

Caches IV

CSE 351 Winter 2020

Instructor: **Teaching Assistants:**

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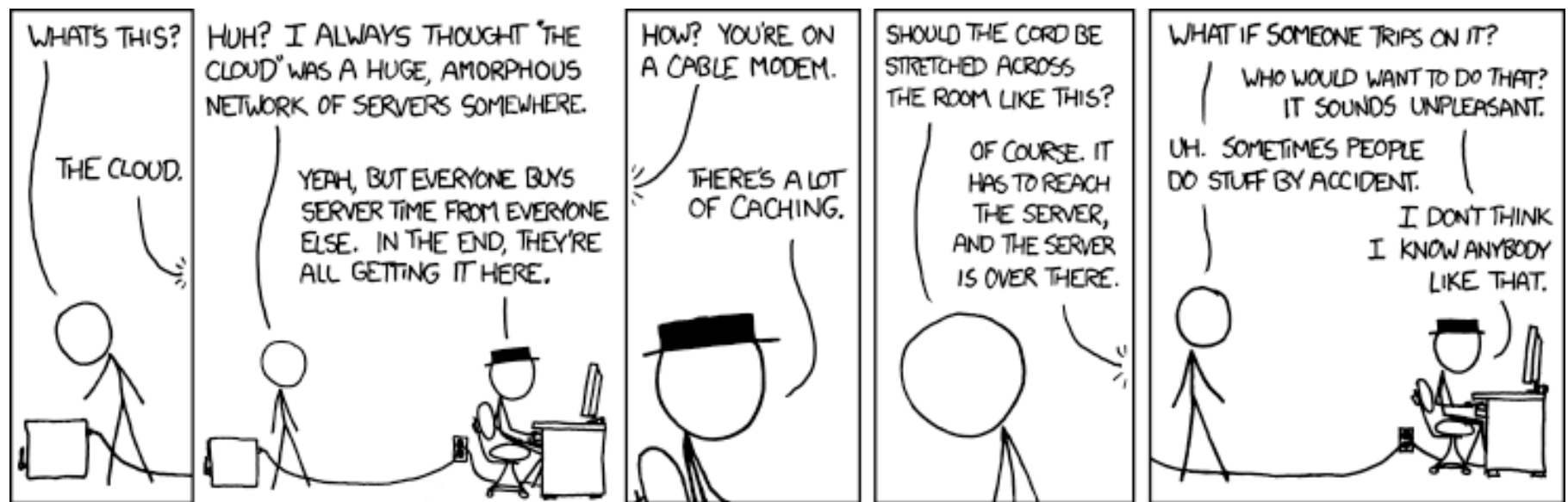
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Josie Lee

Jeffery Tian

Callum Walker

Eddy (Tianyi) Zhou



<http://xkcd.com/908/>

Administrivia

- ❖ Lab 3 due Monday(2/24)
- ❖ hw16 due Monday (2/24)

- ❖ hw17 due Wednesday (2/26)

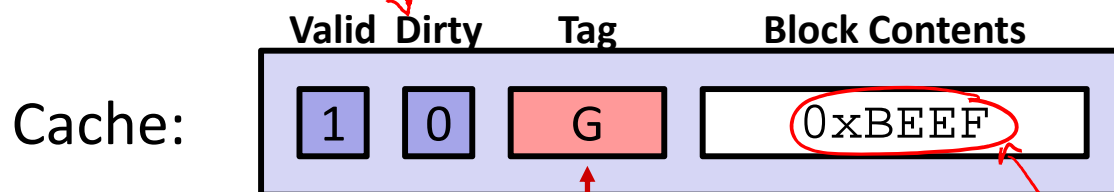
- ❖ Lab 4 coming soon!
 - Cache parameter puzzles and code optimizations

What about writes?

- ❖ Multiple copies of data may exist:
 - multiple levels of cache and main memory
- ❖ What to do on a write-hit? (block/data already in \$)
 - **Write-through**: write immediately to next level
 - ★ **Write-back**: defer write to next level until line is evicted (replaced)
 - Must track which cache lines have been modified ("**dirty bit**") ← extra management bit only for write-back cache
- ❖ What to do on a write-miss? (block/data not currently in \$)
 - **Write allocate**: ("fetch on write") load into cache, then execute the write-hit policy
 - Good if more writes or reads to the location follow
 - **No-write allocate**: ("write around") just write immediately to next level
- ❖ Typical caches:
 - Write-back + Write allocate, usually ★
 - Write-through + No-write allocate, occasionally

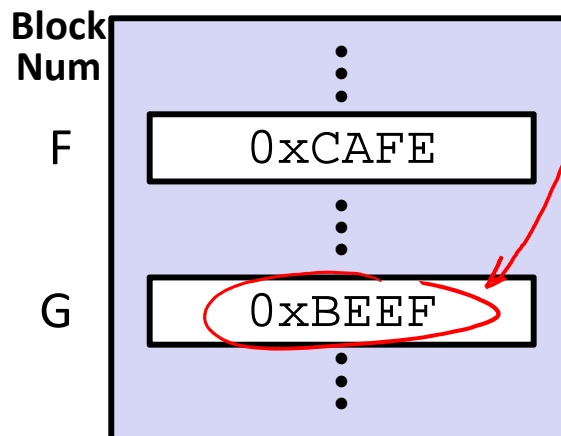
Write-back, Write Allocate Example

Note: While unrealistic, this example assumes that all requests have offset 0 and are for a block's worth of data.



There is only one set in this tiny cache, so the tag is the entire block number!

Memory:

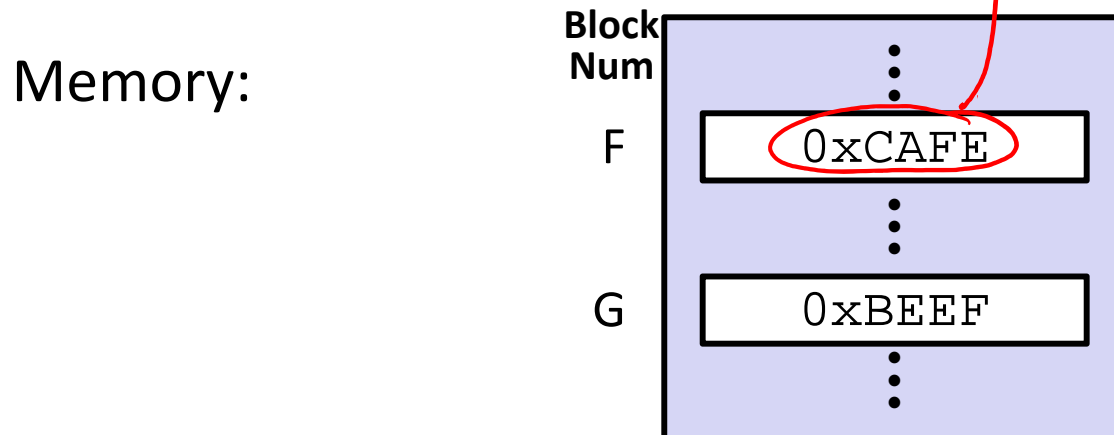
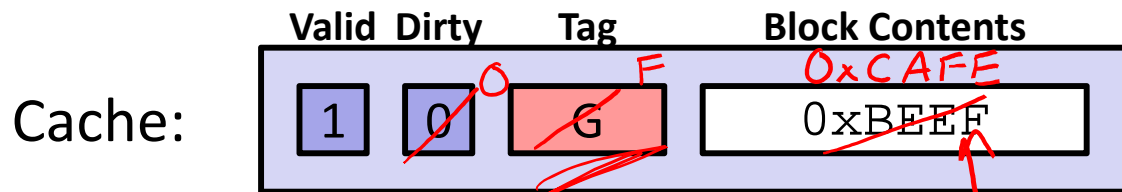


