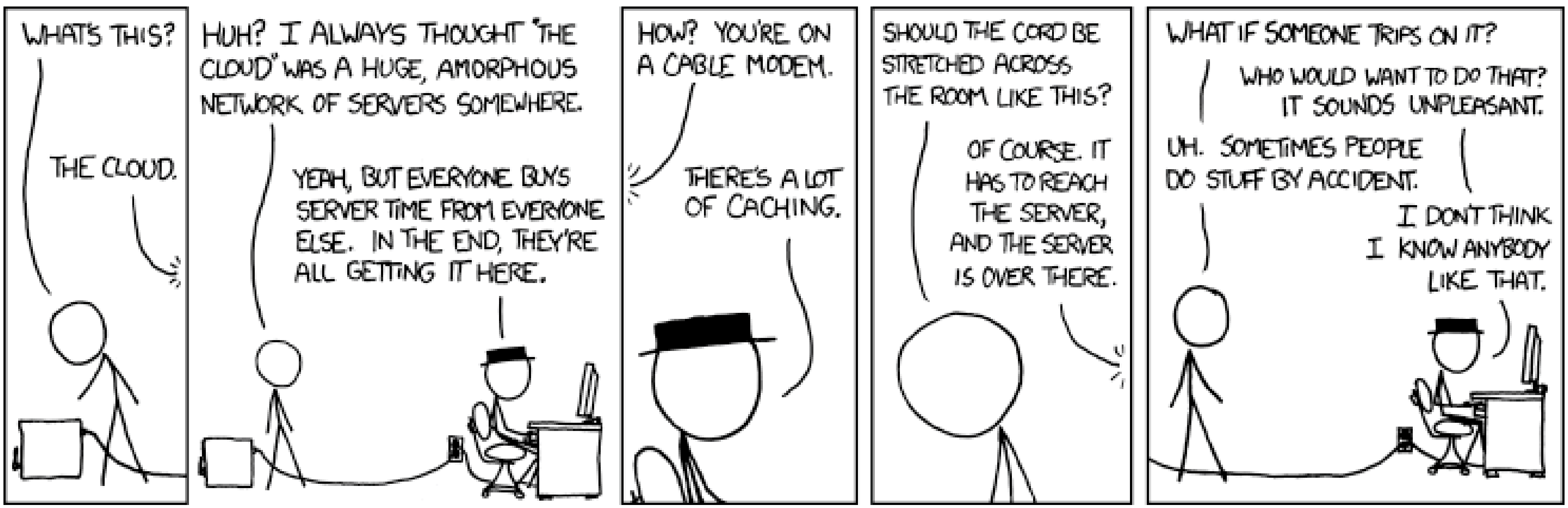
Caches IV

CSE 351 Spring 2021

Instructor:	**Teaching Assistants:**		
Ruth Anderson	Allen Aby	Joy Dang	Alena Dickmann
	Catherine Guevara	Corinne Herzog	Ian Hsiao
	Diya Joy	Jim Limprasert	Armin Magness
	Aman Mohammed	Monty Nitschke	Allie Pfleger
	Neil Ryan	Alex Saveau	Sanjana Sridhar
	Amy Xu		



http://xkcd.com/908/

Administrivia

- hw16 due TONIGHT (5/10)
- hw17 due Wednesday (5/12)
- Lab 3 due Wednesday (5/12)
- hw19 due Friday (5/14)
  - Lab 4 preparation
- Lab 4 coming soon!
  - Cache parameter puzzles and code optimizations

- **Questions Docs**: Use @uw google account to access!!
  - https://tinyurl.com/CSE351-21sp-Questions

Reading Review

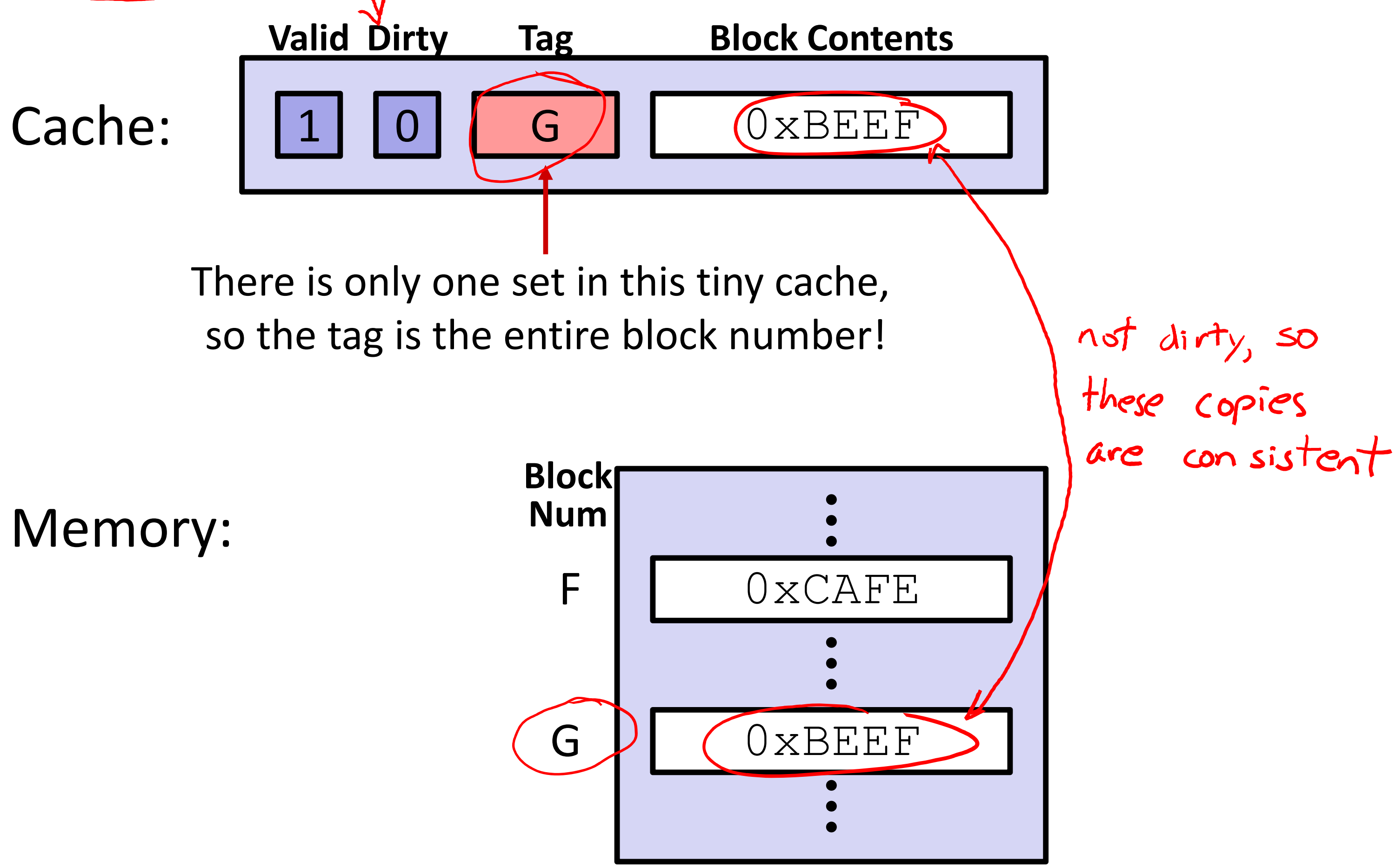
- Terminology:
  - Write-hit policies: write-back, write-through
  - Write-miss policies: write allocate, no-write allocate
  - Cache blocking

What about writes?

- Multiple copies of data may exist:
  - multiple levels of cache and main memory
- What to do on a write-hit? (block/data already in $)
  - Write-through: write immediately to next level
  - Write-back: defer write to next level until line is evicted (replaced)
    - Must track which cache lines have been modified ("*dirty bit*") ← extra management bit only for write-back cache
- What to do on a write-miss? (block/data not currently in $)
  - Write allocate: ("fetch on write") load into cache, then execute the write-hit policy
    - Good if more writes or reads to the location follow
  - No-write allocate: ("write around") just write immediately to next level
- Typical caches:
  - Write-back + Write allocate, usually
  - Write-through + No-write allocate, occasionally

Write-back, Write Allocate Example

Note: While unrealistic, this example assumes that all requests have offset 0 and are for a block's worth of data.

