Data III & Integers I

CSE 351 Autumn 2021

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http://xkcd.com/257/

Relevant Course Information

- hw3 due Friday, hw4 due Monday

- Lab 1a released
  - Workflow:
    1) Edit `pointer.c`
    2) Run the Makefile (`make clean` followed by `make`) and check for compiler errors & warnings
    3) Run ptest (`./ptest`) and check for correct behavior
    4) Run rule/syntax checker (`python3 dlc.py`) and check output
  - Due Monday 10/11, will overlap a bit with Lab 1b
    - We grade just your *last* submission
    - Don’t wait until the last minute to submit – need to check autograder output

Lab Synthesis Questions

- All subsequent labs (after Lab 0) have a "synthesis question" portion
  - Can be found on the lab specs and are intended to be done *after* you finish the lab
  - You will type up your responses in a `.txt` file for submission on Gradescope
  - These will be graded "by hand" (read by TAs)

- Intended to check your understand of what you should have learned from the lab
  - Also great practice for short answer questions on the exams

Reading Review

- Terminology:
  - Bitwise operators (&, |, ^, ~)
  - Logical operators (&&, ||, !)
  - Short-circuit evaluation
  - Unsigned integers
  - Signed integers (Two's Complement)

- Questions from the Reading?

Review Questions

- Compute the result of the following expressions for `char c = 0x81;` // 0b 1000 0001
  - `c ^ c` = $\boxed{0x00}$

    1000 0001
    1000 0001
    0000 0000
  - `~c & 0xA9` = $\boxed{0x28}$

    "true" "true" "true"

    0b0111 1110
    0b 1010 1001
    0010 1000
  - `c || 0x80` = $\boxed{0x01}$

    "true"
  - `!(!c)` = $\boxed{0x01}$

    "false"
    0x0
- Compute the value of `signed char sc = 0xF0;` (Two's Complement)

  = 0b 1111 0000

  = $-2^7+2^6+2^5+2^4$

  $\boxed{= -16}$

  $-sc$ = ~sc + 1 = 0b 0000 1111
  + 1
  0b 0001 0000 = +16

  $\boxed{sc = -16}$

Bitmasks

- Typically binary bitwise operators (&, |, ^) are used with one operand being the "input" and other operand being a specially-chosen bitmask (or *mask*) that performs a desired operation

- Operations for a bit b (answer with 0, 1, b, or $\bar{b}$):

0/1

b & 0 = 0 "set to zero" b & 1 = b "keep as is"

b | 0 = b "keep as is" b | 1 = 1 "set to one"

b ^ 0 = b "keep as is" b ^ 1 = $\bar{b}$ "flip"

0 → 0, 1 → 0 | 0 → 0, 1 → 1
0 → 0, 1 → 1 | 0 → 1, 1 → 1
0 → 0, 1 → 1 | 0 → 1, 1 → 0

Bitmasks

- Typically binary bitwise operators (&, |, ^) are used with one operand being the "input" and other operand being a specially-chosen bitmask (or *mask*) that performs a desired operation

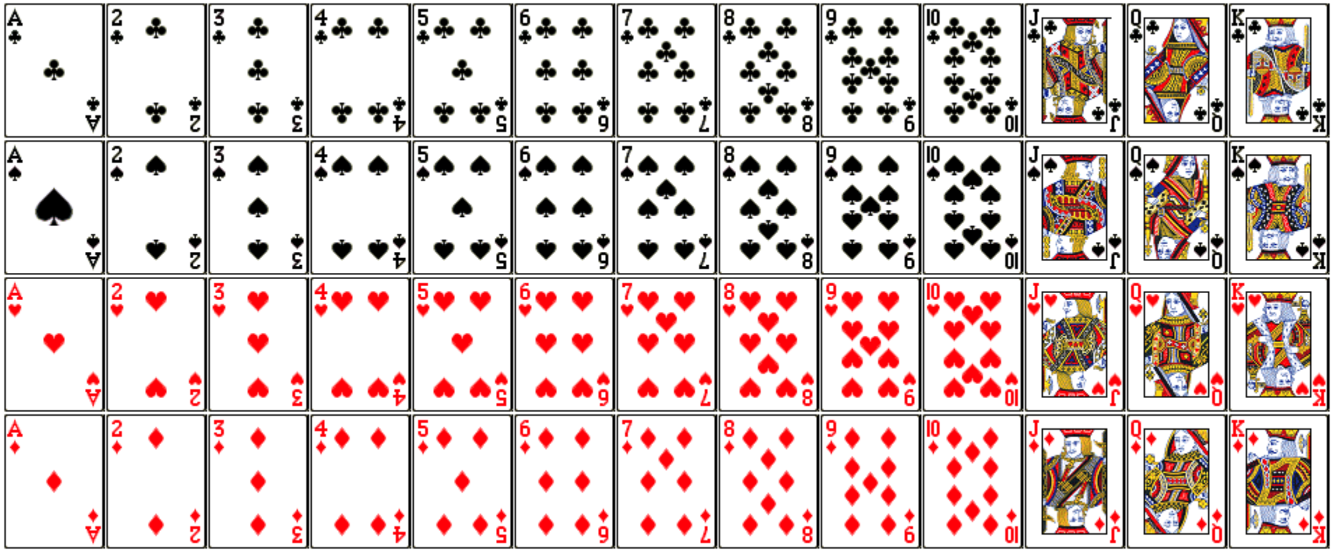
- Example: $b|0 = b,\ b|1 = 1$

```
   01010101  ← input
|  11110000  ← bitmask
-----------
   11110101  ← output
```