not dirty, so these copies are consistent

Write-back, Write Allocate Example

1) mov \$0xFACE, (F)

Write Miss!



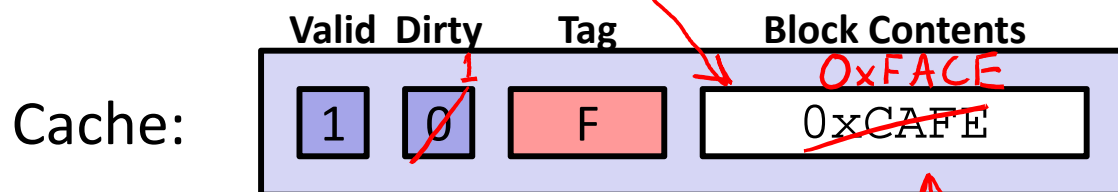
Step 1: Bring F into cache

Write-back, Write Allocate Example

1) `mov $0xFACE, (F)`

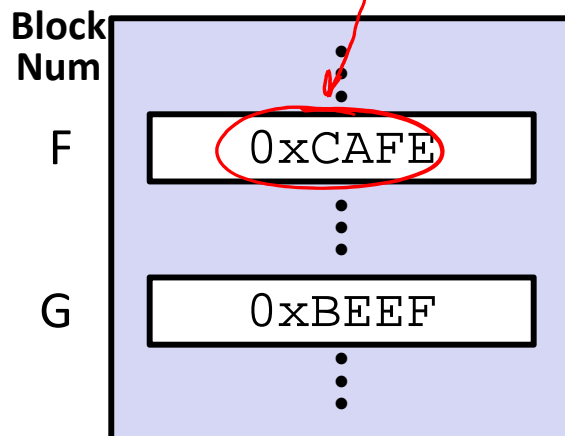
Write Miss

② write data into block



different!

Memory:



Step 1: Bring F into cache

Step 2: Write 0xFACE to cache only and set the dirty bit

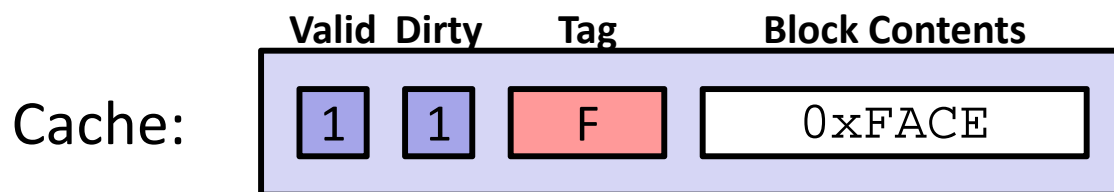
Hit

Miss?

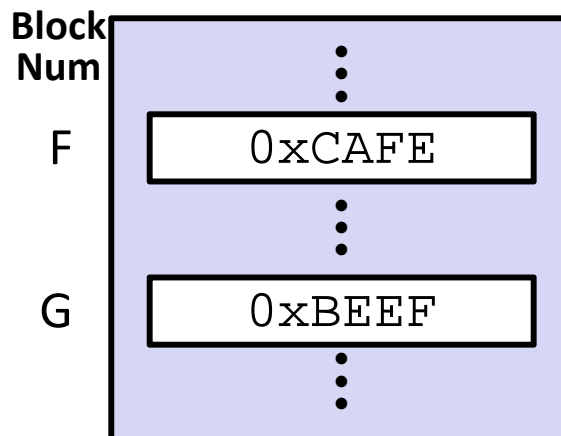
Write-back, Write Allocate Example

1) `mov $0xFACE, (F)`

Write Miss



Memory:

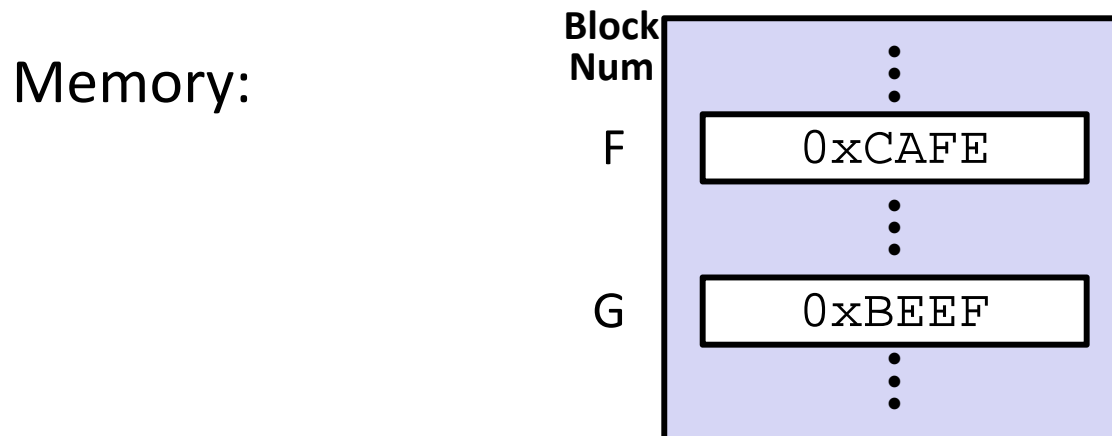
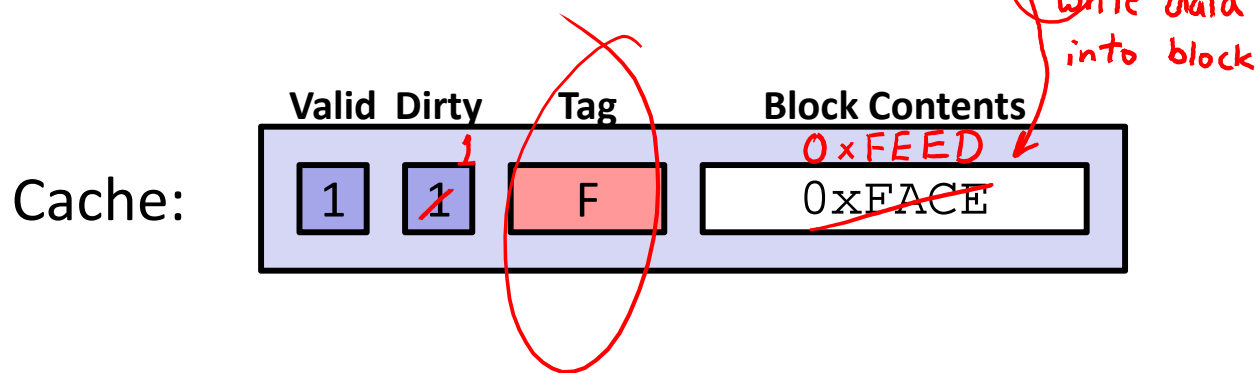


