Data III & Integers I

CSE 351 Spring 2021

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http://xkcd.com/257/

Administrivia

- Lab 0 and hw2 due tonight (4/05) @ 11:59 pm
- hw3 due Wednesday (4/07) @ 11:59 pm
- hw4 due Friday (4/09) @ 11:59 pm
- From here on out, at 11am on day of lecture:
  - Reading for that lecture is DUE at 11am
  - Lecture activities from the previous lecture are DUE at 11am
- **Questions Doc**: You must log on with your @uw google account to access!!
  - https://tinyurl.com/CSE351-21sp-Questions
  - Then open the "TODAY's Lecture Questions" doc for 11:30/2:30
  - See https://edstem.org/us/courses/4805/discussion/339425 for info about turning on your UW G Suite account for your @uw address

Lab 1a released

- Labs can be found linked on our course home page:
  - https://courses.cs.washington.edu/courses/cse351/21sp/labs/lab1a.php
- Workflow:
  1) Edit `pointer.c`
  2) Run the Makefile (`make`) and check for compiler errors & warnings
  3) Run ptest (`./ptest`) and check for correct behavior
  4) Run rule/syntax checker (`python dlc.py`) and check output
- Due Monday 4/12, will overlap a bit with Lab 1b
  - Submit in Gradescope
  - We grade just your *last* submission

Lab Reflections

- All subsequent labs (after Lab 0) have a “reflection” portion
  - The Reflection questions can be found on the lab specs and are intended to be done *after* you finish the lab
  - You will type up your responses in a `.txt` file for submission on Gradescope
  - These will be graded “by hand” (read by TAs)

- Intended to check your understand of what you should have learned from the lab

Memory, Data, and Addressing

- Representing information as bits and bytes
  - Binary, hexadecimal, fixed-widths
- Organizing and addressing data in memory
  - Memory is a byte-addressable array
  - Machine “word” size = address size = register size
  - Endianness – ordering bytes in memory
- Manipulating data in memory using C
  - Assignment
  - Pointers, pointer arithmetic, and arrays
- **Boolean algebra and bit-level manipulations**

Reading Review

- Terminology:
  - Bitwise operators (&, |, ^, ~)
  - Logical operators (&&, ||, !)
  - Short-circuit evaluation
  - Unsigned integers
  - Signed integers (Two's Complement)

Review Questions

- Compute the result of the following expressions for `char c = 0x81;`
  - `c ^ c`
  - `~c & 0xA9`
  - `c || 0x80`
  - `!!c`

- Compute the value of `signed char sc = 0xF0;` (Two's Complement)

Bitmasks

- Typically binary bitwise operators (&, |, ^) are used with one operand being the "input" and other operand being a specially-chosen bitmask (or *mask*) that performs a desired operation

- Operations for a bit b (answer with 0, 1, b, or $\bar{b}$):

$b\ \&\ 0 = \_\_\_$ $b\ \&\ 1 = \_\_\_$

$b\ |\ 0 = \_\_\_$ $b\ |\ 1 = \_\_\_$

$b\ \wedge\ 0 = \_\_\_$ $b\ \wedge\ 1 = \_\_\_$

Bitmasks

- Typically binary bitwise operators (&, |, ^) are used with one operand being the "input" and other operand being a specially-chosen bitmask (or *mask*) that performs a desired operation

- <u>Example</u>: $b|0 = b,\ b|1 = 1$

```
    01010101  ← input
 |  11110000  ← bitmask
 -----------
    11110101
```

Short-Circuit Evaluation

- If the result of a binary logical operator (&&, ||) can be determined by its first operand, then the second operand is never evaluated
  - Also known as *early termination*

- Example: `(p && *p)` for a pointer p to "protect" the dereference
  - Dereferencing NULL (0) results in a segfault

Roadmap

C:

```
car *c = malloc(sizeof(car));
c->miles = 100;
c->gals = 17;
float mpg = get_mpg(c);
free(c);
```

Java:

```
Car c = new Car();
c.setMiles(100);
c.setGals(17);
float mpg =
    c.getMPG();
```

Assembly language:

```
get_mpg:
    pushq   %rbp
    movq    %rsp, %rbp
    ...
    popq    %rbp
    ret
```

Machine code:

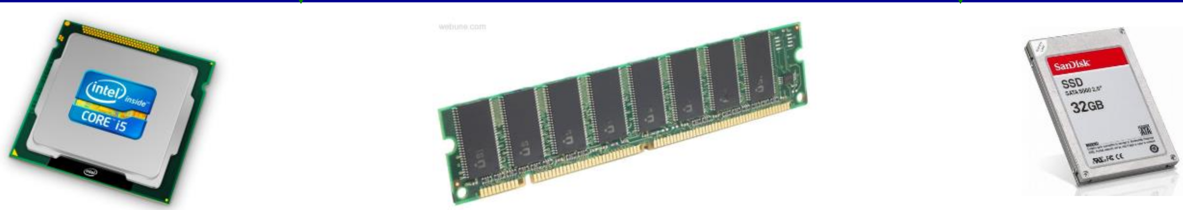
```
0111010000011000
100011010000010000000010
1000100111000010
110000011111101000011111
```

OS:

Windows 10

OS X Yosemite

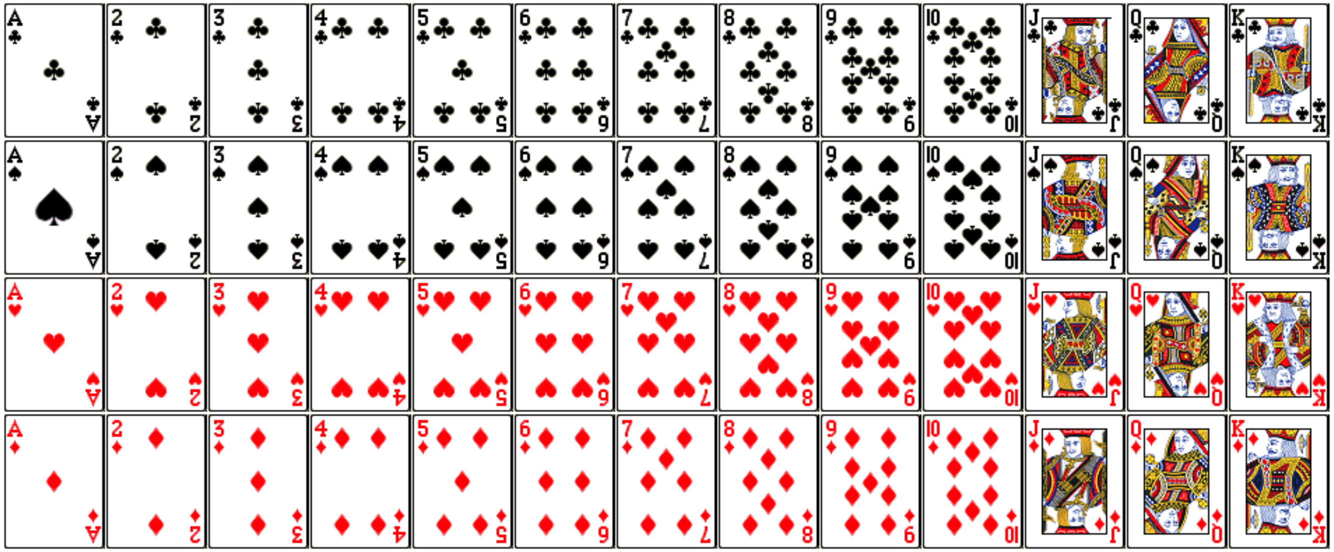
Computer system:



- Memory & data
- Integers & floats
- x86 assembly
- Procedures & stacks
- Executables
- Arrays & structs
- Memory & caches
- Processes
- Virtual memory
- Memory allocation
- Java vs. C

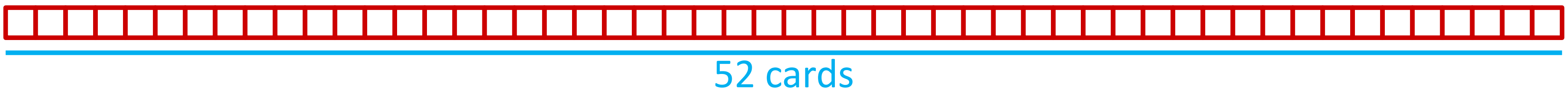
But before we get to integers….

- Encode a standard deck of playing cards
- 52 cards in 4 suits
  - How do we encode suits, face cards?
- What operations do we want to make easy to implement?
  - Which is the higher value card?
  - Are they the same suit?



