Integers I

CSE 351 Summer 2020

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http://xkcd.com/257/

Administrivia

- No lecture on Friday 7/3 (campus holiday)
- hw3 due Wednesday 7/1 – 10:30am
- hw4 due Monday 7/6 – 10:30am
  - As a heads up, hw5 released 7/1, also due 7/6
- Lab 1a released
  - Workflow:
    1) Edit `pointer.c`
    2) Run the Makefile (`make`) and check for compiler errors & warnings
    3) Run ptest (`./ptest`) and check for correct behavior
    4) Run rule/syntax checker (`python dlc.py`) and check output
  - Due Monday 7/6 at 11:59pm **recommended to finish by 7/3** (to give time to complete lab 1b).
    - Lab 1b will be released later this week, due 7/10

Lab Reflections

- All subsequent labs (after Lab 0) have a “reflection” portion
  - The Reflection questions can be found on the lab specs and are intended to be done *after* you finish the lab
  - You will type up your responses in a `.txt` file for submission on Gradescope
  - These will be graded “by hand” (read by TAs)

- Intended to check your understand of what you should have learned from the lab

Poll Everywhere For Credit

- Poll everywhere counts for credit starting today!
  - Remember that you get credit for any answer, not just correct answers.
  - Make sure you enter a poll response before I close the poll.

- Makeup quizzes released after every lecture on Canvas.
  - Only submit this if you did not answer in lecture.
  - Must provide explanation for your answer to receive full credit.
  - Due before the next lecture at 10:30am.

Memory, Data, and Addressing

- Representing information as bits and bytes
  - Binary, hexadecimal, fixed-widths
- Organizing and addressing data in memory
  - Memory is a byte-addressable array
  - Machine "word" size = address size = register size
  - Endianness – ordering bytes in memory
- Manipulating data in memory using C
  - Assignment
  - Pointers, pointer arithmetic, and arrays
- **Boolean algebra and bit-level manipulations**

Boolean Algebra

- Developed by George Boole in 19th Century
  - Algebraic representation of logic (True → 1, False → 0)
  - AND: `A&B=1` when both A is 1 and B is 1
  - OR: `A|B=1` when either A is 1 or B is 1
  - XOR: `A^B=1` when either A is 1 or B is 1, but not both
  - NOT: `~A=1` when A is 0 and vice-versa
  - DeMorgan's Law: `~(A|B) = ~A & ~B`
    `~(A&B) = ~A | ~B`

AND

&	0	1
0	**0**	**0**
1	**0**	**1**

OR

|	0	1
0	**0**	**1**
1	**1**	**1**

XOR

^	0	1
0	**0**	**1**
1	**1**	**0**

NOT

~	
0	**1**
1	**0**

General Boolean Algebras

- Operate on bit vectors
  - Operations applied bitwise
  - All of the properties of Boolean algebra apply

```
   01101001      01101001      01101001
 & 01010101    | 01010101    ^ 01010101    ~ 01010101
```

- Examples of useful operations:

$$x \wedge x = 0$$

```
  01010101
^ 01010101
  00000000
```

"sets to 1" "leaves as is"

$x \mid 1 = 1,$ $x \mid 0 = x$

$0 \mid 1 = 1$ $0 \mid 0 = 0$

$1 \mid 1 = 1$ $1 \mid 0 = 1$

```
  01010101
| 11110000
  11110101
```

Bit-Level Operations in C

- `&` (AND), `|` (OR), `^` (XOR), `~` (NOT)
  - View arguments as bit vectors, apply operations bitwise
  - Apply to any "integral" data type
    - `long, int, short, char, unsigned`
- Examples with `char a, b, c;`
  - ```
    a = (char) 0x41;     // 0x41->0b 0100 0001
    b = ~a;              //       0b           ->0x
    ```
  - ```
    a = (char) 0x69;     // 0x69->0b 0110 1001
    b = (char) 0x55;     // 0x55->0b 0101 0101
    c = a & b;           //       0b           ->0x
    ```
  - ```
    a = (char) 0x41;     // 0x41->0b 0100 0001
    b = a;               //       0b 0100 0001
    c = a ^ b;           //       0b           ->0x
    ```
    ```

Contrast: Logic Operations

- Logical operators in C: `&&` (AND), `||` (OR), `!` (NOT)
  - **0** is False, **anything nonzero** is True
  - **Always** return 0 or 1
  - Early termination (a.k.a. short-circuit evaluation) of `&&`, `||`
- Examples (`char` data type)
  - `!0x41 -> 0x00`
  - `!0x00 -> 0x01`
  - `!!0x41 -> 0x01`
  - `0xCC && 0x33 -> 0x01`
  - `0x00 || 0x33 -> 0x01`
  - `p && *p`
    - If `p` is the null pointer (0x0), then `p` is never dereferenced!

Roadmap

C:

```
car *c = malloc(sizeof(car));
c->miles = 100;
c->gals = 17;
float mpg = get_mpg(c);
free(c);
```

Java:

```
Car c = new Car();
c.setMiles(100);
c.setGals(17);
float mpg =
    c.getMPG();
```

Assembly language:

```
get_mpg:
    pushq   %rbp
    movq    %rsp, %rbp
    ...
    popq    %rbp
    ret
```

Machine code:

```
0111010000011000
100011010000010000000010
1000100111000010
110000011111101000011111
```

OS:

Windows 10

OS X Yosemite

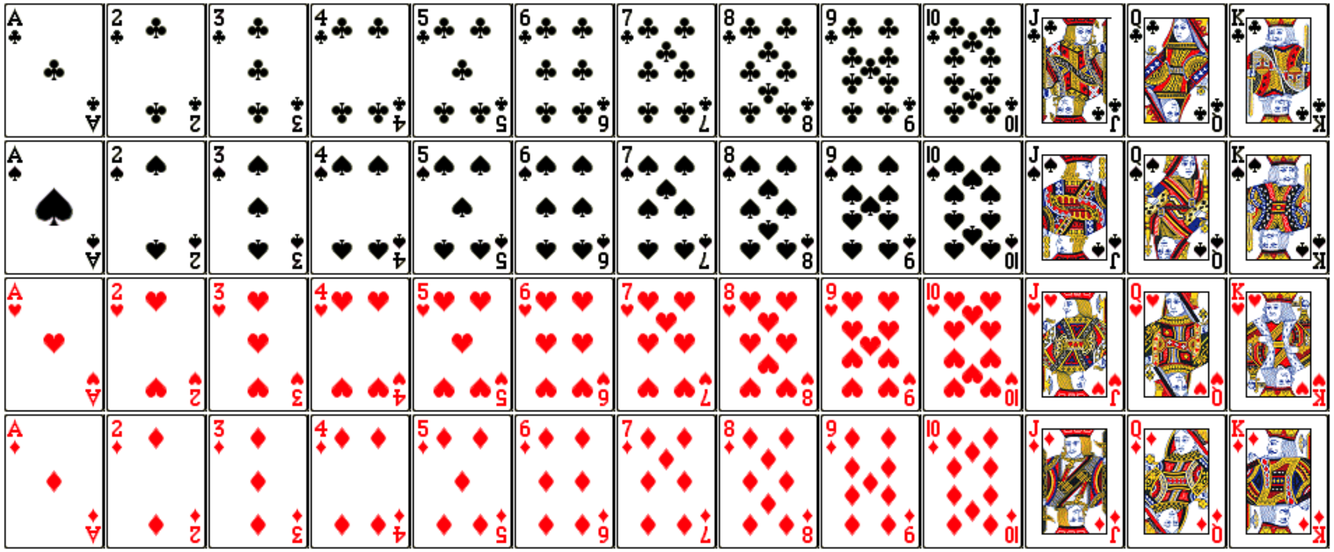
Computer system:



- Memory & data
- **Integers & floats**
- x86 assembly
- Procedures & stacks
- Executables
- Arrays & structs
- Memory & caches
- Processes
- Virtual memory
- Memory allocation
- Java vs. C

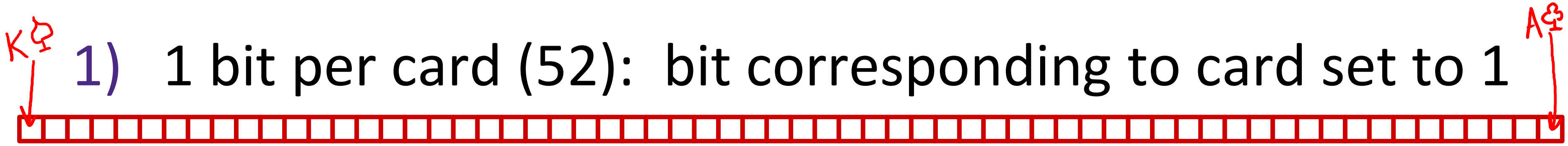
But before we get to integers….

- Encode a standard deck of playing cards
- 52 cards in 4 suits
  - How do we encode suits, face cards?
- What operations do we want to make easy to implement?
  - Which is the higher value card?
  - Are they the same suit?



Two possible representations

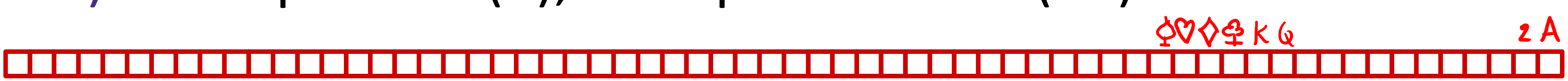
1) 1 bit per card (52): bit corresponding to card set to 1



low-order 52 bits of 64-bit word

- "One-hot" encoding (similar to set notation)
- Drawbacks:
  - Hard to compare values and suits
  - Large number of bits required 52 bits $\xrightarrow{\text{fits in}}$ 7 bytes (56 bits)

2) 1 bit per suit (4), 1 bit per number (13): 2 bits set



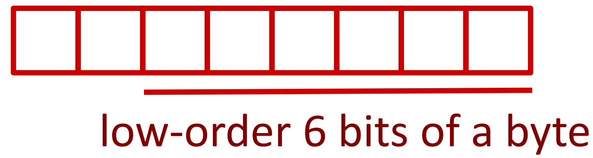
4 suits 13 numbers

17 bits → 3 bytes

- Pair of one-hot encoded values
- Easier to compare suits and values, but still lots of bits used

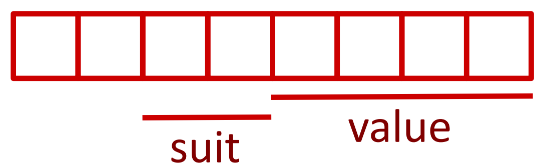
Two better representations

3) Binary encoding of all 52 cards – only 6 bits needed
   - $2^6 = 64 \geq 52$



   - Fits in one byte (smaller than one-hot encodings)
   - How can we make value and suit comparisons easier?

4) Separate binary encodings of suit (2 bits) and value (4 bits)



   - Also fits in one byte, and easy to do comparisons

K	Q	J	. . .	3	2	A
1101	1100	1011	. . .	0011	0010	0001

♣	00
♦	01
♥	10
♠	11

Compare Card Suits

mask: a bit vector designed to achieve a desired behavior when used with a bitwise operator on another bit vector *v*.
Here we turn all *but* the bits of interest in *v* to 0.