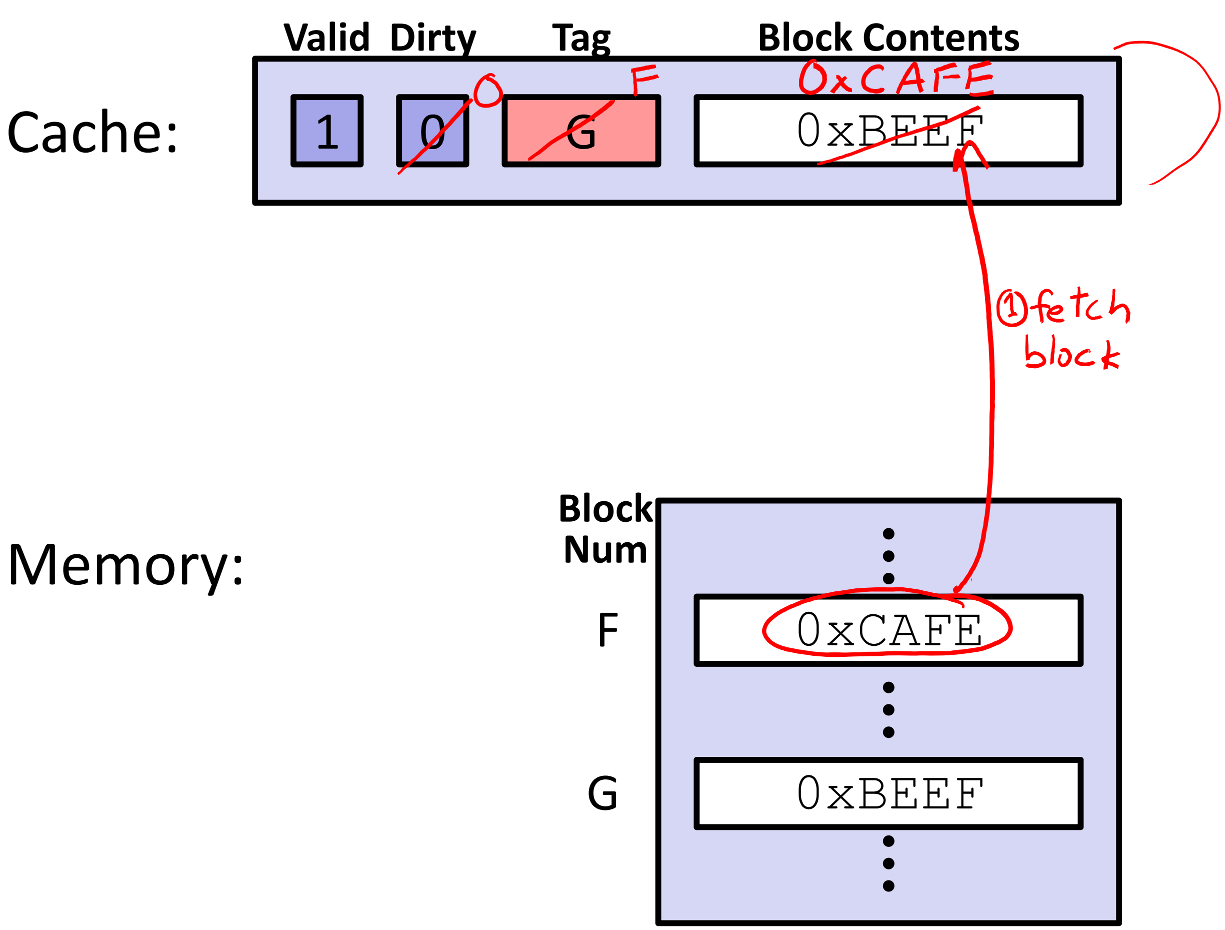


Write-back, Write Allocate Example

Not valid x86, just using block num instead of full byte address to keep the example simple

1) `mov $0xFACE, (F)`

Write Miss!