Step 1: Bring F into cache

Step 2: Write 0xFACE to cache only and set the dirty bit

Write-back, Write Allocate Example

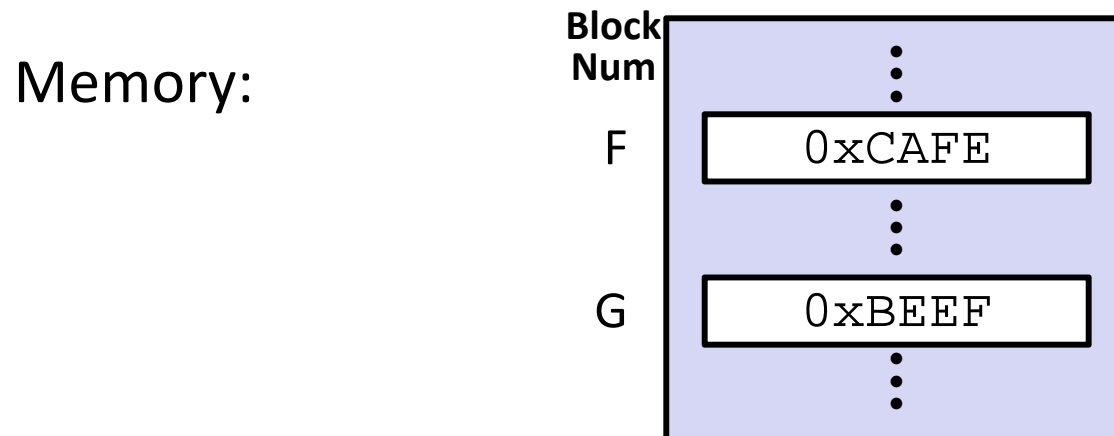
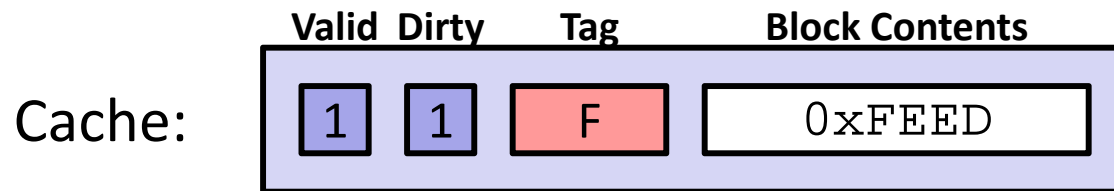
1) `mov $0xFACE, (F)` Write Miss
 2) `mov $0xFEED, (F)` Write Hit!



Step: Write
 0xFEED to cache
 only (and set the
 dirty bit)

Write-back, Write Allocate Example

1) `mov $0xFACE, (F)` 2) `mov $0xFEED, (F)`
 Write Miss Write Hit

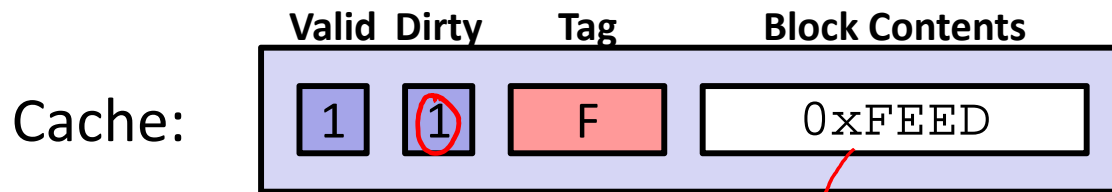


Write-back, Write Allocate Example

1) `mov $0xFACE, (F)`
Write Miss

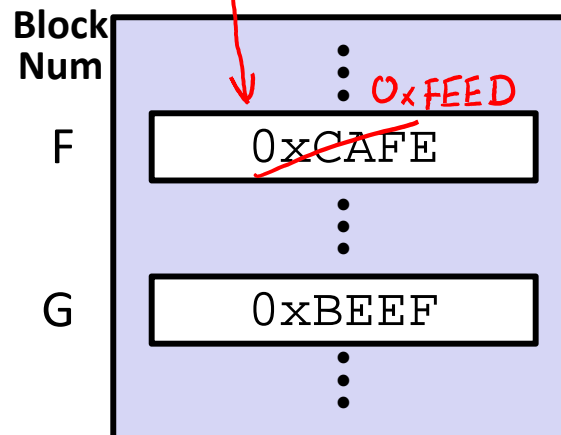
2) `mov $0xFEED, (F)`
Write Hit

3) `mov (G), %ax`
Read Miss!



evicted block was dirty

Memory:



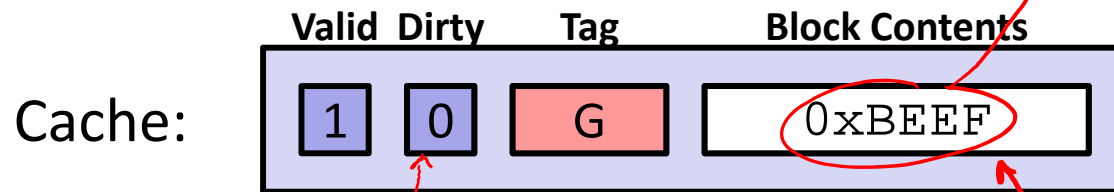
Step 1: Write **F** back to memory since it is dirty

