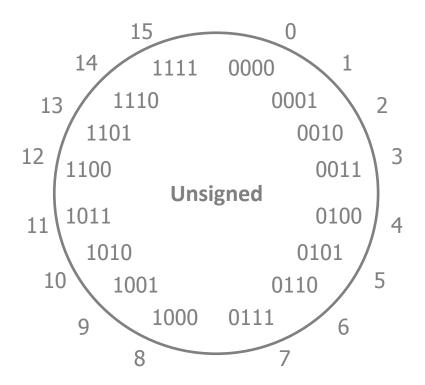
- ∘ Useful:  $2^{N-1} + 2^{N-2} + ... + 2 + 1 = 2^N 1$ 
  - *i.e.* N ones in a row =  $2^N 1$

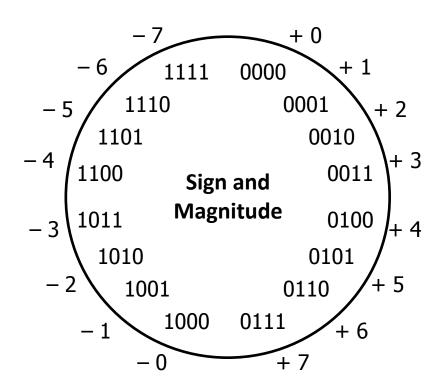
## Sign-Magnitude Encoding Most Significant Bit

- Designate the high-order bit (MSB) as "sign bit"
  - sign=0: positive #'s; sign=1: negative #'s
- Benefits:
  - Using MSB as sign bit matches positive numbers with unsigned
  - All zeros encoding is still = 0
- Examples (8 bits):
  - $0x00 = 000000000_2$  is non-negative; the sign bit is 0
  - $0x7F = 011111111_2$  is non-negative (+127<sub>10</sub>)
  - $0x85 = 10000101_2$  is negative (-5<sub>10</sub>)
  - $0x80 = 10000000_{2}$  is negative.... zero???

#### Sign-Magnitude Encoding

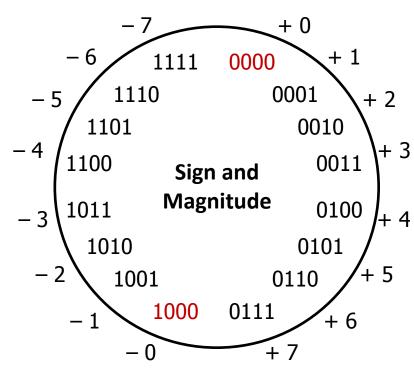
- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks?





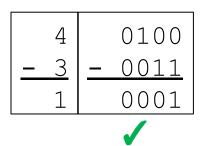
## Sign-Magnitude Encoding

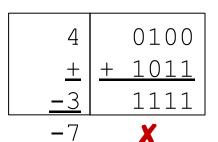
- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks:
  - Two representations of 0 (bad for checking equality)

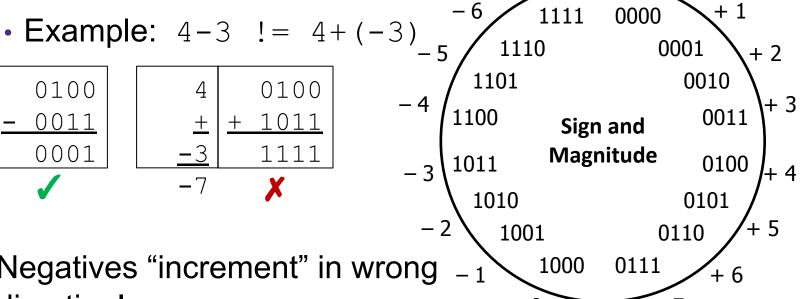


## Sign-Magnitude Encoding

- MSB is the sign bit, rest of the bits are magnitude
- **Drawbacks:** 
  - Two representations of 0 (bad for checking equality)
  - Arithmetic is cumbersome



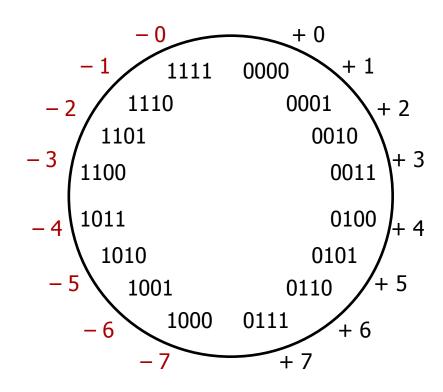




 Negatives "increment" in wrong direction!