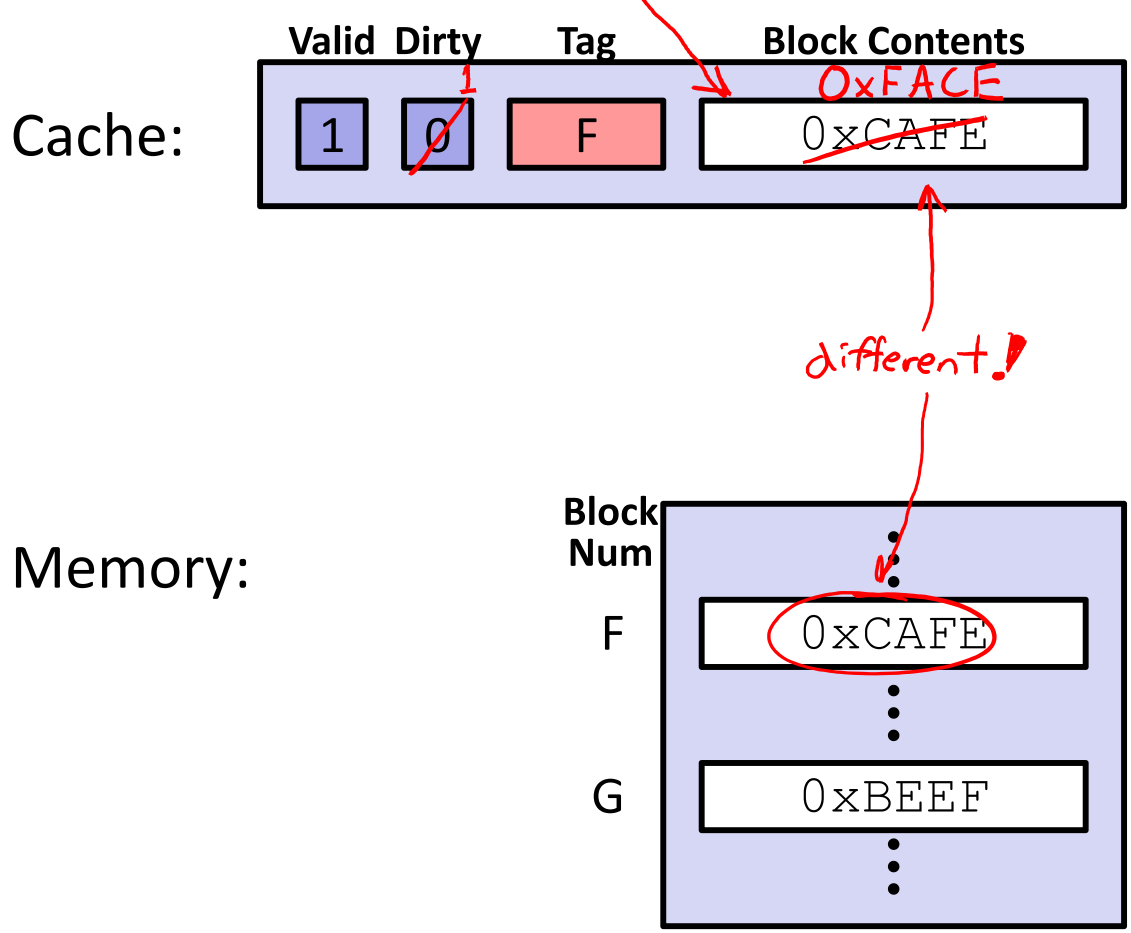


Step 1: Bring **F** into cache

Write-back, Write Allocate Example

1) `mov $0xFACE, (F)`

Write Miss

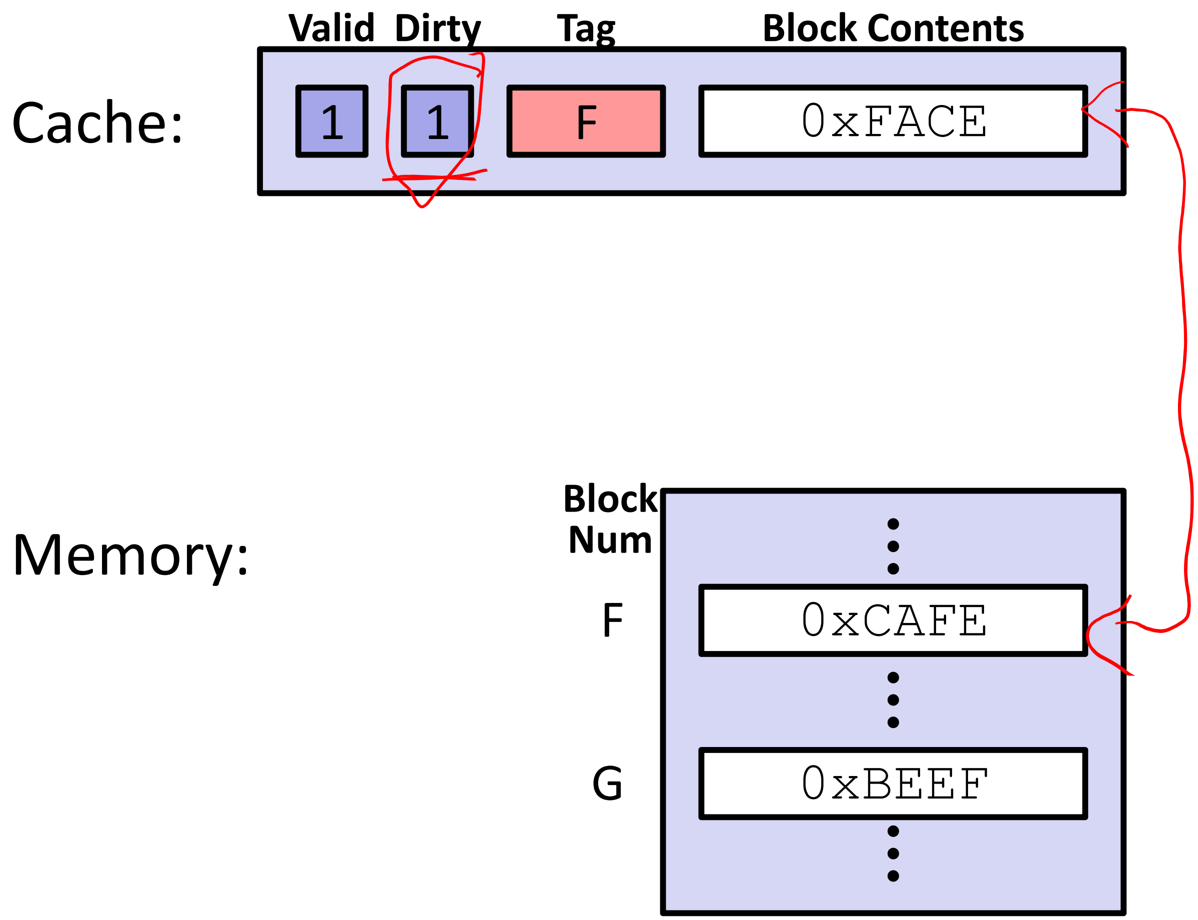


Step 1: Bring **F** into cache

Step 2: Write `0xFACE` to cache only and set the dirty bit

Write-back, Write Allocate Example

1) `mov $0xFACE, (F)`
Write Miss



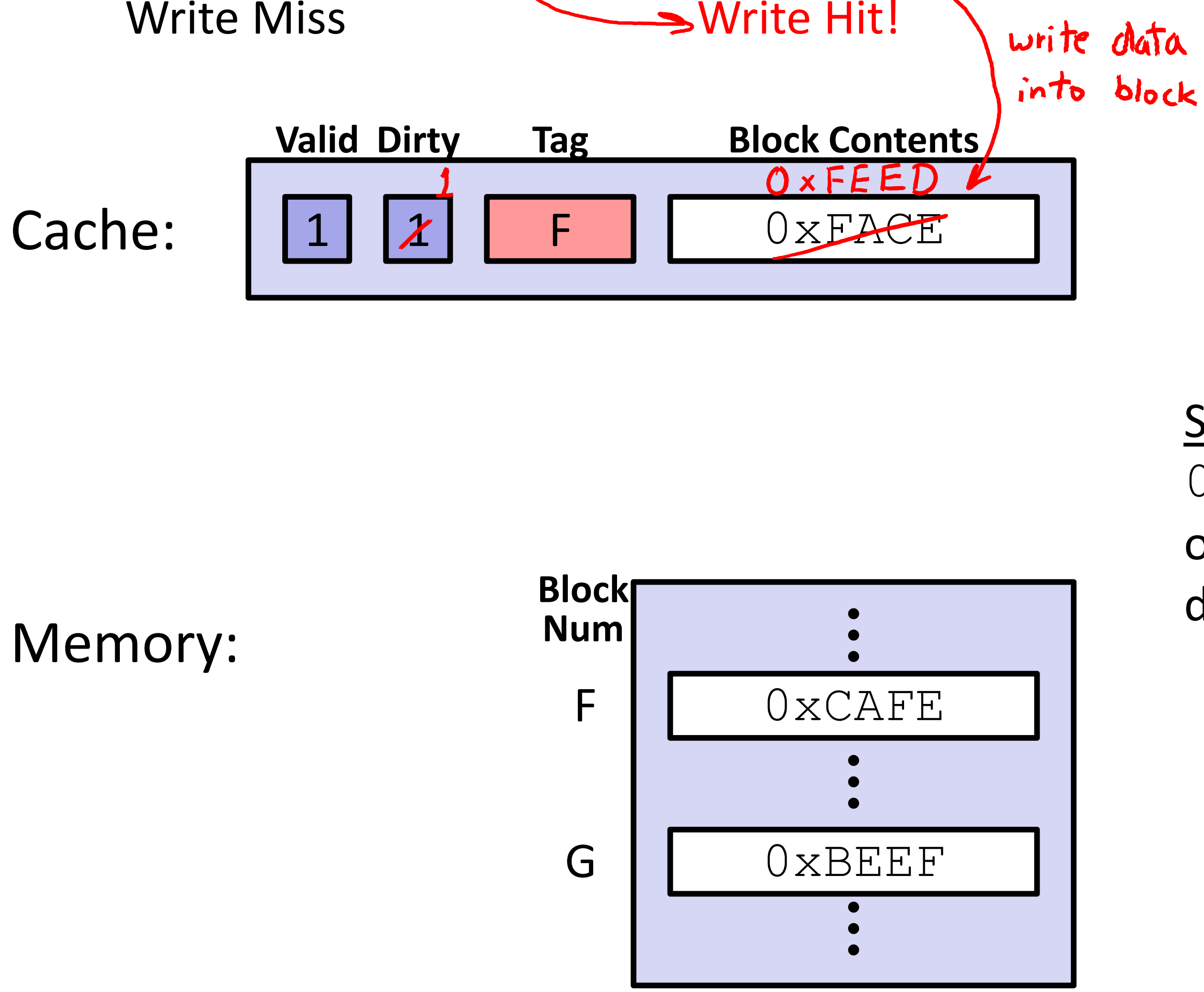
Step 1: Bring **F** into cache

Step 2: Write 0xFACE to cache only and set the dirty bit

Write-back, Write Allocate Example

1) `mov $0xFACE, (F)` Write Miss

2) `mov $0xFEED, (F)` Write Hit!