"set to one" "keep as is"

Numerical Encoding Design Example

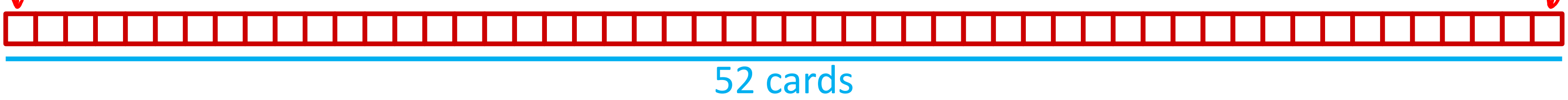
- Encode a standard deck of playing cards
  - 52 cards in 4 suits
- Operations to implement:
  - Which is the higher value card?
  - Are they the same suit?



Representations and Fields

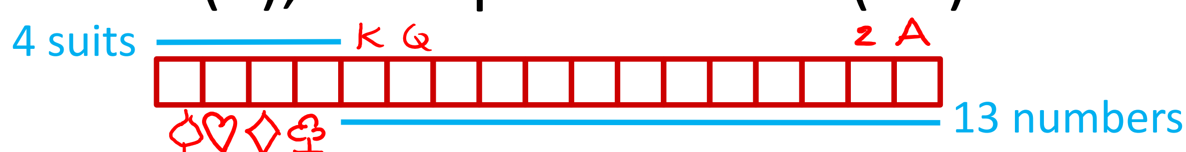
K♠ A♣

1) 1 bit per card (52): bit corresponding to card set to 1



- "One-hot" encoding (similar to set notation)
- Drawbacks:
  - Hard to compare values and suits
  - Large number of bits required 52 bits $\xrightarrow{\text{fits in}}$ 7 bytes (56 bits)

2) 1 bit per suit (4), 1 bit per number (13): 2 bits set



- Pair of one-hot encoded values
- Easier to compare suits and values, but still lots of bits used

17 bits → 3 bytes

Representations and Fields

3) Binary encoding of all 52 cards – only 6 bits needed

- $2^6 = 64 \geq 52$

  $2^5 = 32 < 52$

low-order 6 bits of a byte

- Fits in one byte (smaller than one-hot encodings)
- How can we make value and suit comparisons easier?

4) Separate binary encodings of suit (2 bits) and value (4 bits)

4 suits 13 values

suit value

- Also fits in one byte, and easy to do comparisons

C	♣	00
D	♦	01
H	♥	10
S	♠	11

K	Q	J	. . .	3	2	A
1101	1100	1011	. . .	0011	0010	0001
13			. . .			1

Compare Card Suits

mask: a bit vector designed to achieve a desired behavior when used with a bitwise operator on another bit vector *v*.
Here we turn all *but* the bits of interest in *v* to 0.

```
char hand[5];         // represents a 5-card hand
char card1, card2;    // two cards to compare
card1 = hand[0];
card2 = hand[1];
...
if ( sameSuitP(card1, card2) ) { ... }
```

text substitution

```
#define SUIT_MASK  0x30

int sameSuitP(char card1, char card2) {
  return (!((card1 & SUIT_MASK) ^ (card2 & SUIT_MASK)));
  //return (card1 & SUIT_MASK) == (card2 & SUIT_MASK);
}
```

returns **int**

equivalent

SUIT_MASK = 0x30 =

0	0	1	1	0	0	0	0

suit (keep)

value (discard)

