Data III & Integers I

CSE 351 Autumn 2021

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http://xkcd.com/257/

Relevant Course Information

- hw3 due Friday, hw4 due Monday
- Lab 1a released
  - Workflow:
    1) Edit `pointer.c`
    2) Run the Makefile (`make clean` followed by `make`) and check for compiler errors & warnings
    3) Run ptest (`./ptest`) and check for correct behavior
    4) Run rule/syntax checker (`python3 dlc.py`) and check output
  - Due Monday 10/11, will overlap a bit with Lab 1b
    - We grade just your *last* submission
    - Don't wait until the last minute to submit – need to check autograder output

Lab Synthesis Questions

- All subsequent labs (after Lab 0) have a “synthesis question” portion
  - Can be found on the lab specs and are intended to be done *after* you finish the lab
  - You will type up your responses in a `.txt` file for submission on Gradescope
  - These will be graded “by hand” (read by TAs)

- Intended to check your understand of what you should have learned from the lab
  - Also great practice for short answer questions on the exams

Reading Review

- Terminology:
  - Bitwise operators (&, |, ^, ~)
  - Logical operators (&&, ||, !)
  - Short-circuit evaluation
  - Unsigned integers
  - Signed integers (Two's Complement)

- Questions from the Reading?

Review Questions

- Compute the result of the following expressions for `char c = 0x81;`
  - `c ^ c`
  - `~c & 0xA9`
  - `c || 0x80`
  - `!!c`

- Compute the value of `signed char sc = 0xF0;` (Two’s Complement)

Bitmasks

- Typically binary bitwise operators (&, |, ^) are used with one operand being the "input" and other operand being a specially-chosen bitmask (or *mask*) that performs a desired operation

- Operations for a bit b (answer with 0, 1, b, or $\bar{b}$):