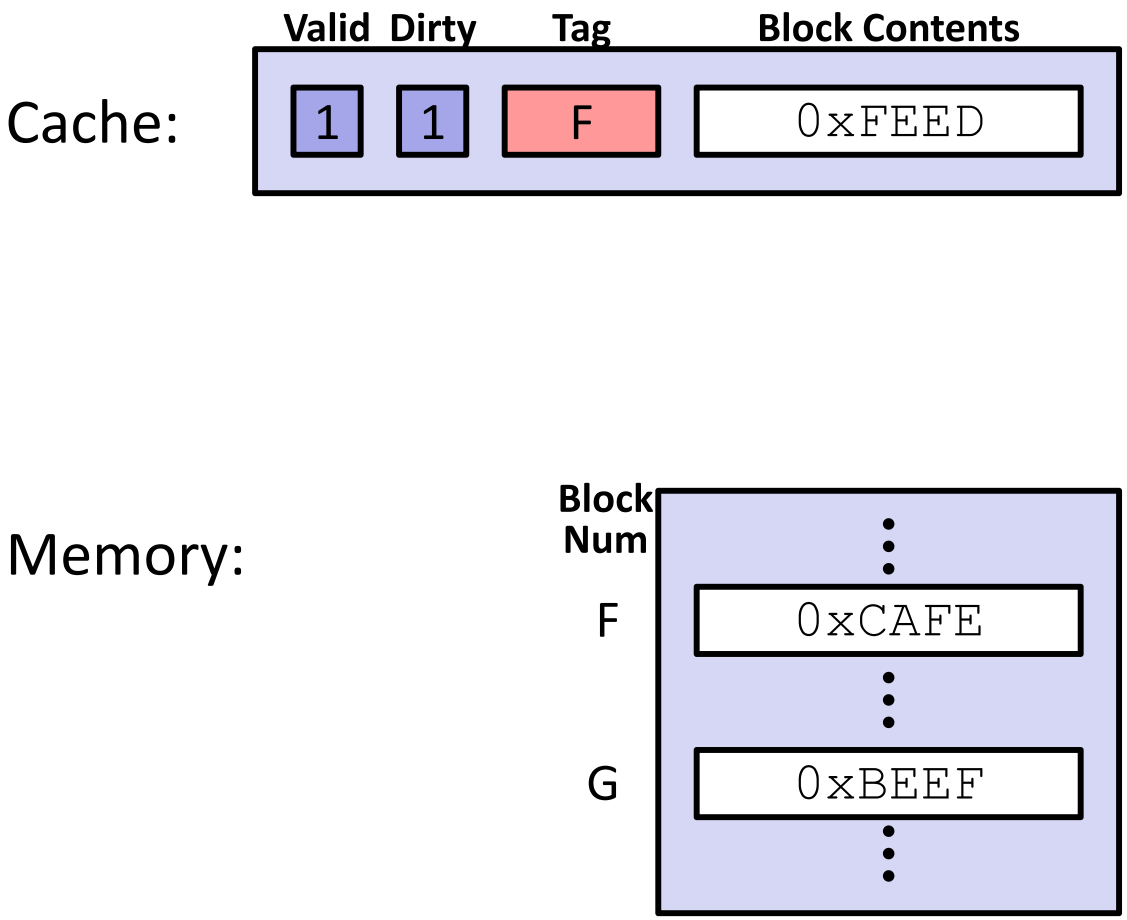


Step: Write `0xFEED` to cache only (and set the dirty bit)

Write-back, Write Allocate Example

1) `mov $0xFACE, (F)` Write Miss

2) `mov $0xFEED, (F)` Write Hit

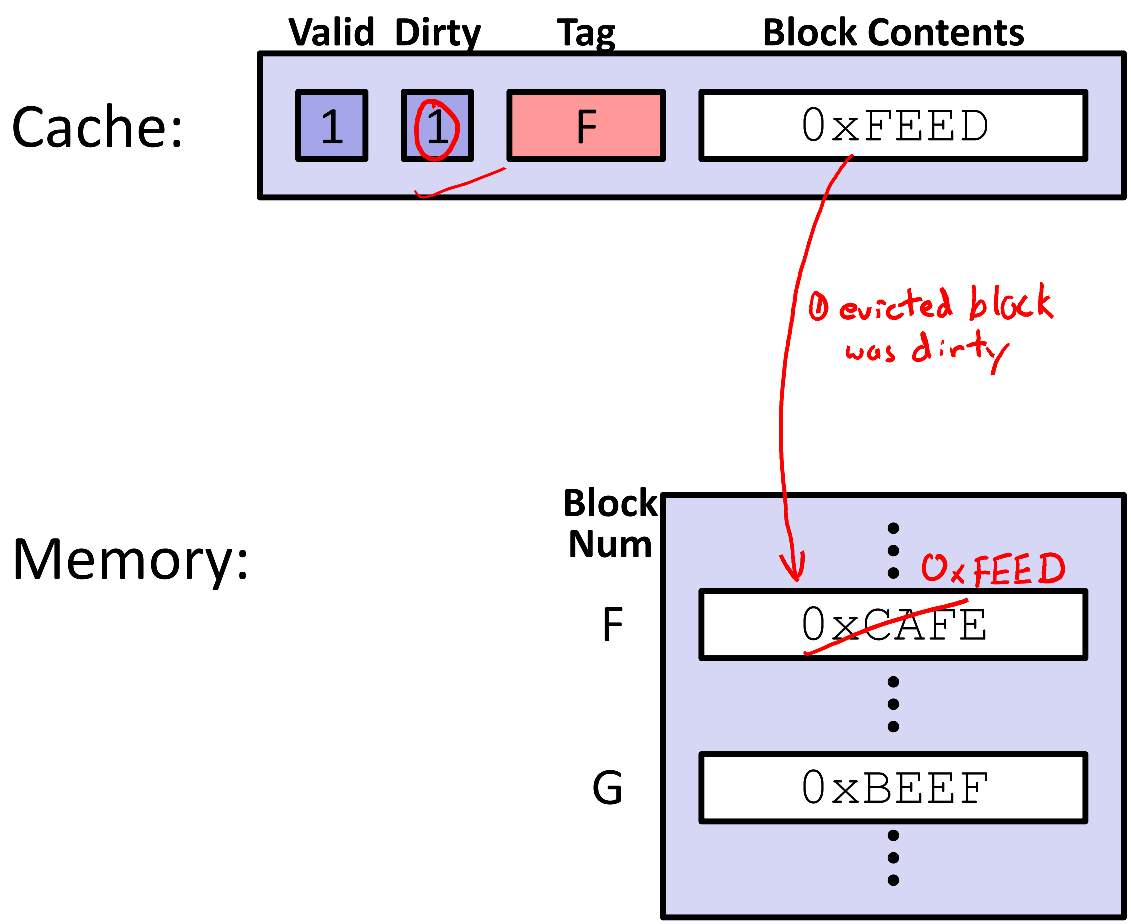


Write-back, Write Allocate Example

1) `mov $0xFACE, (F)` Write Miss

2) `mov $0xFEED, (F)` Write Hit

3) `mov (G), %ax` Read Miss!



Step 1: Write **F** back to memory since it is dirty

%rax

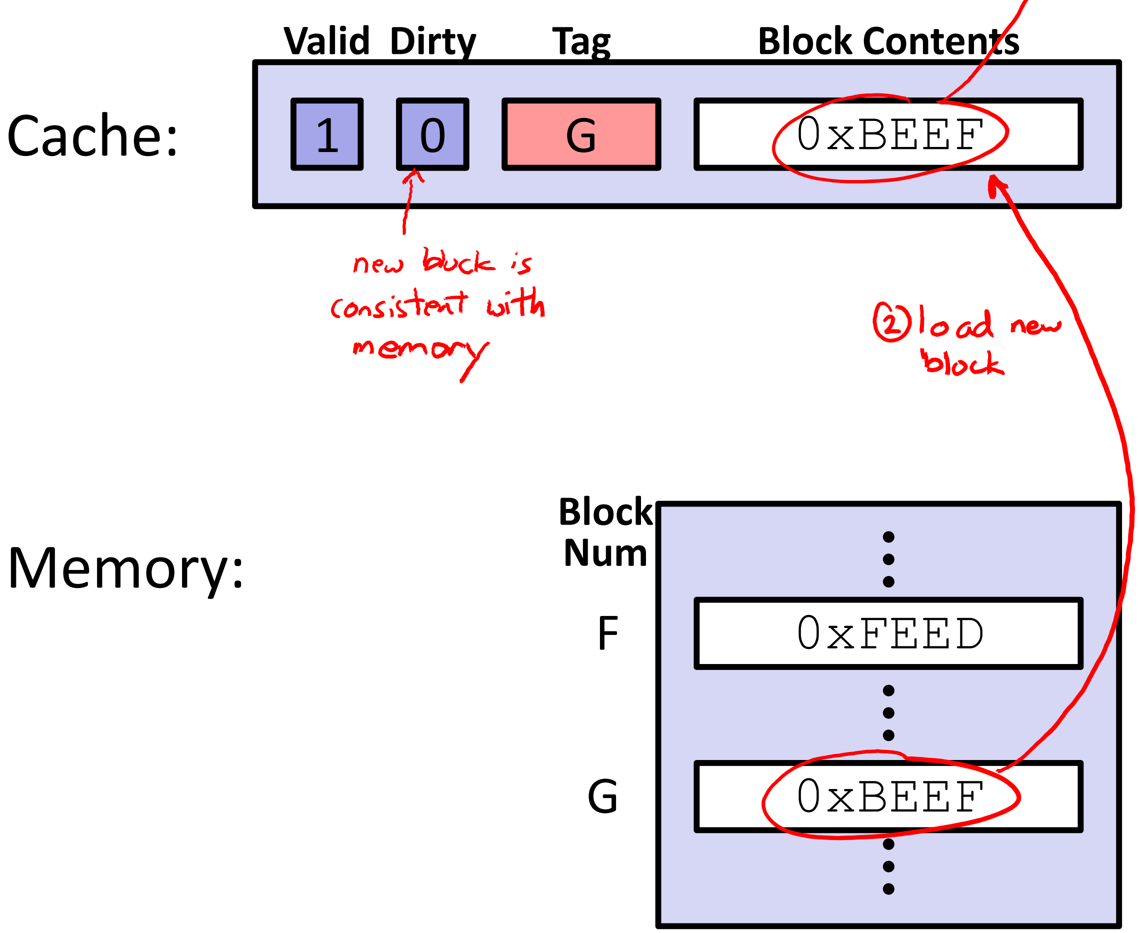
Write-back, Write Allocate Example

1) `mov $0xFACE, (F)` Write Miss

2) `mov $0xFEED, (F)` Write Hit

3) `mov (G), %ax` Read Miss

③ copy into %ax



Step 1: Write **F** back to memory since it is dirty

Step 2: Bring **G** into the cache so that we can copy it into `%ax`

Cache Simulator

- Want to play around with cache parameters and policies? Check out our cache simulator!
  - https://courses.cs.washington.edu/courses/cse351/cachesim/