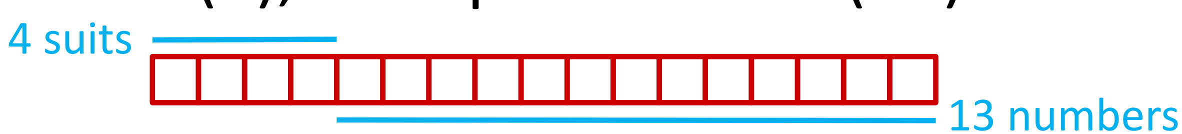
Two possible representations

1) 1 bit per card (52): bit corresponding to card set to 1



- "One-hot" encoding (similar to set notation)
- Drawbacks:
  - Hard to compare values and suits
  - Large number of bits required

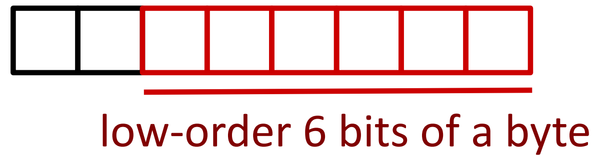
2) 1 bit per suit (4), 1 bit per number (13): 2 bits set



- **Pair** of one-hot encoded values (two fields)
- Easier to compare suits and values, but still lots of bits used

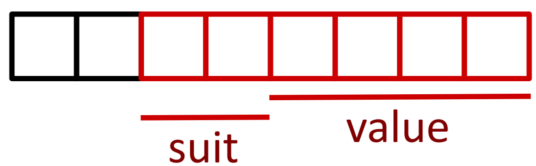
Two better representations

3) Binary encoding of all 52 cards – only 6 bits needed
   - $2^6 = 64 \geq 52$



   - Fits in one byte (smaller than one-hot encodings)
   - How can we make value and suit comparisons easier?

4) Separate binary encodings of suit (2 bits) and value (4 bits)



♣	00
♦	01
♥	10
♠	11

   - Also fits in one byte, and easy to do comparisons

K	Q	J	. . .	3	2	A
1101	1100	1011	. . .	0011	0010	0001

Compare Card Suits

mask: a bit vector designed to achieve a desired behavior when used with a bitwise operator on another bit vector *v*.
Here we turn all *but* the bits of interest in *v* to 0.

```
char hand[5];        // represents a 5-card hand
char card1, card2;   // two cards to compare
card1 = hand[0];
card2 = hand[1];
...
if ( sameSuitP(card1, card2) ) { ... }
```

```
#define SUIT_MASK  0x30

int sameSuitP(char card1, char card2) {
  return (!((card1 & SUIT_MASK) ^ (card2 & SUIT_MASK)));
  //return (card1 & SUIT_MASK) == (card2 & SUIT_MASK);
}
```

returns **int**

equivalent

SUIT_MASK = 0x30 =

0	0	1	1	0	0	0	0

suit (bits 1 1), value (last four bits 0 0 0 0)

Compare Card **Suits**

```
#define SUIT_MASK  0x30

int sameSuitP(char card1, char card2) {
  return (!((card1 & SUIT_MASK) ^ (card2 & SUIT_MASK)));
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}
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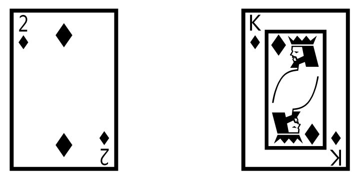

Card 1		Card 2
0 0 0 1 0 0 1 0		0 0 0 1 1 1 0 1
&		&
0 0 1 1 0 0 0 0	SUIT_MASK	0 0 1 1 0 0 0 0
=		=
0 0 0 1 0 0 0 0		0 0 0 1 0 0 0 0

^

0 0 0 0 0 0 0 0

!

0 0 0 0 0 0 0 1

`!(x^y)` equivalent to `x==y`

Compare Card Values

```
char hand[5];          // represents a 5-card hand
char card1, card2;     // two cards to compare
card1 = hand[0];
card2 = hand[1];
...
if ( greaterValue(card1, card2) ) { ... }
```

```
#define VALUE_MASK  0x0F

int greaterValue(char card1, char card2) {
  return ((unsigned int)(card1 & VALUE_MASK) >
          (unsigned int)(card2 & VALUE_MASK));
}
```

VALUE_MASK = 0x0F =

0	0	0	0	1	1	1	1
		suit		value			

Compare Card Values

```
#define VALUE_MASK  0x0F

int greaterValue(char card1, char card2) {
  return ((unsigned int)(card1 & VALUE_MASK) >
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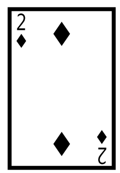
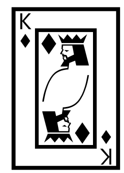
}
```





card1		card2
0 0 1 0 0 0 1 0		0 0 1 0 1 1 0 1
&		&
0 0 0 0 1 1 1 1	VALUE_MASK	0 0 0 0 1 1 1 1
=		=
0 0 0 0 0 0 1 0		0 0 0 0 1 1 0 1

$2_{10} > 13_{10}$

0 (false)

Roadmap

C:

```
car *c = malloc(sizeof(car));
c->miles = 100;
c->gals = 17;
float mpg = get_mpg(c);
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```

Java:

```
Car c = new Car();
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```

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- x86 assembly
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- Virtual memory
- Memory allocation
- Java vs. C

Assembly language:

```
get_mpg:
    pushq   %rbp
    movq    %rsp, %rbp
    ...
    popq    %rbp
    ret
```

Machine code:

```
0111010000011000
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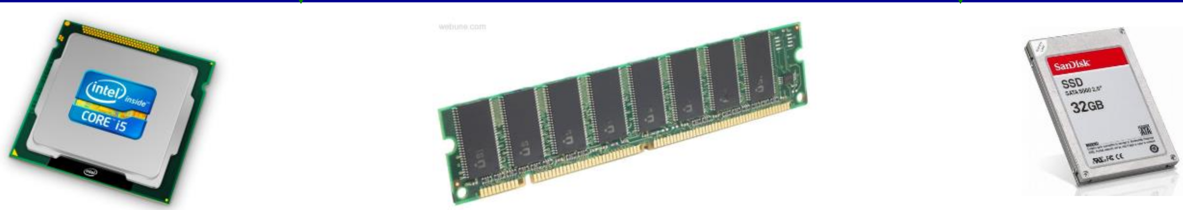
```

OS:

Windows 10 | OS X Yosemite

Computer system:

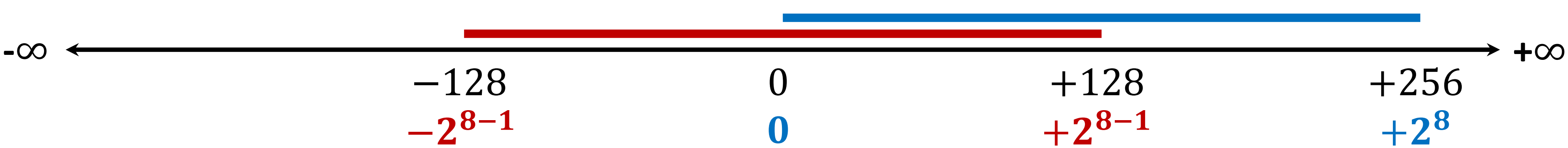


Integers

- **Binary representation of integers**
  - **Unsigned and signed**
- Shifting and arithmetic operations
- In C: Signed, Unsigned and Casting
- Consequences of finite width representations
  - Overflow, sign extension

Encoding Integers