$b \; \& \; 0 = \_\_\_$ $b \; \& \; 1 = \_\_\_$

$b \; | \; 0 = \_\_\_$ $b \; | \; 1 = \_\_\_$

$b \; \wedge \; 0 = \_\_\_$ $b \; \wedge \; 1 = \_\_\_$

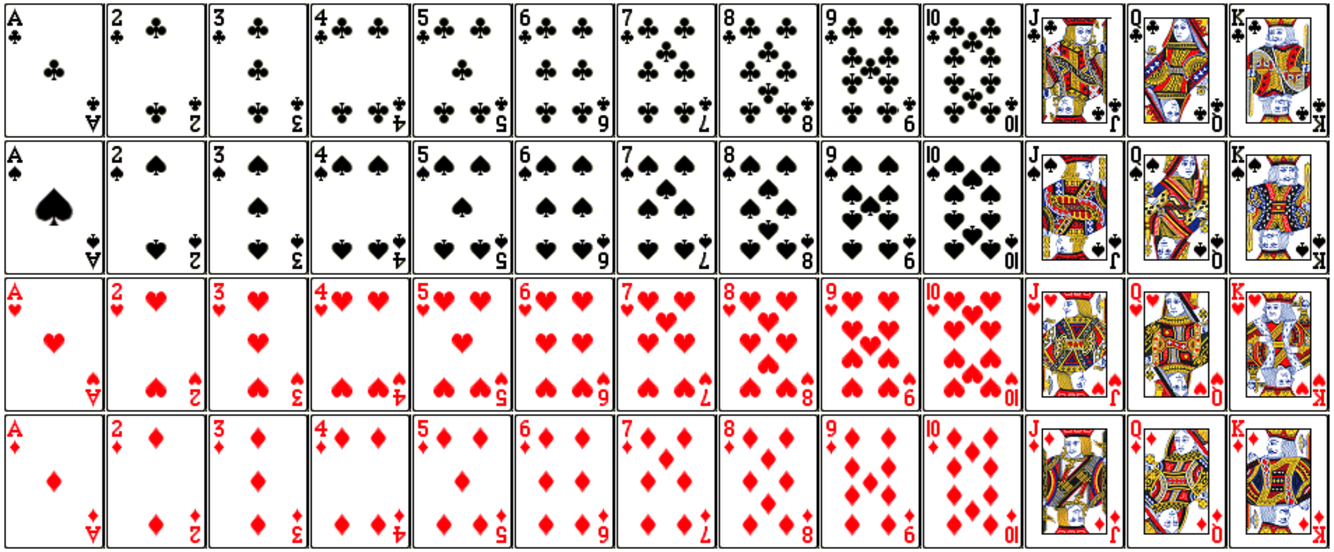
Bitmasks

- Typically binary bitwise operators (&, |, ^) are used with one operand being the "input" and other operand being a specially-chosen bitmask (or *mask*) that performs a desired operation
- Example: $b|0 = b, \ b|1 = 1$

```
  01010101  ← input
| 11110000  ← bitmask
----------
  11110101
```

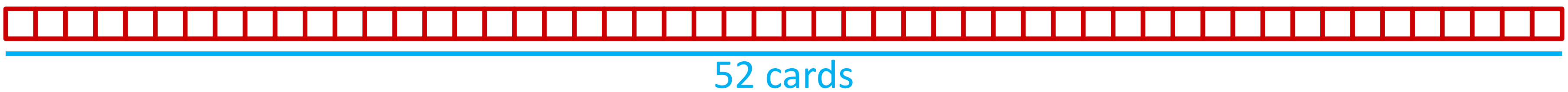
Numerical Encoding Design Example

- Encode a standard deck of playing cards
  - 52 cards in 4 suits
- Operations to implement:
  - Which is the higher value card?
  - Are they the same suit?



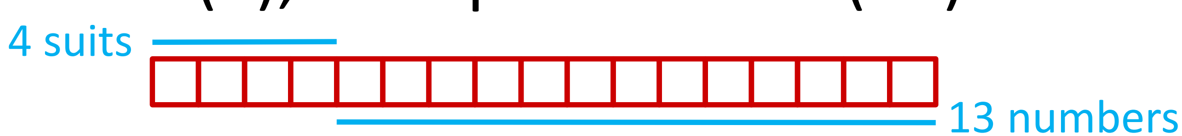
Representations and Fields

1) 1 bit per card (52): bit corresponding to card set to 1



- "One-hot" encoding (similar to set notation)
- Drawbacks:
  - Hard to compare values and suits
  - Large number of bits required

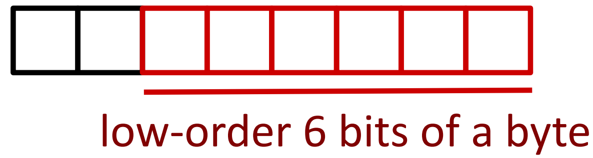
2) 1 bit per suit (4), 1 bit per number (13): 2 bits set



- Pair of one-hot encoded values
- Easier to compare suits and values, but still lots of bits used

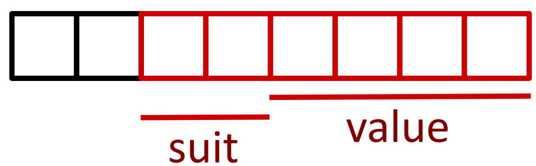
Representations and Fields

3) Binary encoding of all 52 cards – only 6 bits needed
   - $2^6 = 64 \geq 52$



   - Fits in one byte (smaller than one-hot encodings)
   - How can we make value and suit comparisons easier?

4) Separate binary encodings of suit (2 bits) and value (4 bits)



   - Also fits in one byte, and easy to do comparisons

♣	00
♦	01
♥	10
♠	11

K	Q	J	. . .	3	2	A
1101	1100	1011	. . .	0011	0010	0001

Compare Card Suits

mask: a bit vector designed to achieve a desired behavior when used with a bitwise operator on another bit vector *v*.
Here we turn all *but* the bits of interest in *v* to 0.

```
char hand[5];          // represents a 5-card hand
char card1, card2;     // two cards to compare
card1 = hand[0];
card2 = hand[1];
...
if ( sameSuitP(card1, card2) ) { ... }
```

```
#define SUIT_MASK  0x30

int sameSuitP(char card1, char card2) {
  return (!((card1 & SUIT_MASK) ^ (card2 & SUIT_MASK)));
  //return (card1 & SUIT_MASK) == (card2 & SUIT_MASK);
}
```