- Way to use:
  - Take advantage of "explain mode" and navigable history to test your own hypotheses and answer your own questions
  - Self-guided Cache Sim Demo posted along with Section 7
  - Will be used in hw19 – Lab 4 Preparation

Polling Question

- Which of the following cache statements is FALSE?
  - Vote in Ed Lessons

A. We can reduce compulsory misses by decreasing our block size
smaller block size pulls fewer bytes into $ on a miss

B. We can reduce conflict misses by increasing associativity
more options to place blocks before evictions occur

C. A write-back cache will save time for code with good temporal locality on writes
frequently-used blocks rarely get evicted, so fewer write-backs

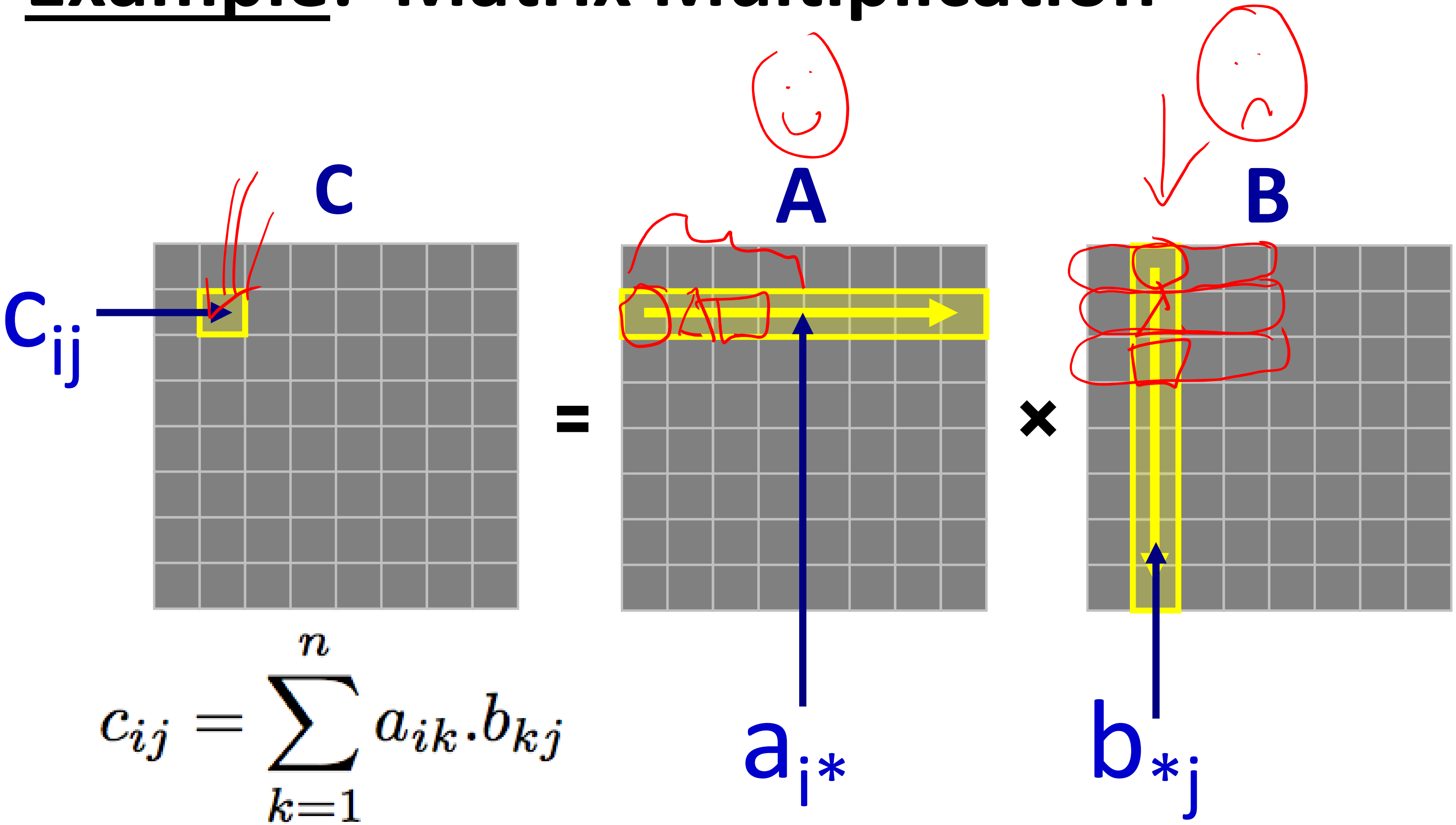
D. A write-through cache will always match data with the memory hierarchy level below it
yes, its main goal is data consistency

E. We're lost…

Optimizations for the Memory Hierarchy

- Write code that has locality!
  - Spatial: access data contiguously
  - Temporal: make sure access to the same data is not too far apart in time
- How can you achieve locality?
  - Adjust memory accesses in *code* (software) to improve miss rate (MR)
    - Requires knowledge of *both* how caches work as well as your system's parameters
  - Proper choice of algorithm
  - Loop transformations