x & 0 = 0

x & 1 = x

Compare Card **Suits**

```
#define SUIT_MASK  0x30

int sameSuitP(char card1, char card2) {
  return (!((card1 & SUIT_MASK) ^ (card2 & SUIT_MASK)));
  //return (card1 & SUIT_MASK) == (card2 & SUIT_MASK);
}
```

card1

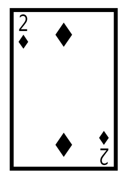
0	0	0	1	0	0	1	0

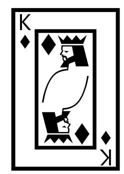
① &

0	0	1	1	0	0	0	0

=

0	0	0	1	0	0	0	0





SUIT_MASK

x & 0 = 0

x & 1 = x

card2

0	0	0	1	1	1	0	1

& ①

0	0	1	1	0	0	0	0

=

0	0	0	1	0	0	0	0

② ^

0	0	0	0	0	0	0	0

③ ! ← logical

0	0	0	0	0	0	0	1

`!(x^y)` equivalent to `x==y`

Compare Card Values

```
char hand[5];          // represents a 5-card hand
char card1, card2;     // two cards to compare
card1 = hand[0];
card2 = hand[1];
...
if ( greaterValue(card1, card2) ) { ... }
```

```
#define VALUE_MASK  0x0F

int greaterValue(char card1, char card2) {
  return ((unsigned int)(card1 & VALUE_MASK) >
          (unsigned int)(card2 & VALUE_MASK));
}
```

VALUE_MASK = 0x0F = | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |

suit (discard) value (keep)

Compare Card Values

```
#define VALUE_MASK  0x0F

int greaterValue(char card1, char card2) {
  return ((unsigned int)(card1 & VALUE_MASK) >
          (unsigned int)(card2 & VALUE_MASK));
}
```

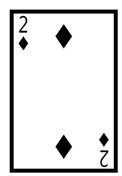
~~0~~	~~0~~	~~1~~	~~0~~	0	0	1	0

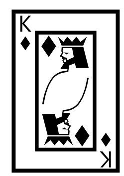
1 &

0	0	0	0	1	1	1	1

=

0	0	0	0	0	0	1	0





VALUE_MASK

~~0~~	~~0~~	~~1~~	~~0~~	1	1	0	1

& 1

0	0	0	0	1	1	1	1

=

0	0	0	0	1	1	0	1

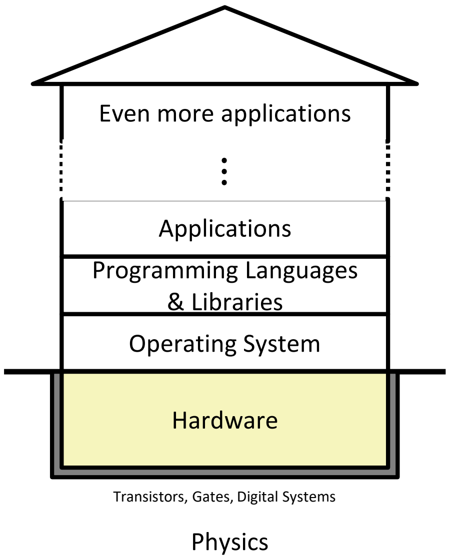
2

$2_{10} > 13_{10}$

0 (false)

The Hardware/Software Interface

- Topic Group 1: **Data**
  - Memory, Data, **Integers**, Floating Point, Arrays, Structs



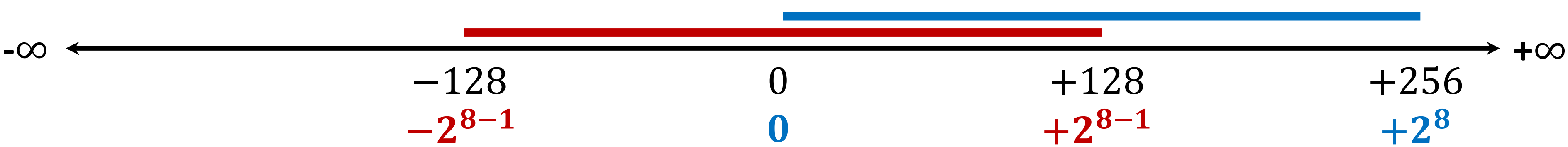
- How do we store information for other parts of the house of computing to access?
  - How do we represent data and what limitations exist?
  - What design decisions and priorities went into these encodings?