#### **Two's Complement**

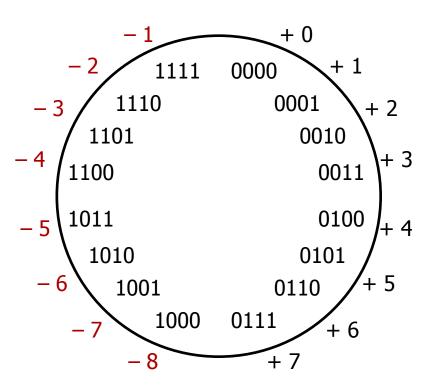
- Let's fix these problems:
  - 1) "Flip" negative encodings so incrementing works



#### **Two's Complement**

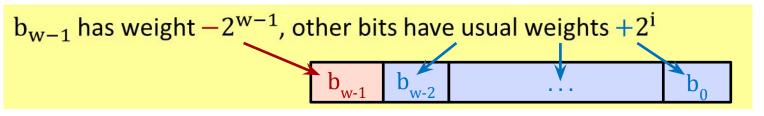
- Let's fix these problems:
  - 1) "Flip" negative encodings so incrementing works
  - 2) "Shift" negative numbers to eliminate -0

- MSB still indicates sign!
  - This is why we represent one more negative than positive number  $(-2^{N-1})$  to  $2^{N-1}$

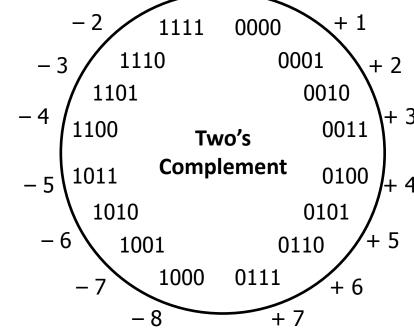


#### **Two's Complement Negatives**

Accomplished with one neat (mathematical) trick!

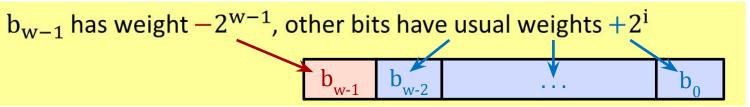


- 4-bit Examples:
  - 1010<sub>2</sub> unsigned:  $1*2^3+0*2^2+1*2^1+0*2^0=10$
  - $1010_2$  two's complement:  $-1*2^3+0*2^2+1*2^1+0*2^0=-6$



#### **Two's Complement Negatives**

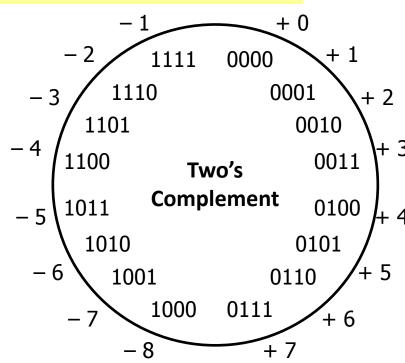
Accomplished with one neat (mathematical) trick!



-1 represented as:

$$1111_2 = -2^3 + (2^3 - 1)$$

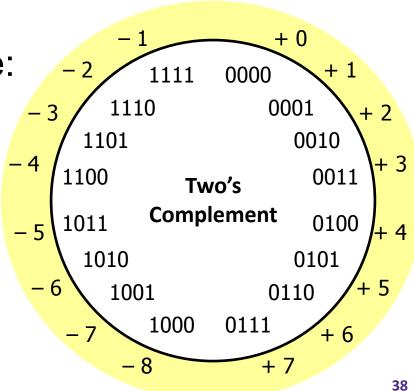
- MSB makes it super negative!
- Add up all the other bits to get back up to -1



#### Why Two's Complement is So Great

- Roughly same number of (+) and (–) numbers
- Positive number encodings match unsigned
- Single zero encoding, all zeros
- Simple negation procedure:
  - Get negative representation of any integer by taking bitwise complement and then adding one!

$$( \sim x + 1 == -x )$$



#### Polling Question [Int I - b]

- $_{\circ}$  Take the **4-bit number encoding x = 0b1011**
- What value would this encoding have in...
  - Unsigned? Sign and Magnitude? Two's Complement?