returns `int`

equivalent

SUIT_MASK = 0x30 = 0 0 1 1 0 0 0 0

suit value

Compare Card **Suits**

```
#define SUIT_MASK  0x30

int sameSuitP(char card1, char card2) {
  return (!((card1 & SUIT_MASK) ^ (card2 & SUIT_MASK)));
  //return (card1 & SUIT_MASK) == (card2 & SUIT_MASK);
}
```

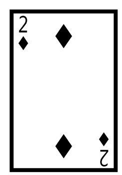
0 0 0 1 0 0 1 0

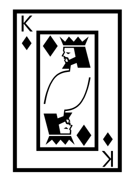
&

0 0 1 1 0 0 0 0

=

0 0 0 1 0 0 0 0





SUIT_MASK

0 0 0 1 1 1 0 1

&

0 0 1 1 0 0 0 0

=

0 0 0 1 0 0 0 0

^

0 0 0 0 0 0 0 0

!

0 0 0 0 0 0 0 1

`!(x^y)` equivalent to `x==y`

Compare Card Values

```c
char hand[5];          // represents a 5-card hand
char card1, card2;     // two cards to compare
card1 = hand[0];
card2 = hand[1];
...
if ( greaterValue(card1, card2) ) { ... }
```

```c
#define VALUE_MASK  0x0F

int greaterValue(char card1, char card2) {
  return ((unsigned int)(card1 & VALUE_MASK) >
          (unsigned int)(card2 & VALUE_MASK));
}
```