Write-back, Write Allocate Example

1) `mov $0xFACE, (F)`
Write Miss

2) `mov $0xFEED, (F)`
Write Hit

3) `mov (G), %ax`
Read Miss



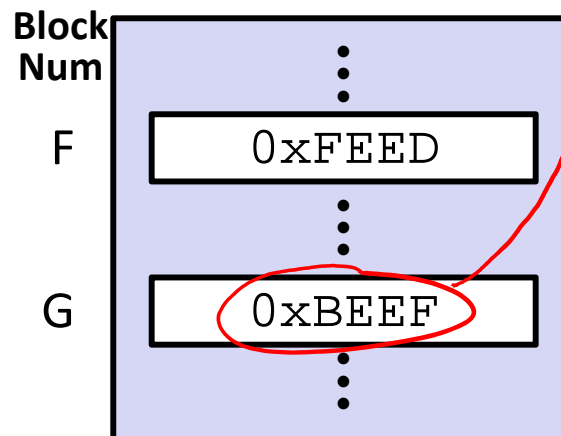
new block is consistent with memory

② load new block

③ copy into %ax

%rax

Memory:



Step 1: Write **F** back to memory since it is dirty

Step 2: Bring **G** into the cache so that we can copy it into `%ax`

Cache Simulator

- ❖ Want to play around with cache parameters and policies? Check out our cache simulator!
 - <https://courses.cs.washington.edu/courses/cse351/cachesim/>
- ❖ Way to use:
 - Take advantage of “explain mode” and navigable history to test your own hypotheses and answer your own questions
 - Self-guided Cache Sim Demo posted along with Section 7
 - Will be used in hw17 – Lab 4 Preparation

Polling Question

❖ Which of the following cache statements is **FALSE**?

▪ Vote at <http://pollev.com/rea>

A. We can reduce compulsory misses by decreasing our block size

*smaller block size pulls fewer bytes into \$
on a miss*

B. We can reduce conflict misses by increasing associativity

*more options to place blocks before
evictions occur*

C. A write-back cache will save time for code with good temporal locality on writes

*frequently-used blocks rarely
get evicted, so fewer write-backs*

D. A write-through cache will always match data with the memory hierarchy level below it

*yes, its main
goal is data
consistency*

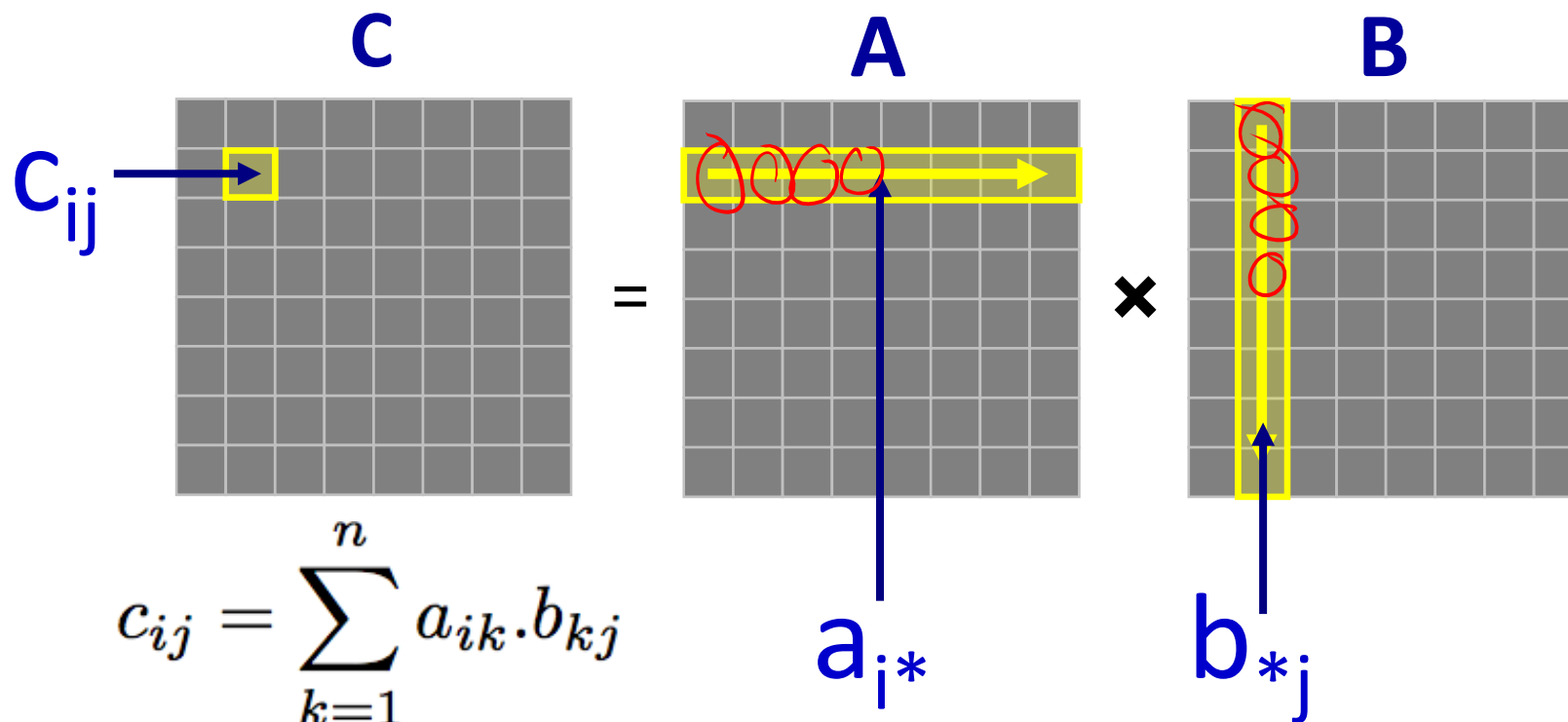
E. We're lost...

Optimizations for the Memory Hierarchy

- ❖ Write code that has locality!
 - Spatial: access data contiguously
 - Temporal: make sure access to the same data is not too far apart in time