-4



-5





-3



#### Integers

- Binary representation of integers
  - Unsigned and signed
- Shifting and arithmetic operations (Lab 1a)
- In C: Signed, Unsigned and Casting
- Consequences of finite width representations
  - Overflow, sign extension

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#### **Shift Operations**

- $_{\circ}$  Left shift (x<<n) bit vector x by n positions
  - Throw away (drop) extra bits on left
  - Fill with 0s on right
- Right shift (x>>n) bit-vector x by n positions
  - Throw away (drop) extra bits on right
  - Logical shift (for unsigned values)
    - Fill with 0s on left
  - Arithmetic shift (for signed values)
    - Replicate most significant bit on left
    - Maintains sign of x

#### **Shift Operations**

- Left Shift (x << n)</li>
  - Fill with 0s on right
- Right shift (x >> n)
  - Logical shift (for unsigned values)
    - Fill with 0s on left
  - Arithmetic shift (for signed values)
    - Replace with MSB on left

Notes

- Shift by n < 0 or n >= w are undefined
- In C: behavior of >> is determined by the compiler
  - Depends on type of x (signed/unsigned)

	Х	0010	0010
	x<<3	0001	0000
logical:	x>>2	0000	1000
arithmetic:	x>>2	0000	1000
`			

Х	1010	0010
x<<3	0001	0000
x>>2	<b>00</b> 10	1000
x>>2	<b>11</b> 10	1000

logical:

arithmetic:

#### **Shifting Arithmetic?**

- What are the following computing?
  - x>>n
    - 0b 0100 >> 1 = 0b 0010
    - 0b 0100 >> 2 = 0b 0001
    - Divide by 2<sup>n</sup>
  - x<<n</li>
    - 0b 0001 << 1 = 0b 0010
    - 0b 0001 << 2 = 0b 0100
    - Multiply by 2<sup>n</sup>
- Shifting "faster" than multiply/divide operations

#### Left Shifting Arithmetic 8-bit Example

- No difference in left shift operation for unsigned and signed numbers (just manipulates bits)
  - Difference comes during interpretation: x\*2<sup>n</sup>?

Signed Unsigned 
$$x = 25$$
;  $00011001 = 25$   $25$   $L1=x<<2$ ;  $0001100100 = 100$   $100$   $L2=x<<3$ ;  $00011001000 = -56$   $200$  signed overflow  $L3=x<<4$ ;  $000110010000 = -112$   $144$  unsigned overflow

### Right Shifting Arithmetic 8-bit Examples

 Reminder: C operator >> does logical shift on unsigned values and arithmetic shift on signed values

```
    Logical Shift: x/2<sup>n</sup>?

xu = 240u;
                                     = 240
                                          30
R1u=xu>>3; 000
R2u=xu>>5; 0000011110000
                                 rounding
                                  (down)
```

### Right Shifting Arithmetic 8-bit Examples

- Reminder: C operator >> does logical shift on unsigned values and arithmetic shift on signed values
  - Arithmetic Shift: x/2<sup>n</sup>?

$$xs = -16;$$
 11110000 = -16  
R1s=xu>>3; 11111110000 = -2

rounding (down)

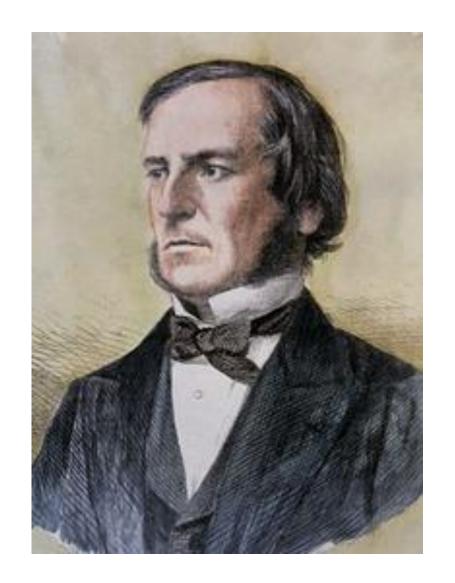
#### Summary

- Bit-level operators allow for fine-grained manipulations of data
  - Bitwise AND (๑), OR (□), and NOT (~) different than logical AND (๑๑), OR (□), and NOT (!)
  - Especially useful with bit masks
- Choice of encoding scheme is important
  - Tradeoffs based on size requirements and desired operations
- Integers represented using unsigned and two's complement representations
  - Limited by fixed bit width
  - We'll examine arithmetic operations next lecture
- Shifting is a useful bitwise operator
  - Right shifting can be arithmetic (sign) or logical (0)
  - Can be used in multiplication with constant or bit masking