VALUE_MASK = 0x0F =

0	0	0	0	1	1	1	1
		suit		value			

Compare Card Values

```
#define VALUE_MASK  0x0F

int greaterValue(char card1, char card2) {
  return ((unsigned int)(card1 & VALUE_MASK) >
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```
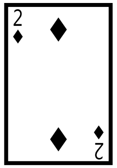
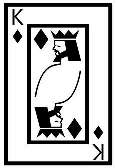





0	0	1	0	0	0	1	0

&

0	0	0	0	1	1	1	1

=

0	0	0	0	0	0	1	0

VALUE_MASK

0	0	1	0	1	1	0	1

&

0	0	0	0	1	1	1	1

=

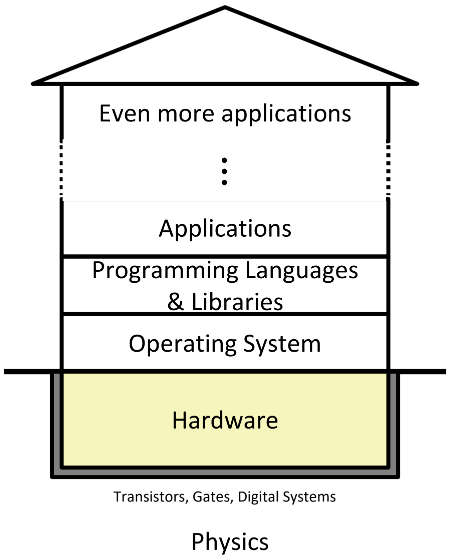
0	0	0	0	1	1	0	1

$2_{10} > 13_{10}$

0 (false)

The Hardware/Software Interface

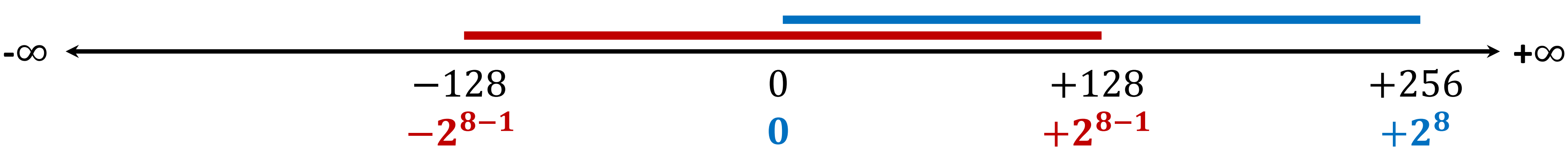
- Topic Group 1: **Data**
  - Memory, Data, **Integers**, Floating Point, Arrays, Structs



- How do we store information for other parts of the house of computing to access?
  - How do we represent data and what limitations exist?
  - What design decisions and priorities went into these encodings?

Encoding Integers

- The hardware (and C) supports two flavors of integers
  - *unsigned* – only the non-negatives
  - *signed* – both negatives and non-negatives