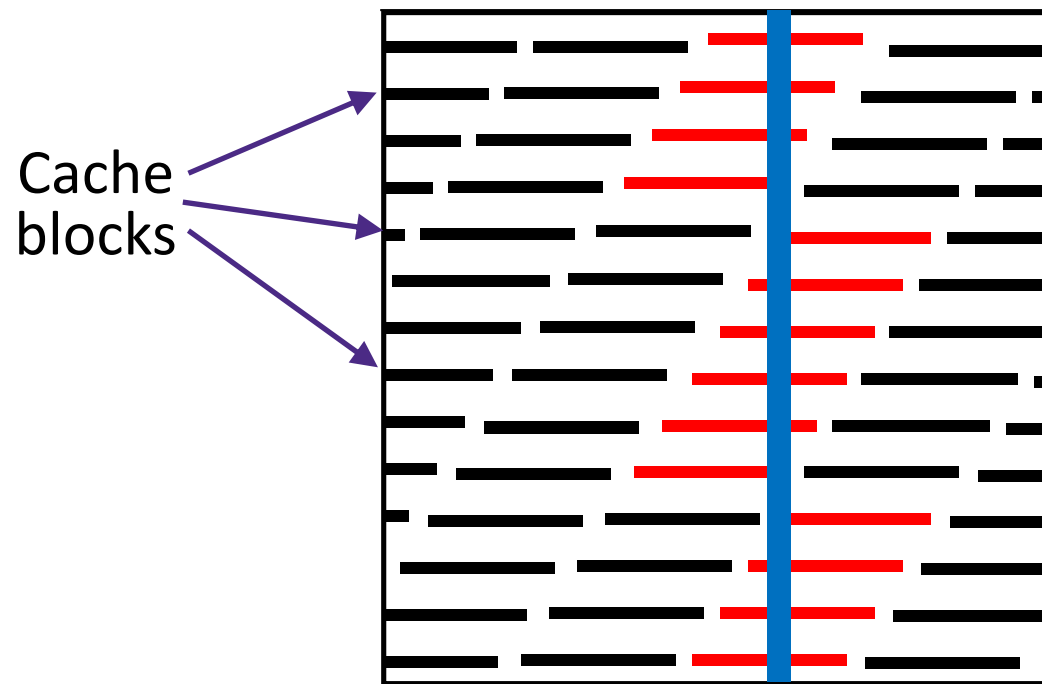
- ❖ How can you achieve locality?
 - Adjust memory accesses in *code* (software) to improve miss rate (MR)
 - Requires knowledge of *both* how caches work as well as your system's parameters
 - Proper choice of algorithm
 - Loop transformations

Example: Matrix Multiplication



Matrices in Memory

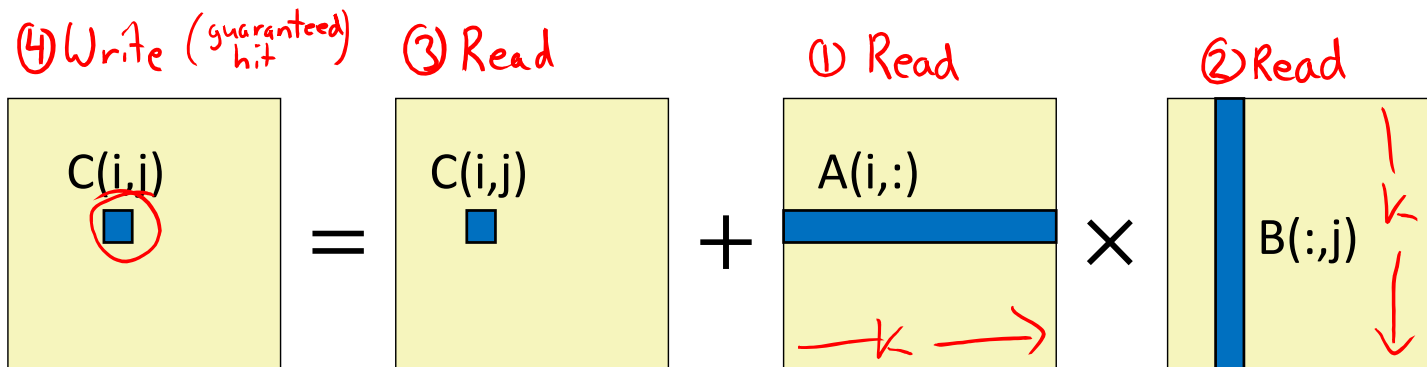
- ❖ How do cache blocks fit into this scheme?
 - Row major matrix in memory:



COLUMN of matrix (blue) is spread
among cache blocks shown in red

Naïve Matrix Multiply

```
# move along rows of A
for (i = 0; i < n; i++)
    # move along columns of B
    for (j = 0; j < n; j++)
        # EACH k loop reads row of A, col of B
        # Also read & write c(i,j) n times
        for (k = 0; k < n; k++)
            c[i*n+j] += a[i*n+k] * b[k*n+j];
```



$$c[i][j] + = a[i][k] * b[k][j]$$

Cache Miss Analysis (Naïve)

Ignoring matrix c

❖ Scenario Parameters:

- Square matrix ($n \times n$), elements are doubles
- Cache block size $K = 64 \text{ B} = \underline{8 \text{ doubles}}$

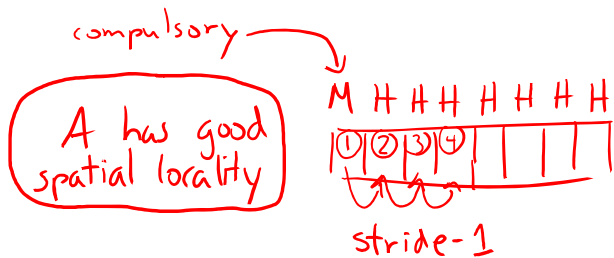
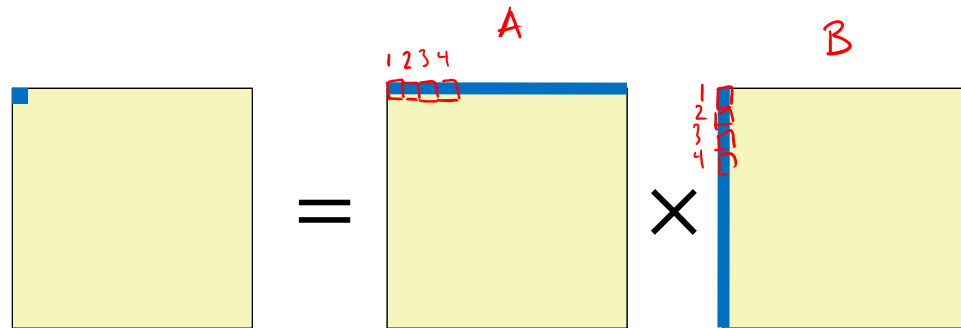
8 matrix elements per cache block

★ Cache size $C \ll n$ (much smaller than n)
key assumption!