Encoding Integers

- The hardware (and C) supports two flavors of integers
  - *unsigned* – only the non-negatives
  - *signed* – both negatives and non-negatives
- Cannot represent all integers with w bits
  - Only 2^w distinct bit patterns
  - Unsigned values: $0 \ldots 2^w-1$
  - Signed values: $-2^{w-1} \ldots 2^{w-1}-1$

same widths, just shifted

- **Example:** 8-bit integers (*e.g.*, `char`)



Unsigned Integers (Review)

- Unsigned values follow the standard base 2 system
  - $b_7b_6b_5b_4b_3b_2b_1b_0 = b_7 2^7 + b_6 2^6 + \cdots + b_1 2^1 + b_0 2^0$

- Useful formula: $2^{N-1} + 2^{N-2} + \ldots + 2 + 1 = 2^N - 1$
  - *i.e.*, N ones in a row = $2^N - 1$
  - *e.g.*, 0b111111 = 63 ← x, 6 1's in a row

$x+1 = 0b1\,000\,000$
$= 2^6$

$x = 2^6 - 1$

Sign and Magnitude

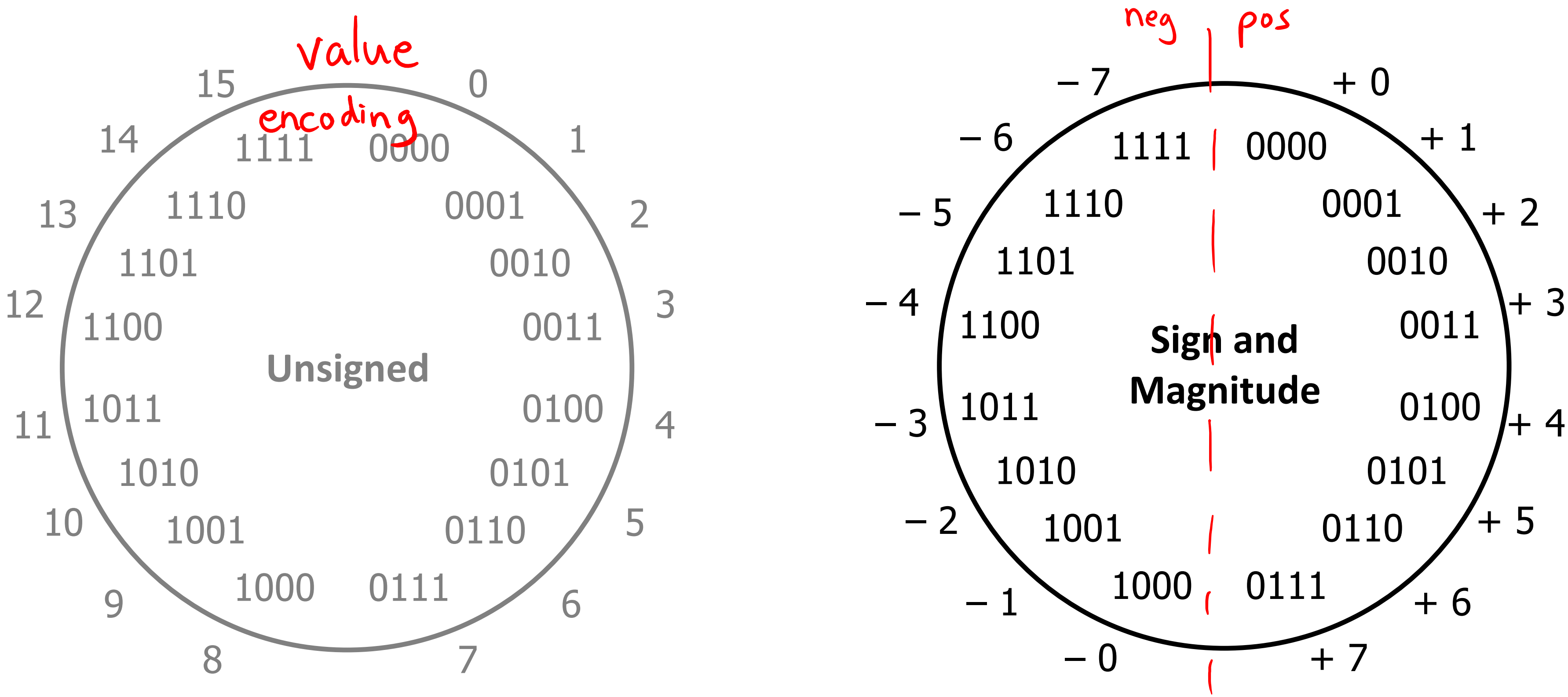
Not used in practice for integers!