- Cannot represent all integers with *w* bits
  - Only 2^w distinct bit patterns
  - Unsigned values: $0 \ldots 2^w-1$
  - Signed values: $-2^{w-1} \ldots 2^{w-1}-1$
- **Example:** 8-bit integers (*e.g.*, `char`)



Unsigned Integers (Review)

- Unsigned values follow the standard base 2 system
  - $b_7b_6b_5b_4b_3b_2b_1b_0 = b_7 2^7 + b_6 2^6 + \cdots + b_1 2^1 + b_0 2^0$
- Useful formula: $2^{N-1} + 2^{N-2} + \ldots + 2 + 1 = 2^N - 1$
  - *i.e.*, N ones in a row = $2^N - 1$
  - *e.g.*, 0b111111 = 63

Sign and Magnitude

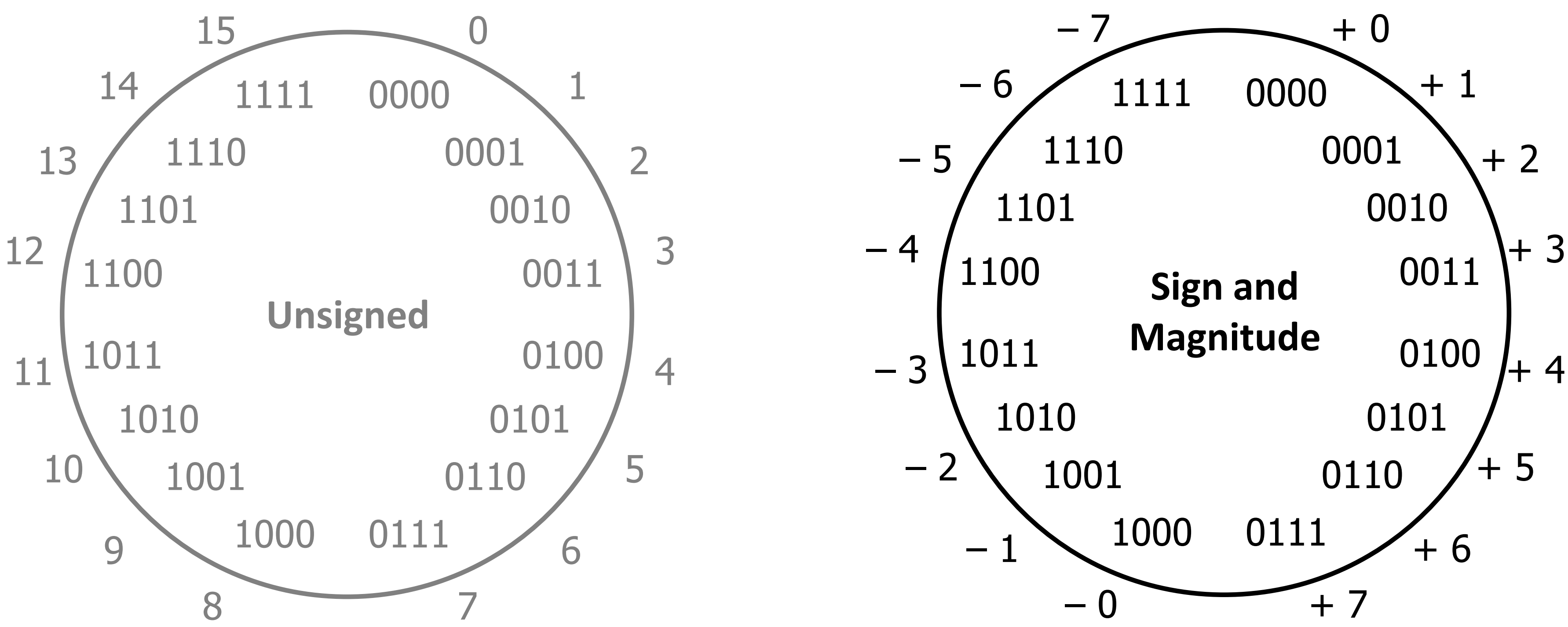
Not used in practice for integers!

- Designate the high-order bit (MSB) as the "sign bit"
  - `sign=0`: positive numbers; `sign=1`: negative numbers
- Benefits:
  - Using MSB as sign bit matches positive numbers with unsigned
  - All zeros encoding is still = 0
- Examples (8 bits):
  - 0x00 = 00000000_2 is non-negative, because the sign bit is 0
  - 0x7F = 01111111_2 is non-negative ($+127_{10}$)
  - 0x85 = 10000101_2 is negative (-5_{10})
  - 0x80 = 10000000_2 is negative... zero???