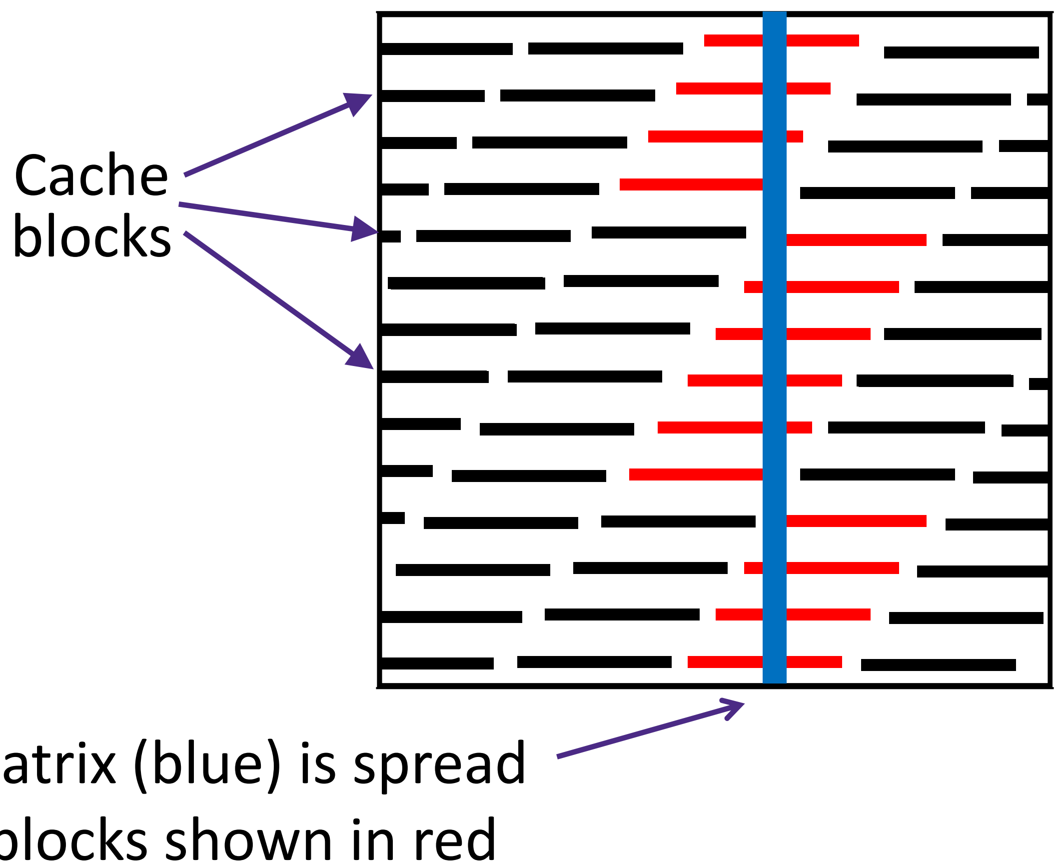
Example: Matrix Multiplication



$$c_{ij} = \sum_{k=1}^{n} a_{ik}.b_{kj}$$

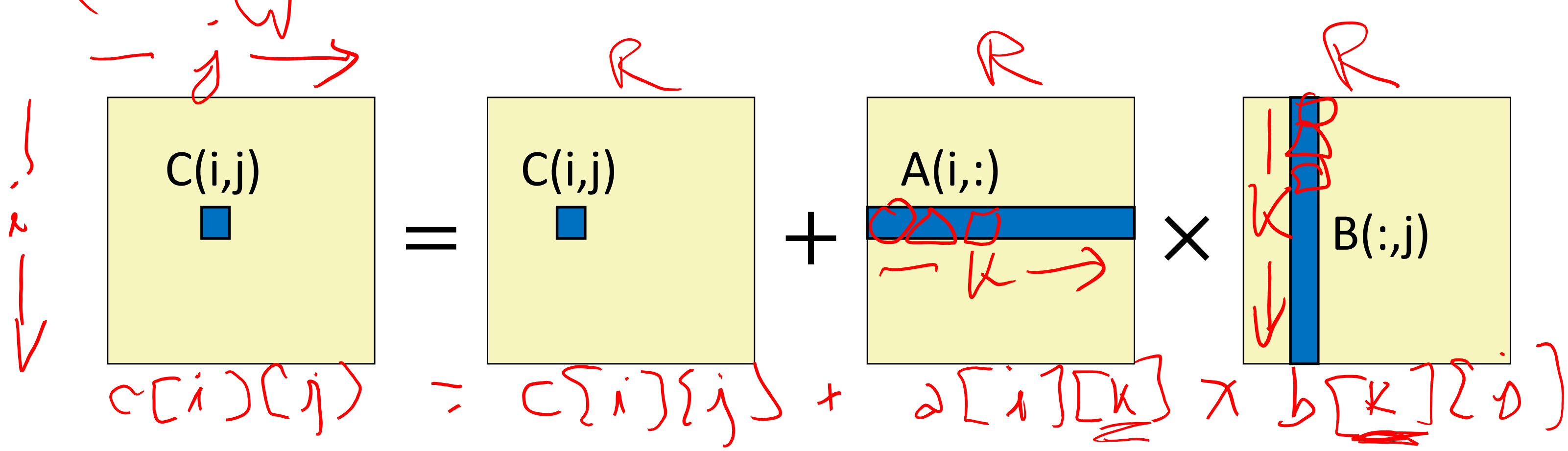
Matrices in Memory

- How do cache blocks fit into this scheme?
  - Row major matrix in memory:



Naïve Matrix Multiply

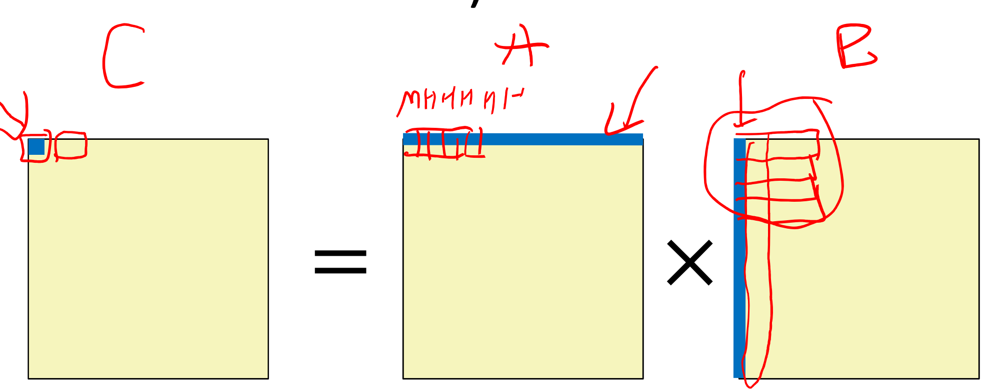
```
# move along rows of A
for (i = 0; i < n; i++)
  # move along columns of B
  for (j = 0; j < n; j++)
    # EACH k loop reads row of A, col of B
    # Also read & write c(i,j) n times
    for (k = 0; k < n; k++)
      c[i*n+j] += a[i*n+k] * b[k*n+j];
```



Cache Miss Analysis (Naïve)

Ignoring matrix c

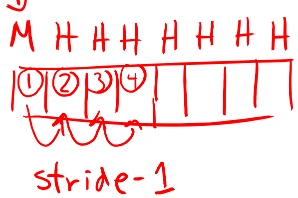
- Scenario Parameters:
  - Square matrix ($n \times n$), elements are doubles
  - Cache block size K = 64 B = 8 doubles ← 8 matrix elements per cache block
  - Cache size $C \ll n$ (much smaller than n) — key assumption!
- Each iteration:
  - $\frac{n}{8} + n = \frac{9n}{8}$ misses (A: $\frac{n}{8}$, B: n)

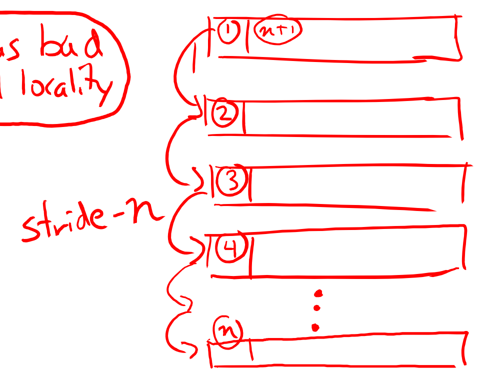


B has bad spatial locality

A has good spatial locality

compulsory





by the time we get to n+1, block has been kicked out of $

Cache Miss Analysis (Naïve)

Ignoring matrix c

- Scenario Parameters:
  - Square matrix ($n \times n$), elements are `double`s
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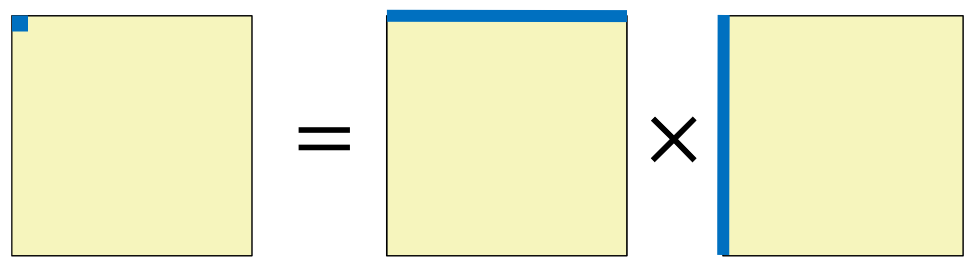
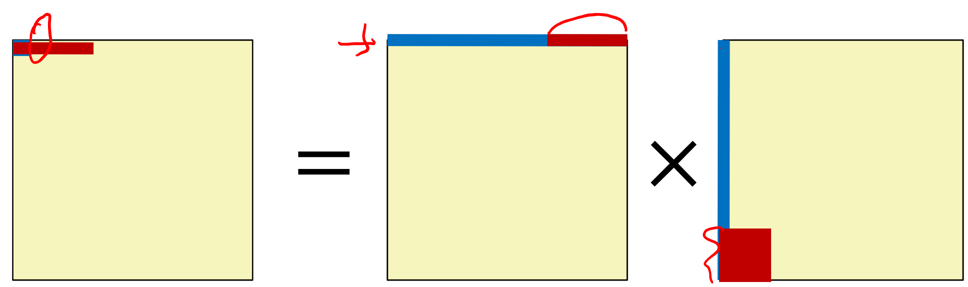
  - $\frac{n}{8} + n = \frac{9n}{8}$ misses



  - Afterwards in cache: (schematic)

red showing blocks remaining in the $



Cache Miss Analysis (Naïve)