- The hardware (and C) supports two flavors of integers
  - *unsigned* – only the non-negatives
  - *signed* – both negatives and non-negatives
- Cannot represent all integers with w bits
  - Only 2^w distinct bit patterns
  - Unsigned values: $0 \ldots 2^w - 1$
  - Signed values: $-2^{w-1} \ldots 2^{w-1} - 1$
- **Example:** 8-bit integers (*e.g.* `char`)



Unsigned Integers

- Unsigned values follow the standard base 2 system
  - $\mathrm{b_7 b_6 b_5 b_4 b_3 b_2 b_1 b_0} = \mathrm{b_7} 2^7 + \mathrm{b_6} 2^6 + \cdots + \mathrm{b_1} 2^1 + \mathrm{b_0} 2^0$

- Useful formula: $2^{\mathrm{N}-1} + 2^{\mathrm{N}-2} + \ldots + 2 + 1 = 2^{\mathrm{N}} - 1$
  - *i.e.*, N ones in a row = $2^{\mathrm{N}} - 1$
  - *e.g.*, 0b111111 = 63

Sign and Magnitude

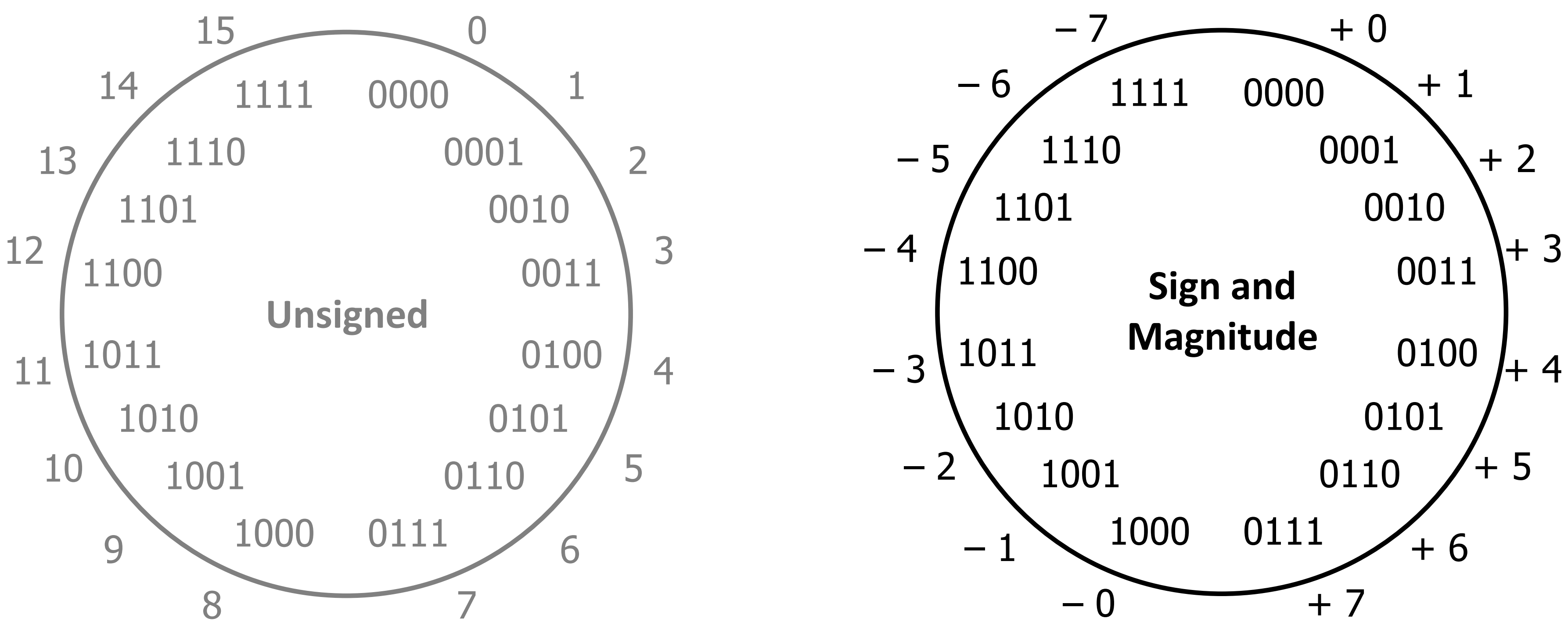
Not used in practice!

- Designate the high-order bit (MSB) as the "sign bit"
  - `sign=0`: positive numbers; `sign=1`: negative numbers
- Benefits:
  - Using MSB as sign bit matches positive numbers with unsigned
  - All zeros encoding is still = 0