Sign and Magnitude

Not used in practice for integers!

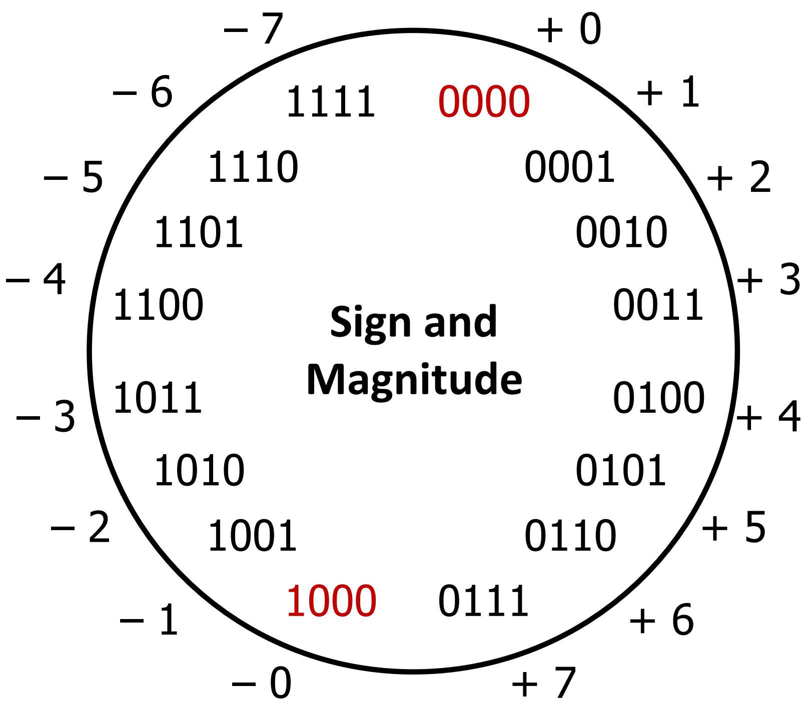
- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks?



Sign and Magnitude

Not used in practice for integers!

- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks:
  - Two representations of 0 (bad for checking equality)



Sign and Magnitude

Not used in practice for integers!

- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks:
  - Two representations of 0 (bad for checking equality)
  - Arithmetic is cumbersome
    - Example: `4-3 != 4+(-3)`

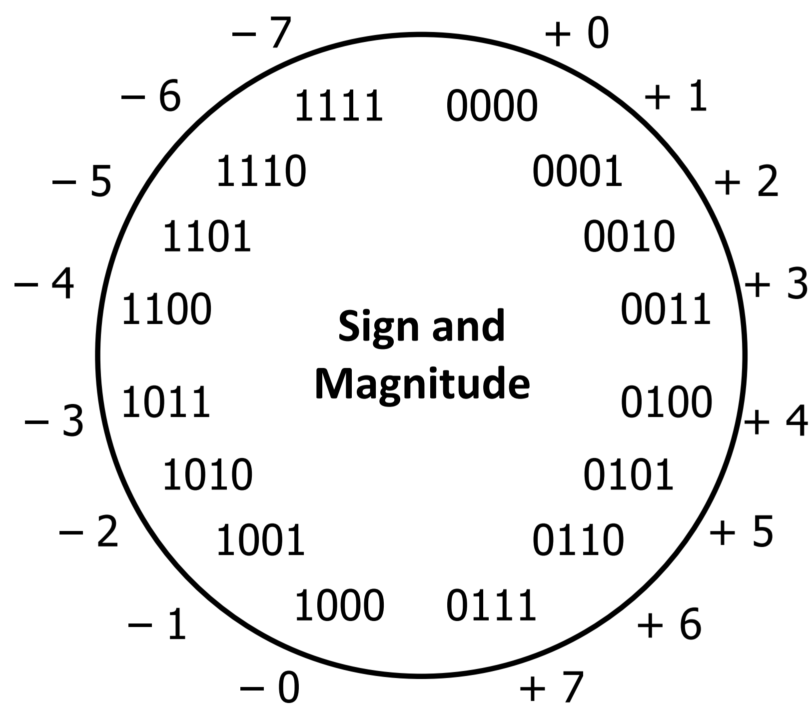
4	0100
– 3	– 0011
1	0001

✓

4	0100
+ -3	+ 1011
-7	1111

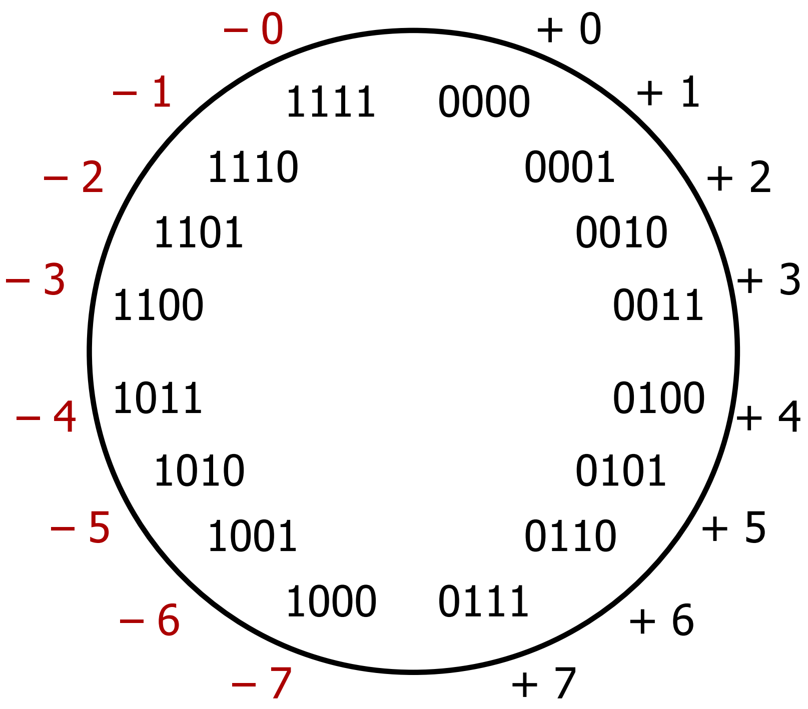
✗

    - Negatives "increment" in wrong direction!



Two's Complement

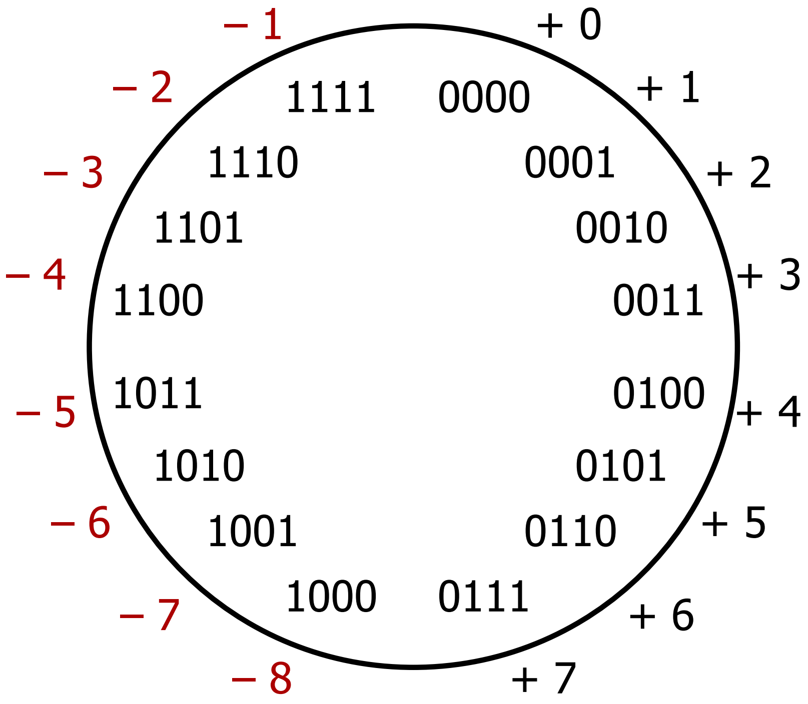
- Let's fix these problems:
  1) "Flip" negative encodings so incrementing works



Two's Complement

- Let's fix these problems:
  1) "Flip" negative encodings so incrementing works
  2) "Shift" negative numbers to eliminate –0

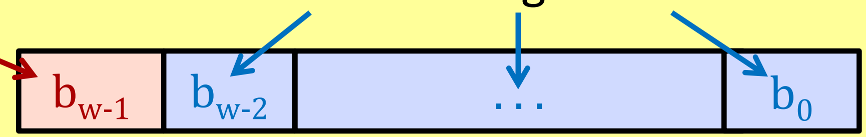