#### Knowledge & Insulation

#### George Boole

- A committed educator!
  - Walked in the rain to teach, taught in wet clothes, wife wrapped him in wet blankets to cure him
- Philosophy, logic
  - Pseudo-religious doctrine of psychology
  - HEAVILY influenced by Indian systems of logic



#### Colonialism in 19<sup>th</sup> Century

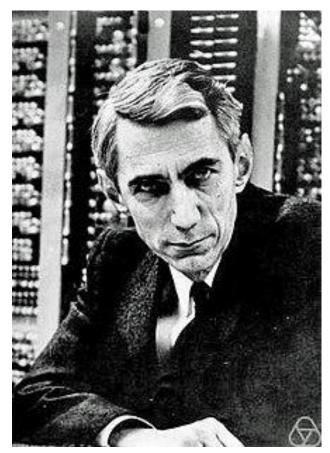
- <sub>o</sub> 18<sup>th</sup> /19<sup>th</sup> century India under colonial rule
  - Re-education camps, the whole business

"On examining this work I saw in it, not merely merit worthy of encouragement, but merit of a peculiar kind, the encouragement of which, as it appeared to me, was likely to promote native effort towards the restoration of the native mind in India."

DeMorgan, prefacing: "A treatise on problems of maxima and minima, solved by algebra." by Ram Chundra, European Publication in 1859

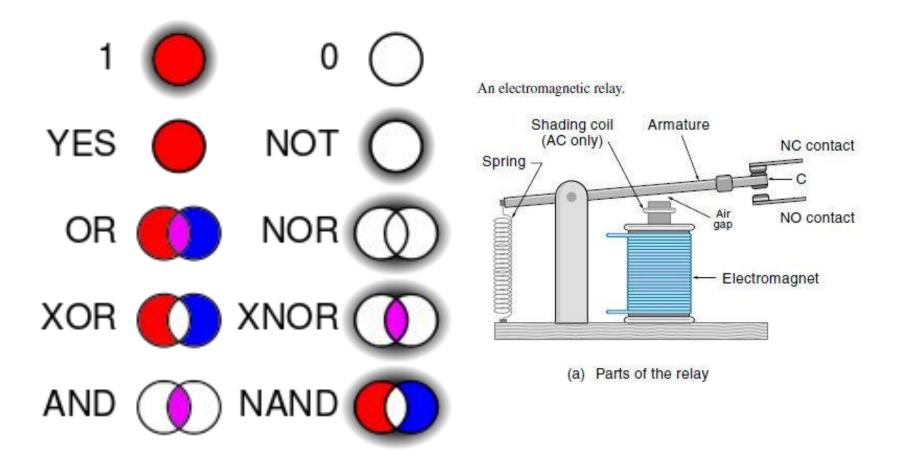
#### **Claude Shannon (1916-2001)**

- Mathematician
  - Took a philosophy class! Not CS!
  - Learned Boole's algebra, realized relevance to digital logic
- 1937 Master's Thesis
  - "Founded" information theory



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#### I mean, it's a close connection