Ignoring matrix `c`

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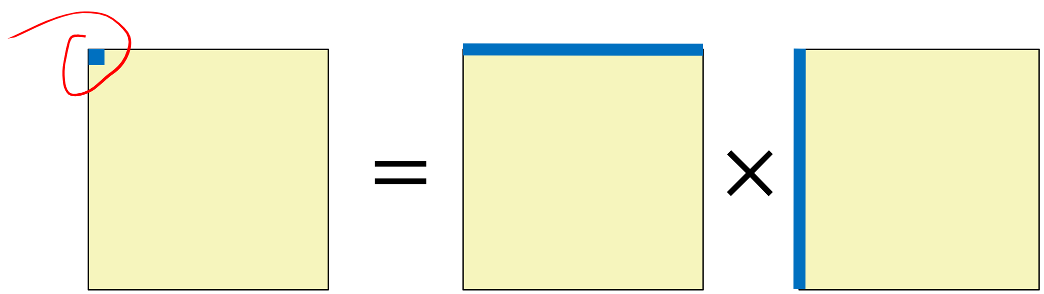


- Total misses: $\frac{9n}{8} \times n^2 = \frac{9}{8}n^3$

once per product matrix element

Linear Algebra to the Rescue (1)

This is extra (non-testable) material

- Can get the same result of a matrix multiplication by splitting the matrices into smaller submatrices (matrix "blocks")

- For example, multiply two 4×4 matrices:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \text{ with } B \text{ defined similarly.}$$

$$AB = \begin{bmatrix} (A_{11}B_{11} + A_{12}B_{21}) & (A_{11}B_{12} + A_{12}B_{22}) \\ (A_{21}B_{11} + A_{22}B_{21}) & (A_{21}B_{12} + A_{22}B_{22}) \end{bmatrix}$$

This is extra (non-testable) material

Linear Algebra to the Rescue (2)

C_{11}	C_{12}	C_{13}	C_{14}
C_{21}	C_{22}	C_{23}	C_{24}
C_{31}	C_{32}	C_{43}	C_{34}
C_{41}	C_{42}	C_{43}	C_{44}

A_{11}	A_{12}	A_{13}	A_{14}
A_{21}	A_{22}	A_{23}	A_{24}
A_{31}	A_{32}	A_{33}	A_{34}
A_{41}	A_{42}	A_{43}	A_{144}

B_{11}	B_{12}	B_{13}	B_{14}
B_{21}	B_{22}	B_{23}	B_{24}
B_{32}	B_{32}	B_{33}	B_{34}
B_{41}	B_{42}	B_{43}	B_{44}

Matrices of size $n \times n$, split into 4 blocks of size r (n=4r)

$$C_{22} = A_{21}B_{12} + A_{22}B_{22} + A_{23}B_{32} + A_{24}B_{42} = \sum_k A_{2k}*B_{k2}$$

- Multiplication operates on small "block" matrices
  - Choose size so that they fit in the cache!
  - This technique called *"cache blocking"*

Blocked Matrix Multiply

r*r

- Blocked version of the naïve algorithm:

```
# move by rxr BLOCKS now
for (i = 0; i < n; i += r)
  for (j = 0; j < n; j += r)
    for (k = 0; k < n; k += r)
      # block matrix multiplication
      for (ib = i; ib < i+r; ib++)
        for (jb = j; jb < j+r; jb++)
          for (kb = k; kb < k+r; kb++)
            c[ib*n+jb] += a[ib*n+kb]*b[kb*n+jb];
```

loop over block matrices

loop within block matrices

  - r = block matrix size (assume r divides n evenly)

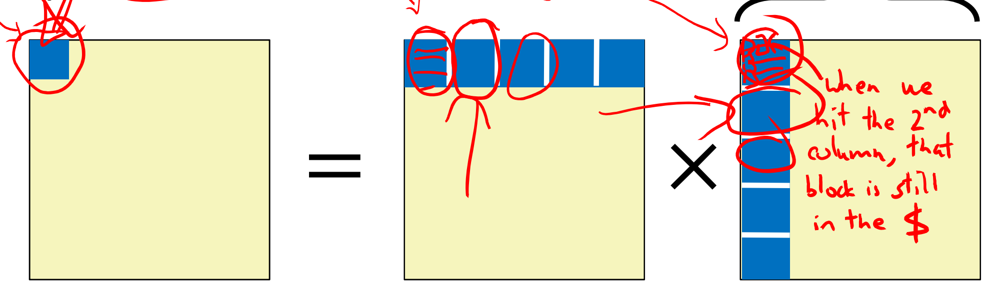
Cache Miss Analysis (Blocked)

Ignoring matrix c

- Scenario Parameters:
  - Cache block size K = 64 B = 8 doubles
  - Cache size $C \ll n$ (much smaller than n)
  - Three blocks ■ $(r \times r)$ fit into cache: $3r^2 < C$

r^2 elements per block, 8 per cache block

n/r blocks



- Each *block* iteration:
  - $\frac{r^2}{8}$ misses per block
  - $\frac{2n}{r} \times \frac{r^2}{8} = \frac{nr}{4}$

n/r blocks in row and column

Cache Miss Analysis (Blocked)

Ignoring matrix `c`

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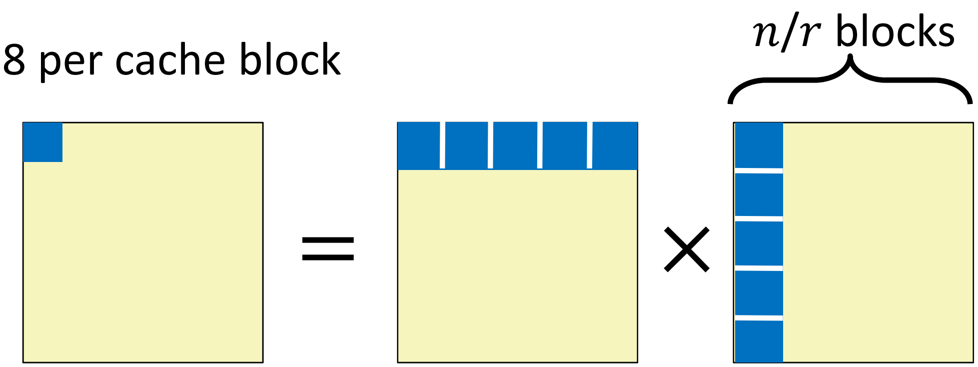
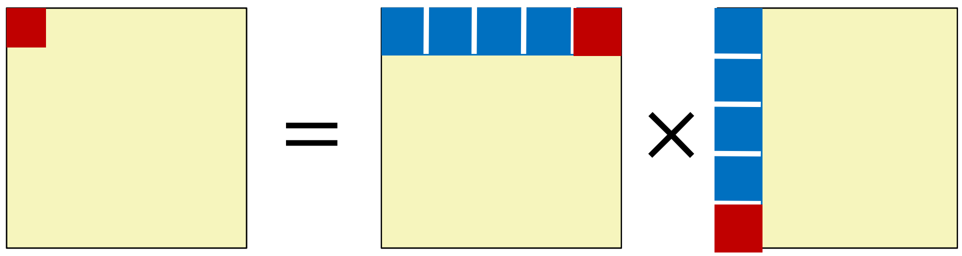


  - Afterwards in cache (schematic)



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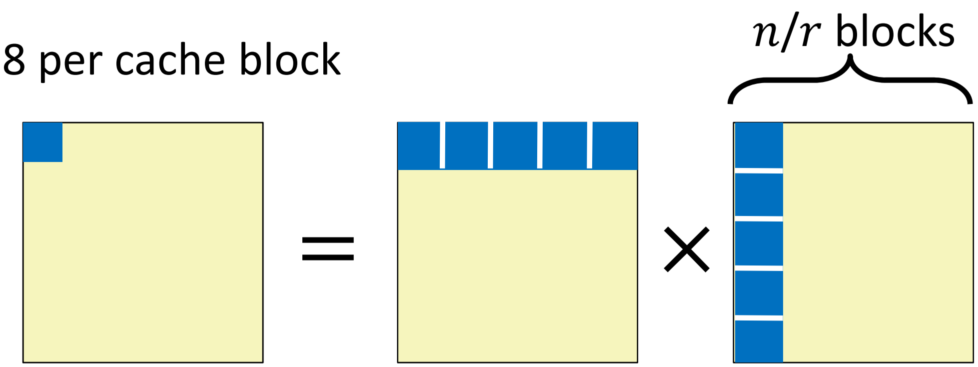