- Designate the high-order bit (MSB) as the "sign bit"
  - `sign=0`: positive numbers; `sign=1`: negative numbers
- Benefits:
  - Using MSB as sign bit matches positive numbers with unsigned unsigned: $0b\,0010 = 2^1 = 2$; sign + mag: $0b\,0010 = +2^1 = 2$ ☑
  - All zeros encoding is still = 0
- Examples (8 bits):
  - 0x00 = 00000000_2 is non-negative, because the sign bit is 0 (+)
  - 0x7F = 01111111_2 is non-negative ($+127_{10}$) (+) 2^7-1
  - 0x85 = 10000101_2 is negative (-5_{10}) (−)
  - 0x80 = 10000000_2 is negative... zero???

Sign and Magnitude

Not used in practice for integers!

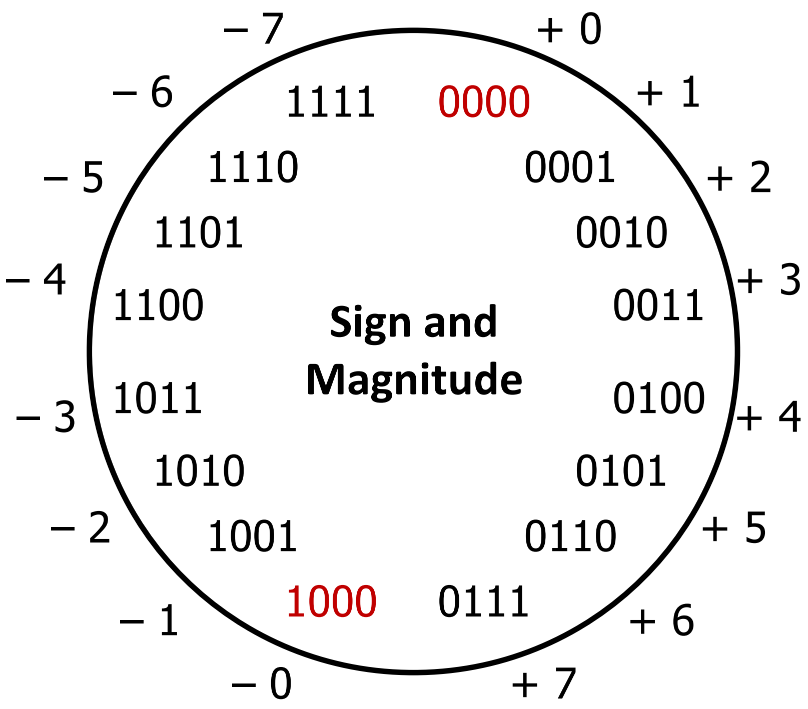
- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks?



Sign and Magnitude

Not used in practice for integers!

- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks:
  - Two representations of 0 (bad for checking equality)



Sign and Magnitude

Not used in practice for integers!

- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks:
  - Two representations of 0 (bad for checking equality)
  - Arithmetic is cumbersome
    - Example: `4-3 != 4+(-3)`