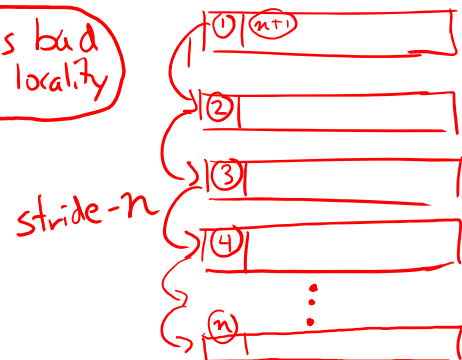
To calculate a single value in c

❖ ~~Each iteration:~~

▪ $\frac{n}{8} + n = \frac{9n}{8}$ misses



B has bad spatial locality



by the time we get to nt+1, block has been kicked out of \$

Cache Miss Analysis (Naïve)

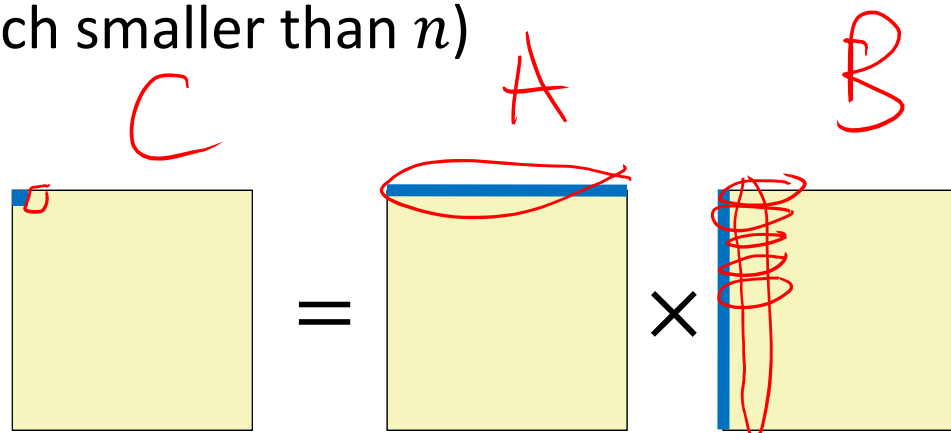
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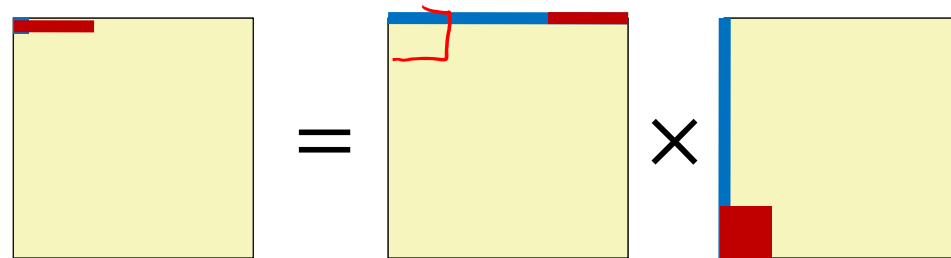
❖ Each iteration:

- $\frac{n}{8} + n = \frac{9n}{8}$ misses



- Afterwards **in cache:** (schematic)

red showing blocks remaining in the \$



8 doubles wide

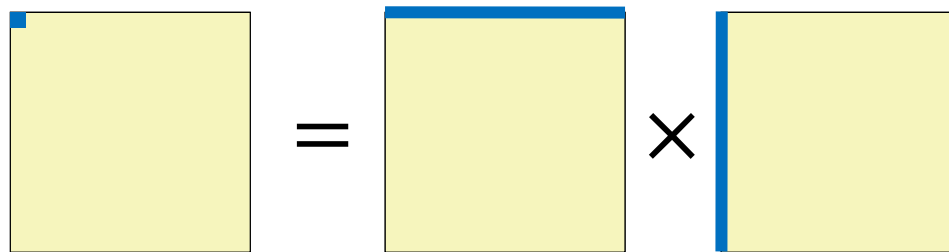
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❖ Each iteration:

- $\frac{n}{8} + n = \frac{9n}{8}$ misses



❖ Total misses: $\frac{9n}{8} \times n^2 = \frac{9}{8}n^3$ once per product matrix element

Linear Algebra to the Rescue (1)

This is extra
(non-testable)
material

- ❖ Can get the same result of a matrix multiplication by splitting the matrices into smaller submatrices (matrix “blocks”)
- ❖ For example, multiply two 4×4 matrices:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \text{ with } B \text{ defined similarly.}$$

$$AB = \begin{bmatrix} (A_{11}B_{11} + A_{12}B_{21}) & (A_{11}B_{12} + A_{12}B_{22}) \\ (A_{21}B_{11} + A_{22}B_{21}) & (A_{21}B_{12} + A_{22}B_{22}) \end{bmatrix}$$

Linear Algebra to the Rescue (2)

This is extra (non-testable) material

C_{11}	C_{12}	C_{13}	C_{14}
C_{21}	C_{22}	C_{23}	C_{24}
C_{31}	C_{32}	C_{43}	C_{34}
C_{41}	C_{42}	C_{43}	C_{44}

A_{11}	A_{12}	A_{13}	A_{14}
A_{21}	A_{22}	A_{23}	A_{24}
A_{31}	A_{32}	A_{33}	A_{34}
A_{41}	A_{42}	A_{43}	A_{144}

B_{11}	B_{12}	B_{13}	B_{14}
B_{21}	B_{22}	B_{23}	B_{24}
B_{32}	B_{32}	B_{33}	B_{34}
B_{41}	B_{42}	B_{43}	B_{44}

Matrices of size $n \times n$, split into 4 blocks of size r ($n=4r$)

$$C_{22} = A_{21}B_{12} + A_{22}B_{22} + A_{23}B_{32} + A_{24}B_{42} = \sum_k A_{2k} * B_{k2}$$

- ❖ Multiplication operates on small “block” matrices
 - Choose size so that they fit in the cache!
 - This technique called “cache blocking” ★

Blocked Matrix Multiply

- ❖ Blocked version of the naïve algorithm:

6 nested loops may seem less efficient, but leads to much faster code!

```

# move by rxr BLOCKS now
for (i = 0; i < n; i += r)
  for (j = 0; j < n; j += r)
    for (k = 0; k < n; k += r)
      # block matrix multiplication
      for (ib = i; ib < i+r; ib++)
        for (jb = j; jb < j+r; jb++)
          for (kb = k; kb < k+r; kb++)
            c[ib*n+jb] += a[ib*n+kb]*b[kb*n+jb];

```

loop over block matrices

looping over entire matrix a blocks at a time

loop within block matrices

looping over a single block

- r = block matrix size (assume r divides n evenly)

Cache Miss Analysis (Blocked)

Ignoring matrix c

❖ Scenario Parameters:

- Cache block size $K = 64 \text{ B} = 8 \text{ doubles}$
- Cache size $C \ll n$ (much smaller than n)

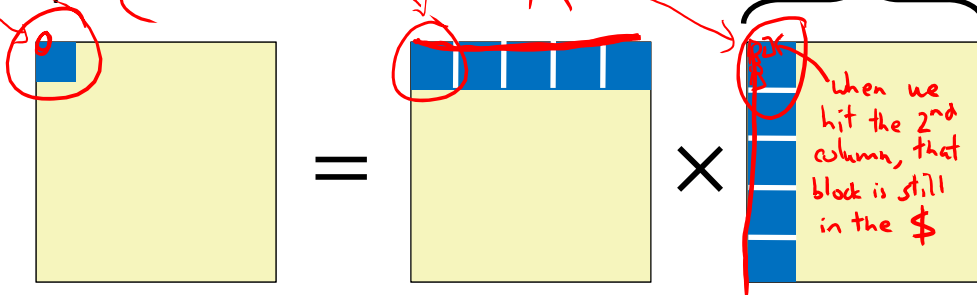
- Three blocks \blacksquare ($r \times r$) fit into cache: $3r^2 < C$

r^2 elements per block, 8 per cache block

❖ Each block iteration:

- $r^2/8$ misses per block
- $2n/r \times r^2/8 = nr/4$

n/r blocks in row and column



Cache Miss Analysis (Blocked)

Ignoring matrix c

❖ Scenario Parameters:

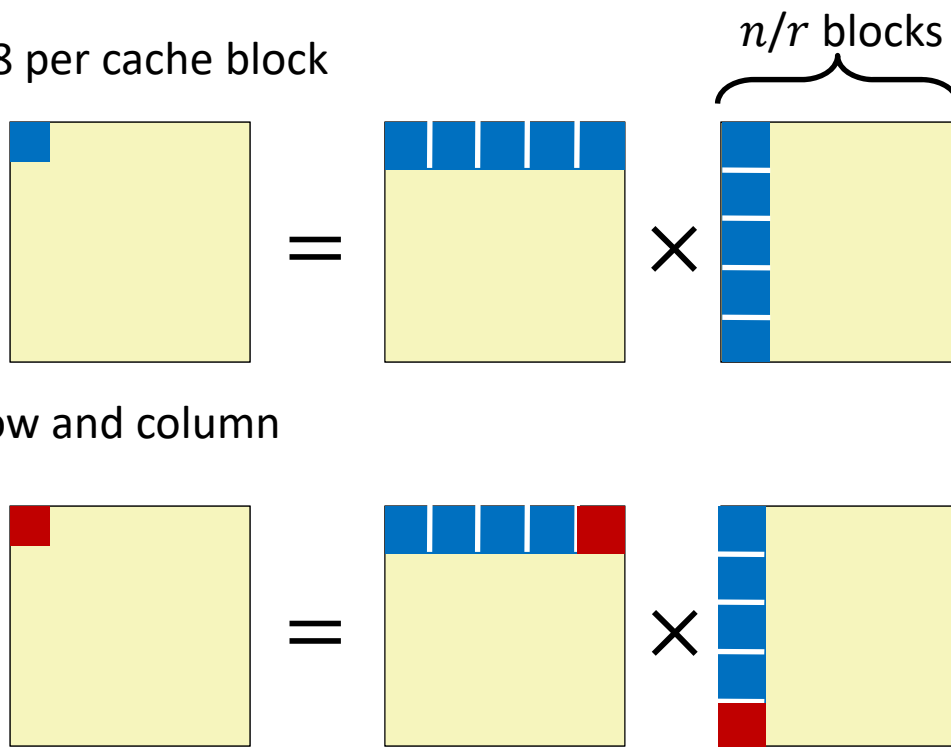
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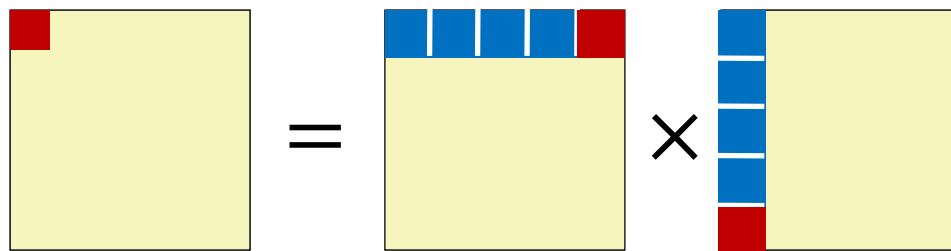
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n/r blocks in row and column



- Afterwards in cache (schematic)



Cache Miss Analysis (Blocked)

Ignoring matrix c

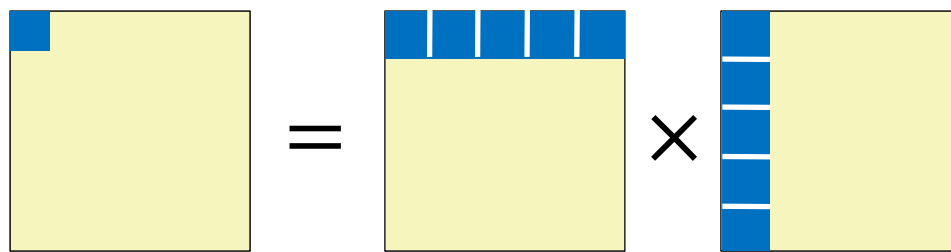
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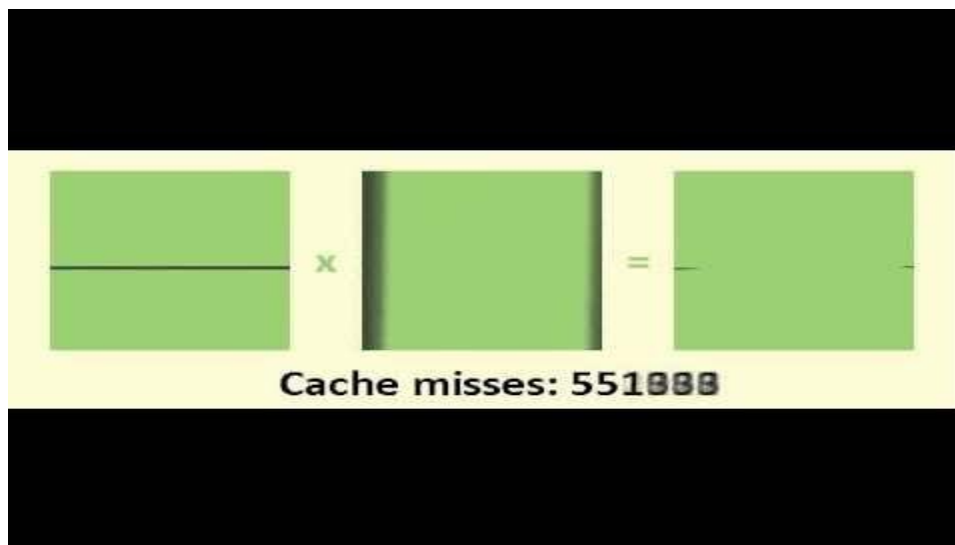
❖ Total misses:

- $nr/4 \times (n/r)^2 = n^3/(4r)$ vs. $9n^3/8$

Matrix Multiply Visualization

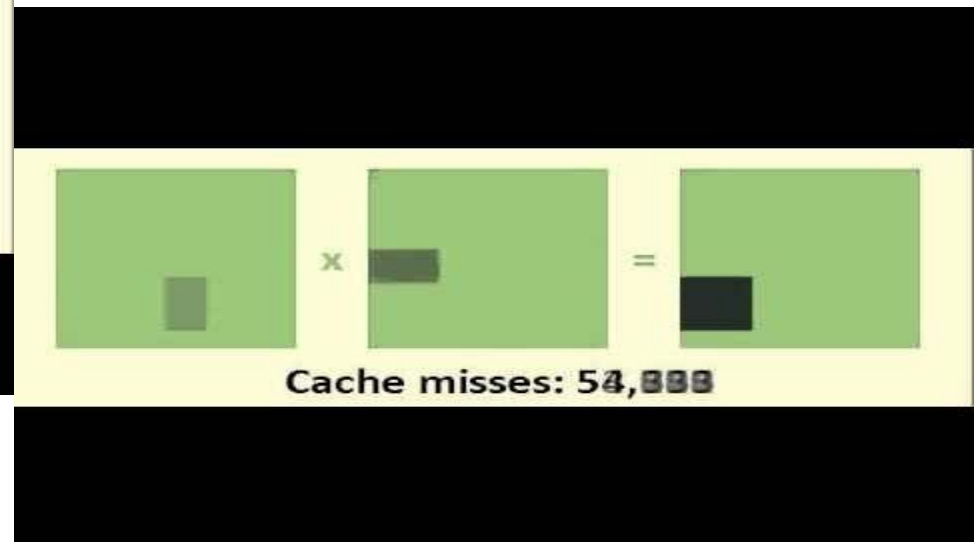
❖ Here $n = 100$, $C = 32$ KiB, $r = 30$

Naïve:



$\approx 1,020,000$
cache misses

Blocked:



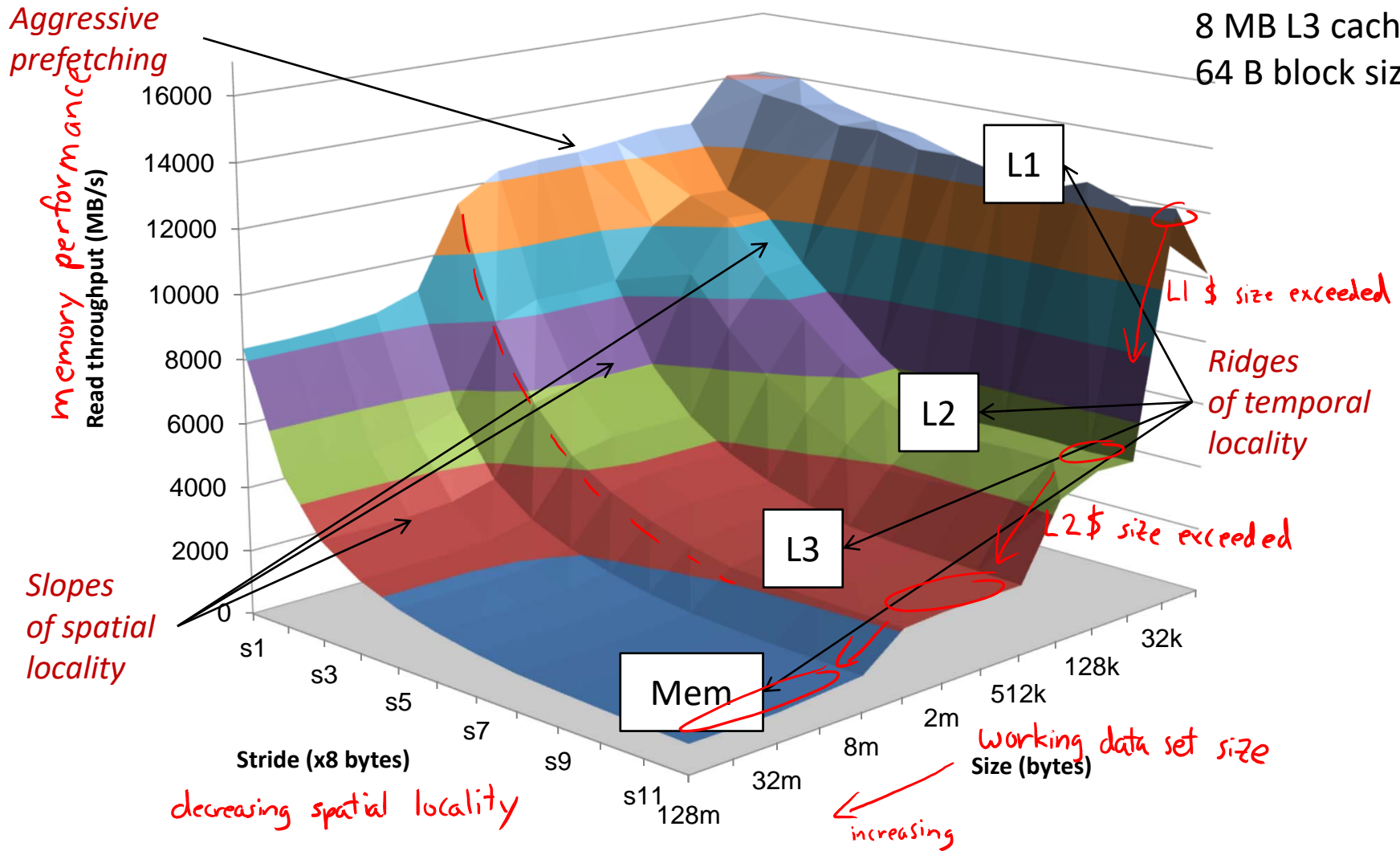
$\approx 90,000$
cache misses

Cache-Friendly Code

- ❖ Programmer can optimize for cache performance
 - How data structures are organized
 - How data are accessed
 - Nested loop structure ←
 - Blocking is a general technique
- ❖ All systems favor “cache-friendly code”
 - Getting absolute optimum performance is very platform specific
 - Cache size, cache block size, associativity, etc.
 - Can get most of the advantage with generic code
 - Keep working set reasonably small (temporal locality)
 - Use small strides (spatial locality)
 - Focus on inner loop code

The Memory Mountain

Core i7 Haswell
 2.1 GHz
 32 KB L1 d-cache
 256 KB L2 cache
 8 MB L3 cache
 64 B block size



Learning About Your Machine

❖ Linux:

- `lscpu`
- `ls /sys/devices/system/cpu/cpu0/cache/index0/`
 - Example: `cat /sys/devices/system/cpu/cpu0/cache/index*/size`

❖ Windows:

- `wmic memcache get <query>` (all values in KB)
- Example: `wmic memcache get MaxCacheSize`

- ❖ Modern processor specs: <http://www.7-cpu.com/>