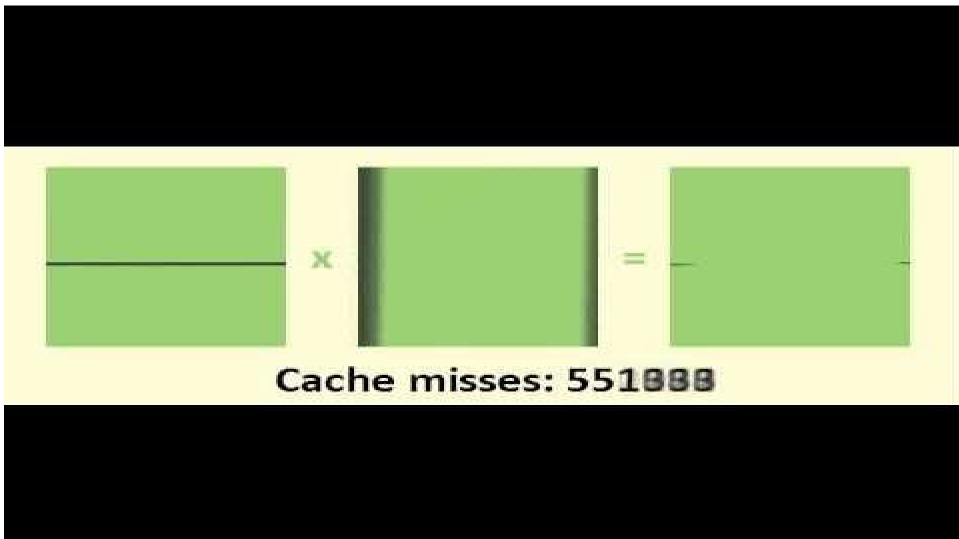
- Total misses:
  - $\left(\frac{nr}{4}\right) \times \left(\frac{n}{r}\right)^2 = \frac{n^3}{4r}$ vs. $9n^3/8$

Matrix Multiply Visualization

- Here $n = 100$, $C = 32$ KiB, $r = 30$

shaded areas show blocks stored in the $

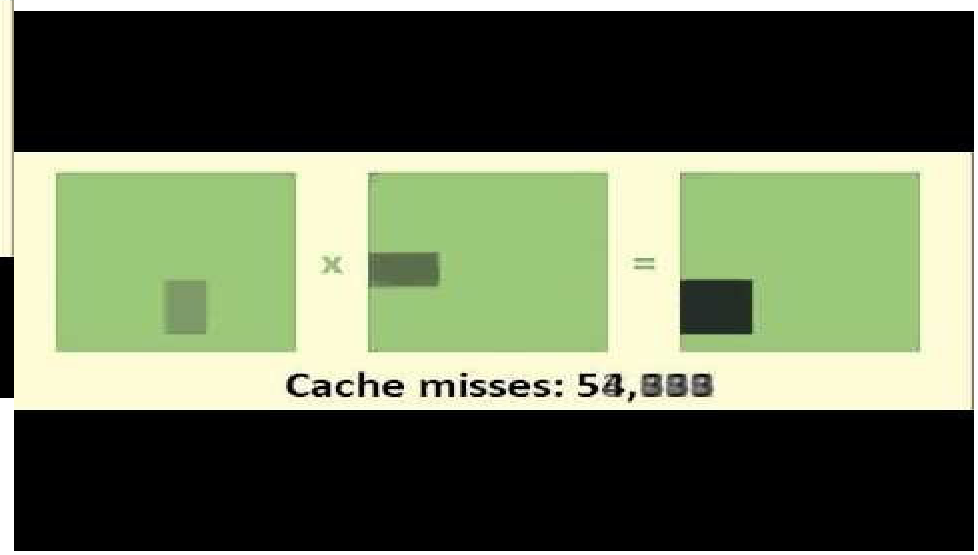
Naïve:



A C

≈ 1,020,000 cache misses

Blocked:



≈ 90,000 cache misses

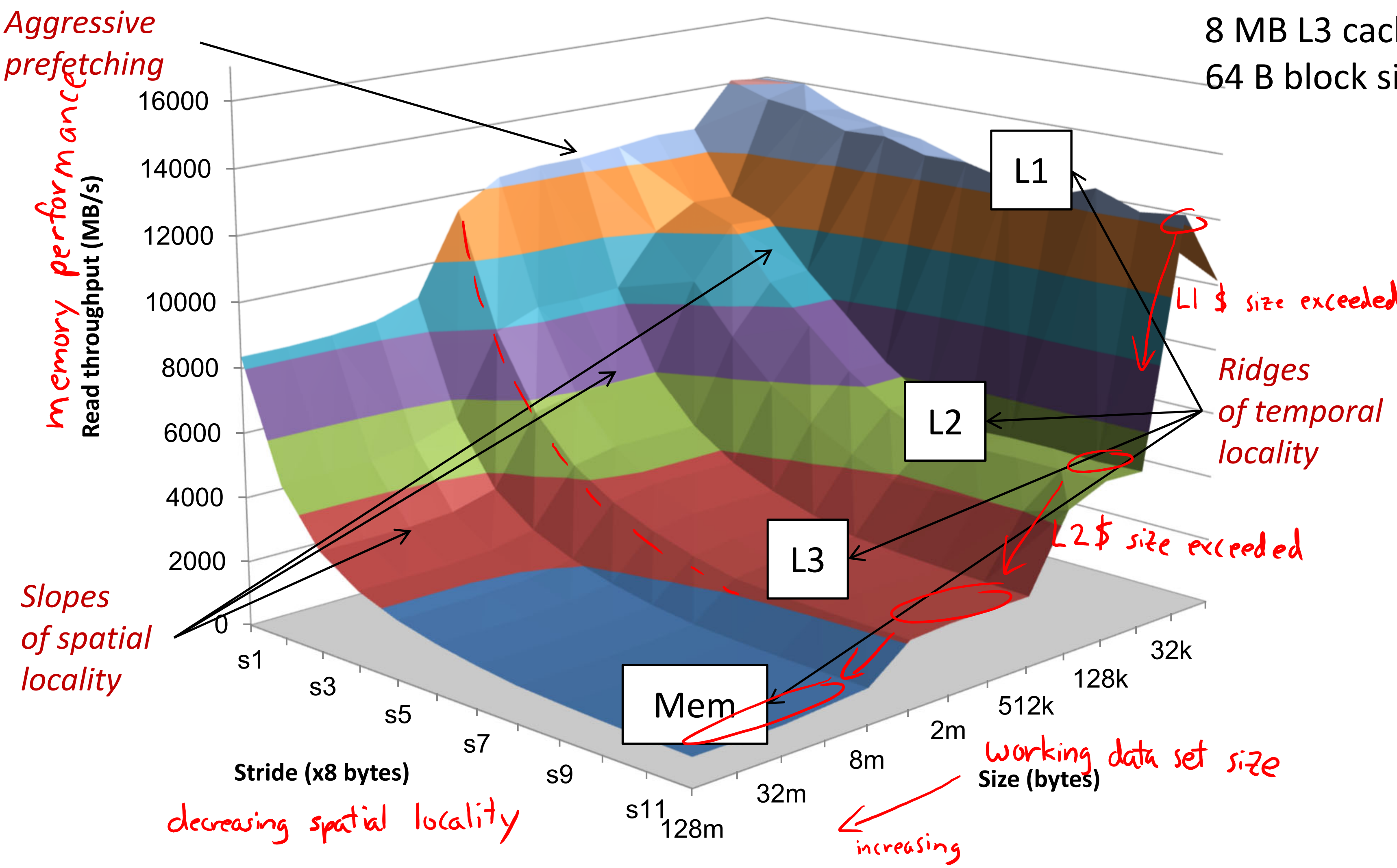
Cache-Friendly Code

- Programmer can optimize for cache performance
  - How data structures are organized
  - How data are accessed
    - Nested loop structure
    - Blocking is a general technique
- All systems favor "cache-friendly code"
  - Getting absolute optimum performance is very platform specific
    - Cache size, cache block size, associativity, etc.
  - Can get most of the advantage with generic code
    - Keep working set reasonably small (temporal locality)
    - Use small strides (spatial locality)
    - Focus on inner loop code

great general rules of thumb!

The Memory Mountain

Core i7 Haswell
2.1 GHz
32 KB L1 d-cache
256 KB L2 cache
8 MB L3 cache
64 B block size



Learning About Your Machine

- **Linux:**
  - `lscpu`
  - ls /sys/devices/system/cpu/cpu0/cache/index0/
    - Example: cat /sys/devices/system/cpu/cpu0/cache/index*/size
- **Windows:**
  - `wmic memcache get <query>` (all values in KB)
  - Example: `wmic memcache get MaxCacheSize`
- Modern processor specs: http://www.7-cpu.com/