- Examples (8 bits):
  - 0x00 = 00000000_2 is non-negative, because the sign bit is 0
  - 0x7F = 01111111_2 is non-negative ($+127_{10}$)
  - 0x85 = 10000101_2 is negative (-5_{10})
  - 0x80 = 10000000_2 is negative... zero???

Sign and Magnitude

Not used in practice!

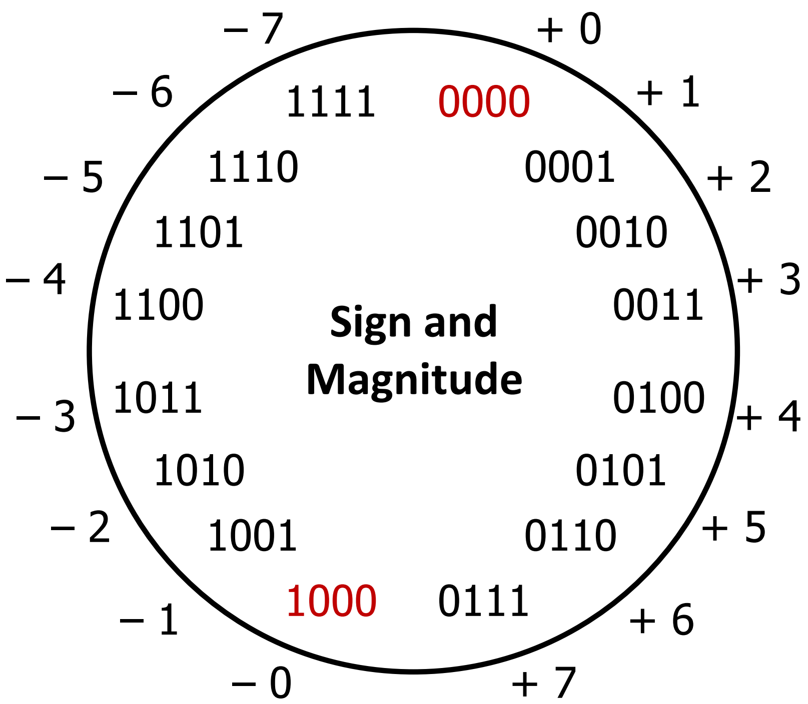
- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks?



Sign and Magnitude

Not used in practice!

- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks:
  - Two representations of 0 (bad for checking equality)



Sign and Magnitude

Not used in practice!

- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks:
  - Two representations of 0 (bad for checking equality)
  - Arithmetic is cumbersome
    - Example: `4-3 != 4+(-3)`

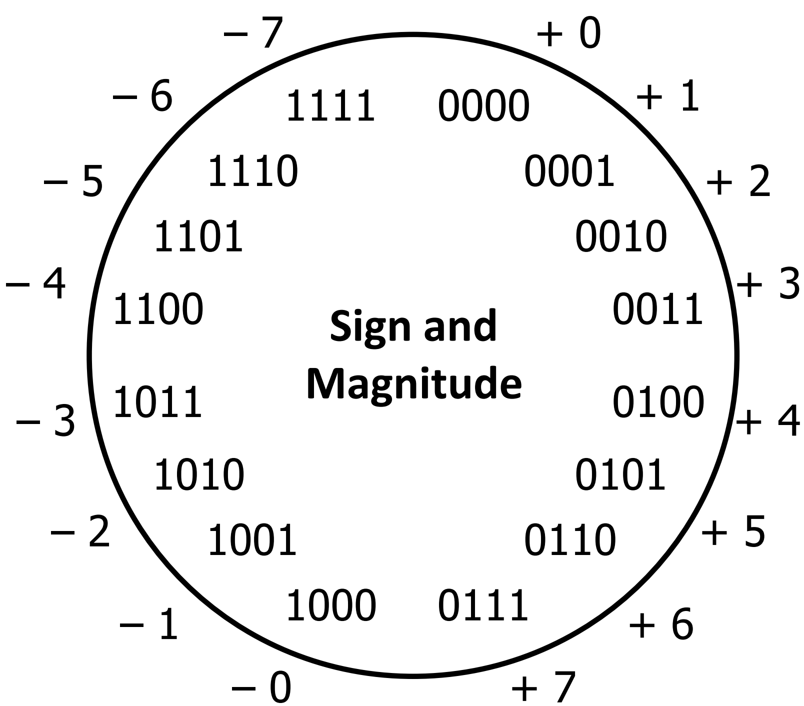
4	0100
− 3	− 0011
1	0001

✓

4	0100
+ −3	+ 1011
−7	1111

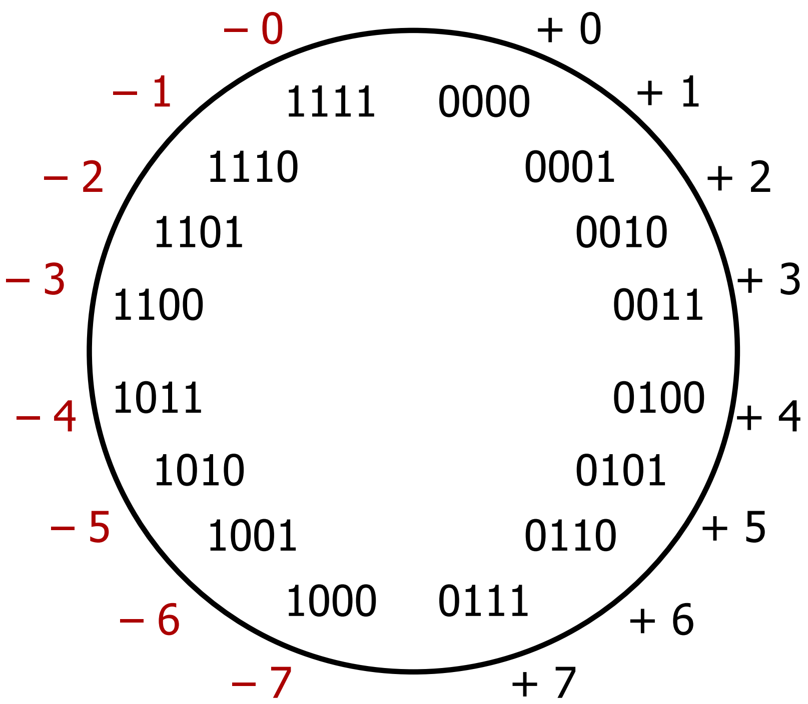
✗

    - Negatives "increment" in wrong direction!



Two's Complement

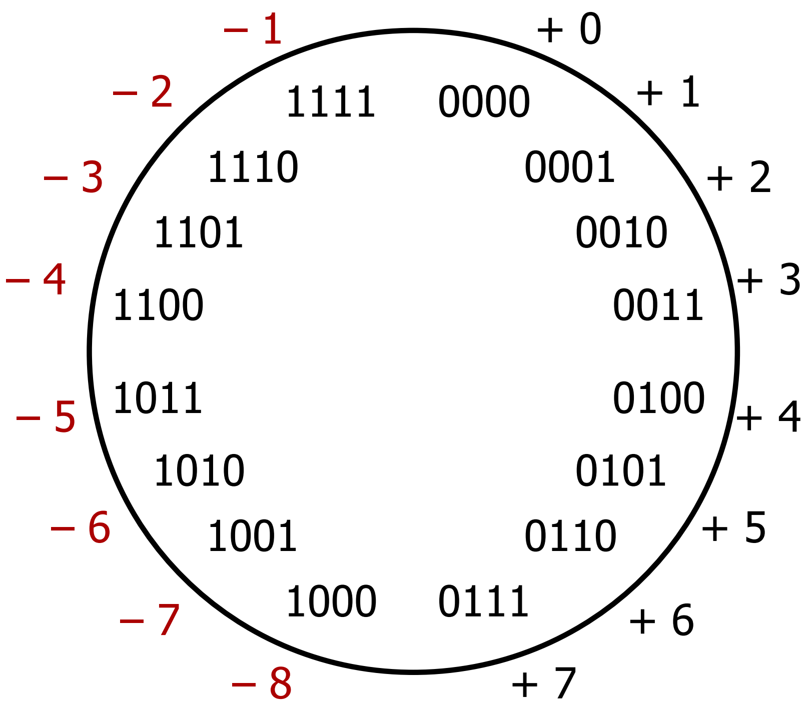