- MSB *still* indicates sign!
  - This is why we represent one more negative than positive number (-2^{N-1} to $2^{N-1}-1$)



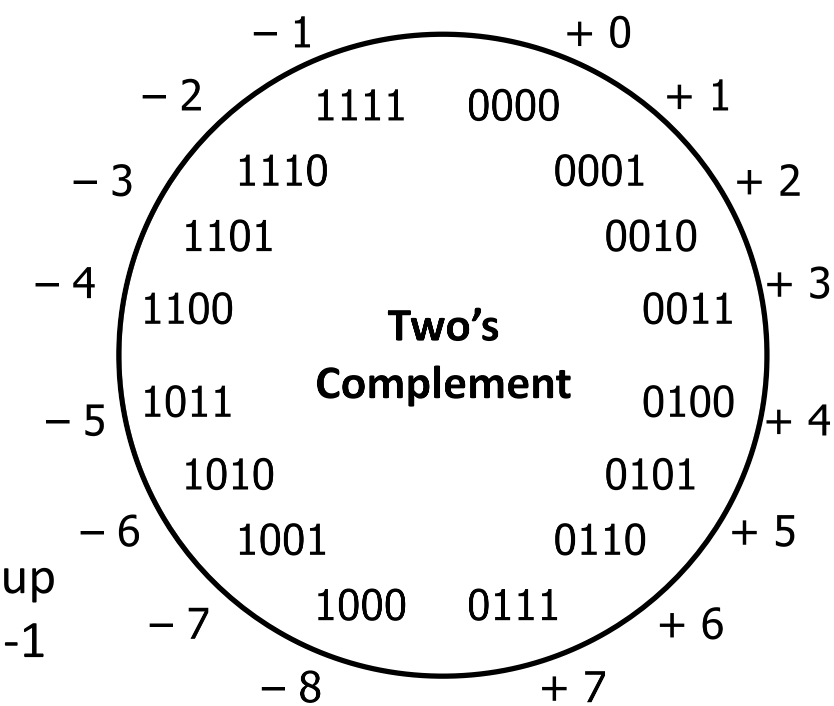
Two's Complement Negatives (Review)

- Accomplished with one neat mathematical trick!

b_{w-1} has weight -2^{w-1}, other bits have usual weights $+2^{i}$



- 4-bit Examples:
  - 1010_2 unsigned:
    $1*2^3+0*2^2+1*2^1+0*2^0 =$ **10**
  - 1010_2 two's complement:
    $-1*2^3+0*2^2+1*2^1+0*2^0 =$ **–6**
- -1 represented as:
  $1111_2 = -2^3+(2^3 - 1)$
  - MSB makes it super negative, add up all the other bits to get back up to -1



Polling Question

- Take the 4-bit number encoding **`x = 0b1011`**
- Which of the following numbers is NOT a valid interpretation of `x` using any of the number representation schemes discussed today?
  - Unsigned, Sign and Magnitude, Two's Complement
  - Vote in Ed Lessons

A. **-4**

B. **-5**

C. **11**

D. **-3**

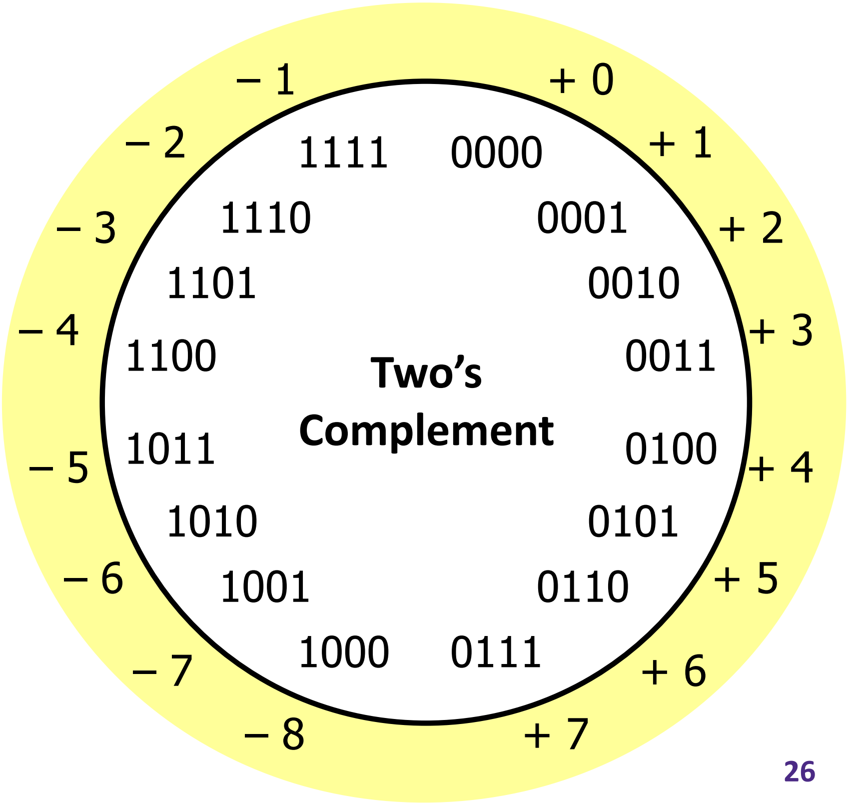
E. **We're lost…**

Two's Complement is Great (Review)

- Roughly same number of (+) and (–) numbers
- Positive number encodings match unsigned
- Single zero
- All zeros encoding = 0

- Simple negation procedure:
  - Get negative representation of any integer by taking bitwise complement and then adding one!

    `( ~x + 1 == -x )`



Integer Representation Design Decisions

- Some explicitly mentioned and some implicit:
  - Represent consecutive integers
    - What would happen if we had gaps?
  - Represent the same number of positives and negatives
    - Could we choose a different shift/bias?
  - Positive number encodings match unsigned
    - What's the advantage here?

Summary

- Bit-level operators allow for fine-grained manipulations of data
  - Bitwise AND (`&`), OR (`|`), and NOT (`~`) different than logical AND (`&&`), OR (`||`), and NOT (`!`)
  - Especially useful with bit masks
- Choice of *encoding scheme* is important
  - Tradeoffs based on size requirements and desired operations
- Integers represented using unsigned and two's complement representations
  - Limited by fixed bit width
  - We'll examine arithmetic operations next lecture