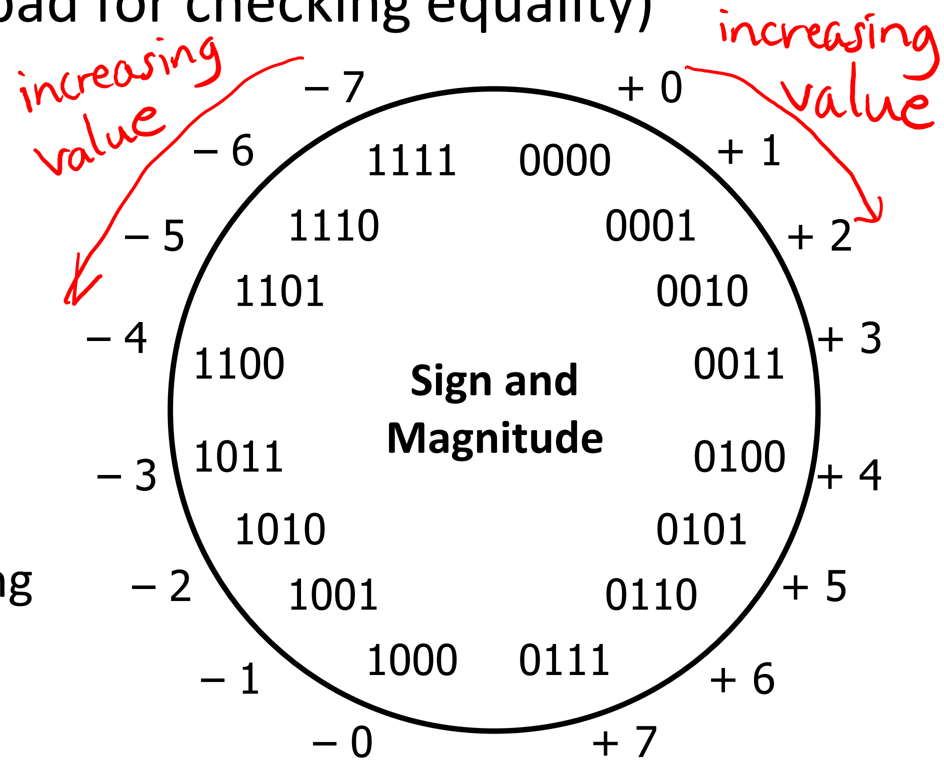
4	0100
– 3	– 0011
1	0001

✓

4	0100
+ -3	+ 1011
-7	1111

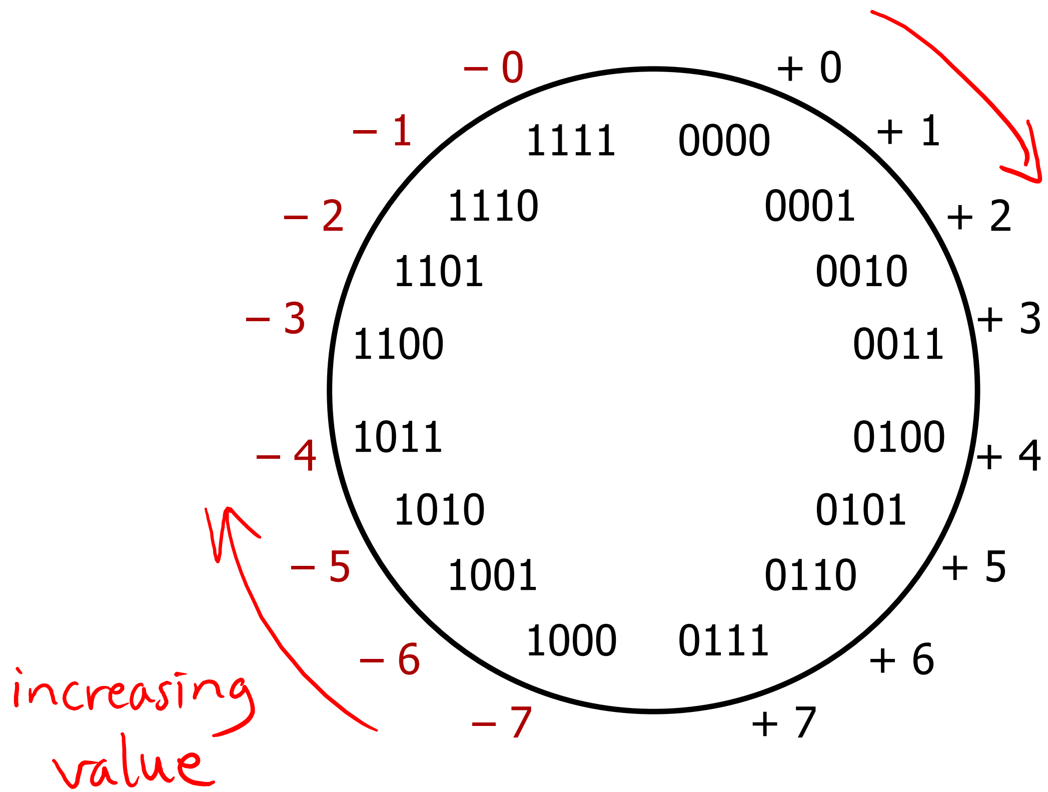
✗

    - Negatives "increment" in wrong direction!