- Let's fix these problems:
  1) "Flip" negative encodings so incrementing works



Two’s Complement

- Let’s fix these problems:
  1) “Flip” negative encodings so incrementing works
  2) “Shift” negative numbers to eliminate –0

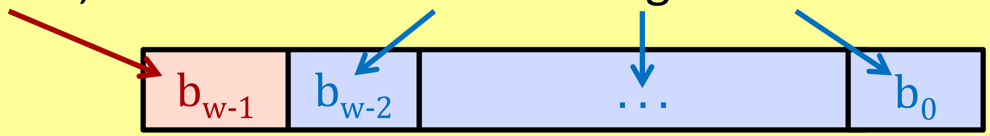
- MSB *still* indicates sign!
  - This is why we represent one more negative than positive number (-2^{N-1} to $2^{N-1}-1$)



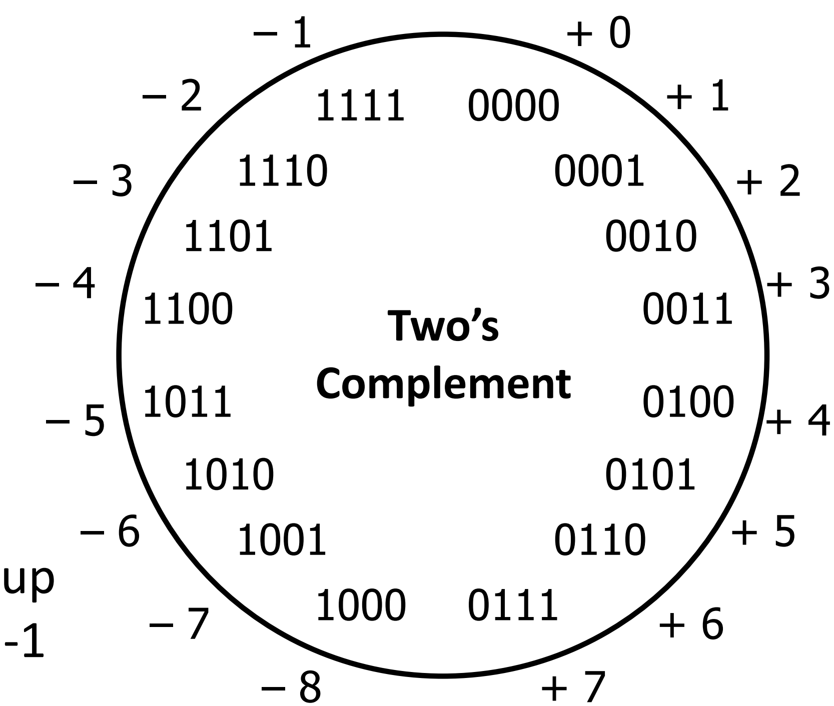
Two's Complement Negatives

- Accomplished with one neat mathematical trick!

b_{w-1} has weight -2^{w-1}, other bits have usual weights $+2^i$

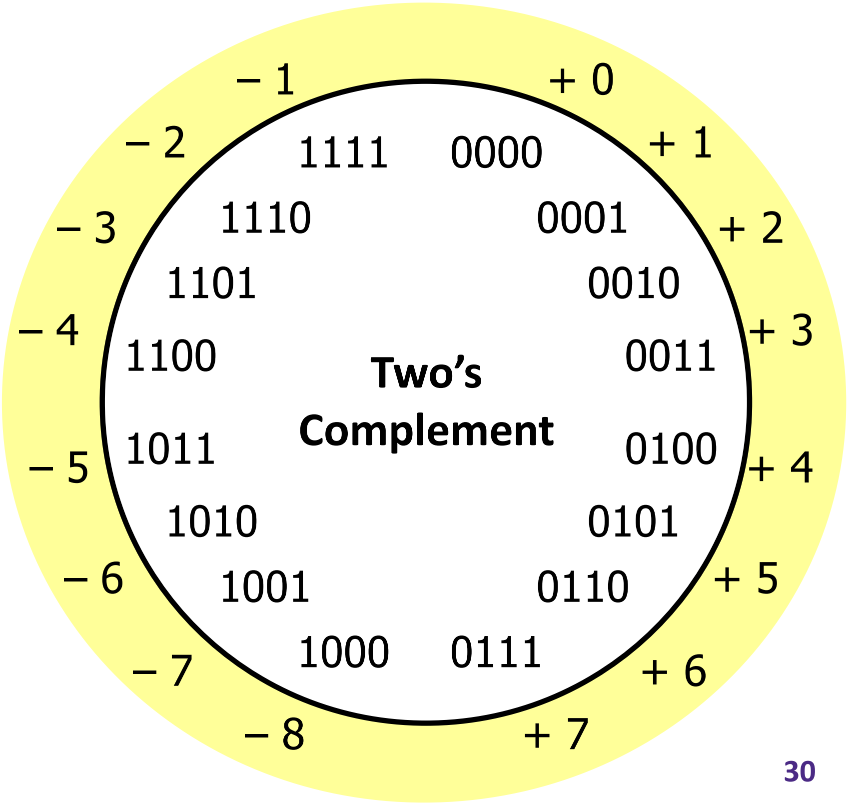


- 4-bit Examples:
  - 1010_2 unsigned:
    $1*2^3+0*2^2+1*2^1+0*2^0$ = **10**
  - 1010_2 two's complement:
    $-1*2^3+0*2^2+1*2^1+0*2^0$ = **–6**
- -1 represented as:
  $1111_2 = -2^3+(2^3 - 1)$
  - MSB makes it super negative, add up all the other bits to get back up to -1



Why Two's Complement is So Great

- Roughly same number of (+) and (–) numbers
- Positive number encodings match unsigned
- Single zero
- All zeros encoding = 0
- Simple negation procedure:
  - Get negative representation of any integer by taking bitwise complement and then adding one!
    `( ~x + 1 == -x )`



Polling Question

- Take the 4-bit number encoding `x = 0b1011`
- Which of the following numbers is NOT a valid interpretation of `x` using any of the number representation schemes discussed today?
  - Unsigned, Sign and Magnitude, Two's Complement
  - Vote in Ed Lessons

A. -4

B. -5

C. 11

D. -3

E. We're lost…

Summary

- Bit-level operators allow for fine-grained manipulations of data
  - Bitwise AND (`&`), OR (`|`), and NOT (`~`) different than logical AND (`&&`), OR (`||`), and NOT (`!`)
  - Especially useful with bit masks
- Choice of *encoding scheme* is important
  - Tradeoffs based on size requirements and desired operations
- Integers represented using unsigned and two's complement representations
  - Limited by fixed bit width
  - We'll examine arithmetic operations next lecture