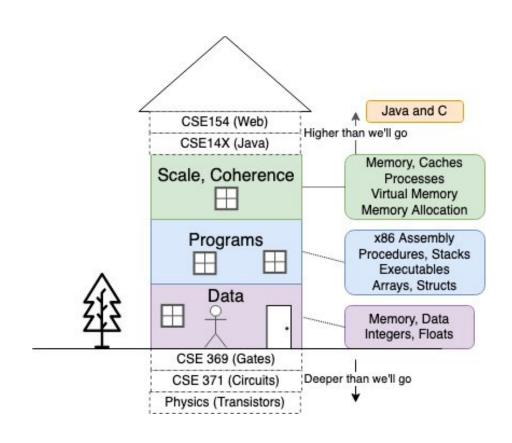


Ba dum **dum** 

## The "best" ideas come from outside CS!

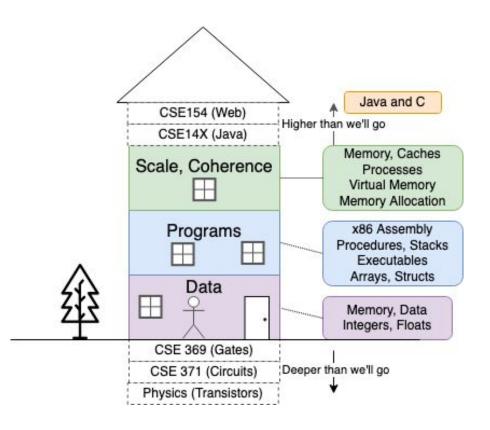


#### Insulation: a metaphor



#### Insulation: a metaphor

- It keeps us warm!
  - It protects us from the outside!
- It can be harmful!
  - Asbestos, clearly
  - Fiberglass is still bad
  - Modern alternatives still harmful



# Insulation keeps us comfortable and protected.

# I'd argue that CS is insulated! But, why?

#### Amy's Keynote

- Briefly, CS has been a safe space for:
  - Women, whose jobs were automated by computers
  - Bullied kids that liked calculators
  - Wealthy "young geeks" (me)
  - Closeted trans kids (me)
  - Autistic people (me)
  - Queer folks trying to move to more progressive cities (SF/Seattle)
  - Those seeking social mobility

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  - Autistic people (me)
  - Queer folks trying to move to more progressive cities (SF/Seattle)
  - Those seeking social mobility
- Y'all aren't the target audience, what'd you think?

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#### Autism, extremely briefly



In the last few decades, we have detected thousands of planets outside of our solar system. They did not suddenly appear at the time we found them. No one calls it an "exoplanet epidemic". The planets were always there, just previously undiscovered. This is a tweet about autism.

More information @ https://autisticadvocacy.org/about-asan/about-autism/

#### Autism, extremely briefly

- Special interests: intense, passionate fixations on topics, "little professors"
  - Turing was "obsessed" with codes and ciphers, didn't care about much else
- Theory of mind (some low, some high):
  - Understanding that the brains of others are different than yours
- Empathy (some low, some high):
  - I have to be careful who I'm holding space for, others not so much
- Communication
  - Neurodiverse folks tend to communicate well with each other, less well with neurotypical folks

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#### Autism, and computing

- Attention to detail highly valued in CS
- Historically, CS is an intellectual space centering rules (theory) and routines (algorithms)
- Little social interaction required, "don't bother me" assumed
- Lots of space to pursue special interests without interruption!
- Most of the world expect neurotypical conformity, most of CS emphasizes conformity along some\* aspects of neurodiversity
  - Again, UW is much better than my undergrad

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#### Insulation and me

- I identify as neurodiverse (autistic), trans, enby
- CS is was a great closet...
  - Encouraged to be "in my head", outside my body
  - Focus on patterns, abstract concepts
  - Completely ignore society
- $_{\circ}$  ...but, really, that's no way to live
  - I promise, it would've killed me
- At some point, leave the house and go exploring!
  - Though camping can still be a stretch

#### I promise, I'm not alone

CSE351, Summer 2021

- Generally, transgender rate of 0.5% to 1%
- Autism Spectrum Disorder (ASD) around 1.5%
- 8-15% ASD among trans\* folks!
- Higher rates of ASD in STEM than elsewhere
  - Most CS folks tend not to recognize neurodiversity

 I got to undergrad and realized that lots of folks were like me, realized later that most of them were autistic.

#### Making Space in CS

- Foundations of computing:
  - Insulating for safety (i.e. being trans in Seattle, at UW)
  - Exploring new frontiers (even just philosophy)
- Both foundational!
  - Though, many focus on one or the other
  - Technical/Socio-technical; you need both!
- CS needs both to survive and thrive!
  - New ideas and safe spaces both necessary

#### Making Space in CS



#### **Exploring Space, Making Space**

- Goodness, if you can, take classes outside CS
  - Even better if they're non-technical!
  - Philosophy, Art, History, Education??
- Claude Shannon, Founder of Information Theory
  - Go take a class because it looks interesting!
  - Maybe it'll be relevant to computing?
  - More likely, you'll learn something about yourself!

## Insulation is Necessary and Deadly!

## Try to make space when you can!