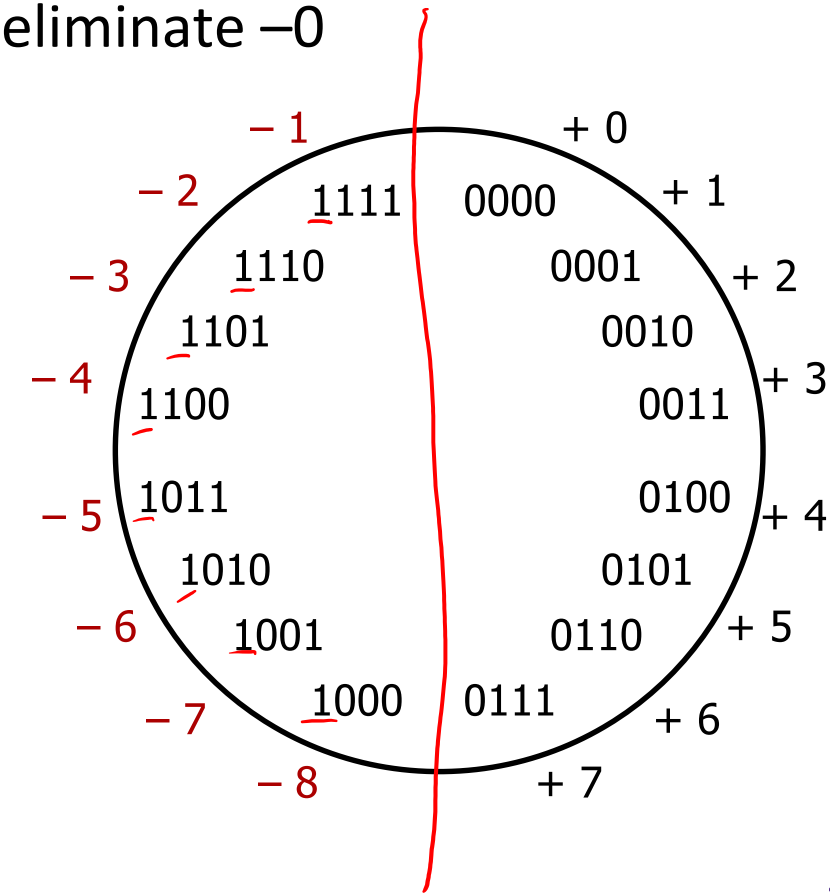
Two's Complement

- Let's fix these problems:
  1) "Flip" negative encodings so incrementing works



Two's Complement

- Let's fix these problems:
  1) "Flip" negative encodings so incrementing works
  2) "Shift" negative numbers to eliminate –0

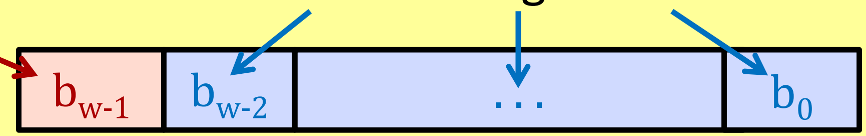


- MSB *still* indicates sign!
  - This is why we represent one more negative than positive number (-2^{N-1} to $2^{N-1}-1$)

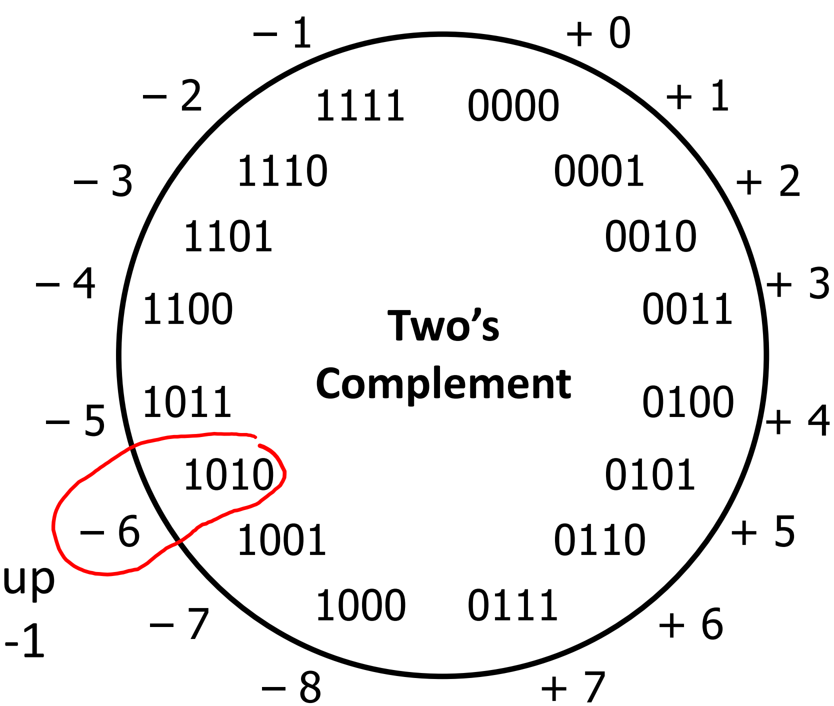
Two’s Complement Negatives (Review)

- Accomplished with one neat mathematical trick!

b_{w-1} has weight -2^{w-1}, other bits have usual weights $+2^i$



- 4-bit Examples:
  - 8 2
  - 1010_2 unsigned:
    $1*2^3+0*2^2+1*2^1+0*2^0$ = **10**
  - -8
  - 1010_2 two’s complement:
    $-1*2^3+0*2^2+1*2^1+0*2^0$ = **–6**
- -1 represented as:
  $1111_2 = -2^3+(2^3 - 1)$
  3 one's in a row
  - MSB makes it super negative, add up all the other bits to get back up to -1



Polling Question

MSB

- Take the 4-bit number encoding **x = 0b1011**
- Which of the following numbers is NOT a valid interpretation of x using any of the number representation schemes discussed today?
  - Unsigned, Sign and Magnitude, Two's Complement
  - Vote in Ed Lessons

A. $\boxed{\textbf{A. } -4}$

~~B. -5~~

~~C. 11~~

~~D. -3~~

E. We're lost…

unsigned: $8+2+1=11$

sign + mag: $1011 \rightarrow -(2+1) = -3$

two's: $-8+2+1=-5$

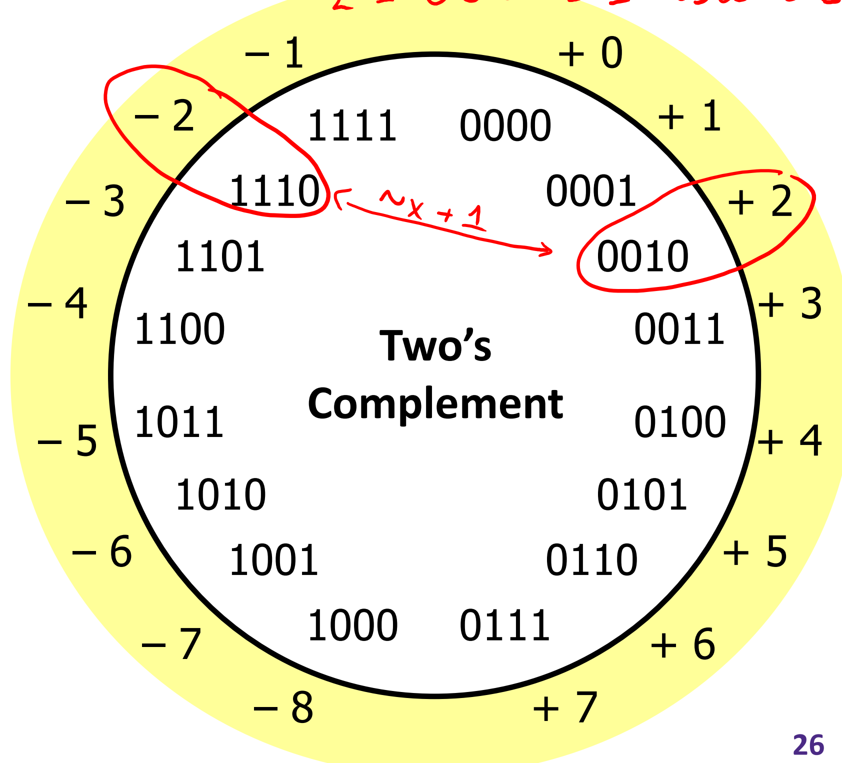
$-x = 0b\ 0100 + 1 = 5 \rightarrow x = -5$

Two's Complement is Great (Review)

- Roughly same number of (+) and (–) numbers
- Positive number encodings match unsigned
- Single zero
- All zeros encoding = 0

2 = 0b0010
−2 = 0b1101 +1 = 0b1110 ✓
2 = 0b0001+1 = 0b0010 ✓

- Simple negation procedure:
  - Get negative representation of any integer by taking bitwise complement and then adding one!
    `( ~x + 1 == -x )`



Integer Representation Design Decisions

- Some explicitly mentioned and some implicit:
  - Represent consecutive integers
    - What would happen if we had gaps?

      example, if only representing even integers, what should happen when we compute 6/2 ?
  - Represent the same number of positives and negatives
    - Could we choose a different shift/bias? Sure! "biased notation"

      the bias should make sense in the context of our application

      arithmetic might get weird again...
  - Positive number encodings match unsigned
    - What's the advantage here?

      no need to convert anything when changing interpretations

      example: int x

      (unsigned int) x

Summary

- Bit-level operators allow for fine-grained manipulations of data
  - Bitwise AND (`&`), OR (`|`), and NOT (`~`) different than logical AND (`&&`), OR (`||`), and NOT (`!`)
  - Especially useful with bit masks
- Choice of *encoding scheme* is important
  - Tradeoffs based on size requirements and desired operations
- Integers represented using unsigned and two's complement representations
  - Limited by fixed bit width
  - We'll examine arithmetic operations next lecture