```
char hand[5];         // represents a 5-card hand
char card1, card2;    // two cards to compare
card1 = hand[0];
card2 = hand[1];
...
if ( sameSuitP(card1, card2) ) { ... }
```

```
#define SUIT_MASK  0x30

int sameSuitP(char card1, char card2) {
  return (!((card1 & SUIT_MASK) ^ (card2 & SUIT_MASK)));
  //return (card1 & SUIT_MASK) == (card2 & SUIT_MASK);
}
```

returns **int**

equivalent

SUIT_MASK = 0x30 =

0	0	1	1	0	0	0	0

suit (keep)

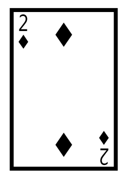
value (discard)

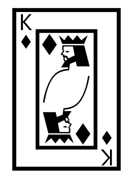
Compare Card **Suits**

mask: a bit vector designed to achieve a desired behavior when used with a bitwise operator on another bit vector *v*.
Here we turn all *but* the bits of interest in *v* to 0.

```
#define SUIT_MASK  0x30

int sameSuitP(char card1, char card2) {
  return (!((card1 & SUIT_MASK) ^ (card2 & SUIT_MASK)));
  //return (card1 & SUIT_MASK) == (card2 & SUIT_MASK);
}
```





0	0	0	1	0	0	1	0

&

0	0	1	1	0	0	0	0

=

0	0	0	1	0	0	0	0

SUIT_MASK

0	0	0	1	1	1	0	1

&

0	0	1	1	0	0	0	0

=

0	0	0	1	0	0	0	0

^

0	0	0	0	0	0	0	0

!

0	0	0	0	0	0	0	1

`!(x^y)` equivalent to `x==y`

Compare Card Values

mask: a bit vector designed to achieve a desired behavior when used with a bitwise operator on another bit vector *v*.

```
char hand[5];          // represents a 5-card hand
char card1, card2;     // two cards to compare
card1 = hand[0];
card2 = hand[1];
...
if ( greaterValue(card1, card2) ) { ... }
```

```
#define VALUE_MASK  0x0F

int greaterValue(char card1, char card2) {
  return ((unsigned int)(card1 & VALUE_MASK) >
          (unsigned int)(card2 & VALUE_MASK));
}
```

VALUE_MASK = 0x0F =

0	0	0	0	1	1	1	1
		suit	suit	value	value	value	value

Compare Card Values

mask: a bit vector designed to achieve a desired behavior when used with a bitwise operator on another bit vector *v*.

```
#define VALUE_MASK  0x0F

int greaterValue(char card1, char card2) {
  return ((unsigned int)(card1 & VALUE_MASK) >
          (unsigned int)(card2 & VALUE_MASK));
}
```

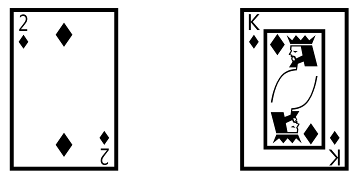
0	0	1	0	0	0	1	0

&

0	0	0	0	1	1	1	1

=

0	0	0	0	0	0	1	0



VALUE_MASK

0	0	1	0	1	1	0	1

&

0	0	0	0	1	1	1	1

=

0	0	0	0	1	1	0	1

$2_{10} > 13_{10}$

0 (false)

Integers

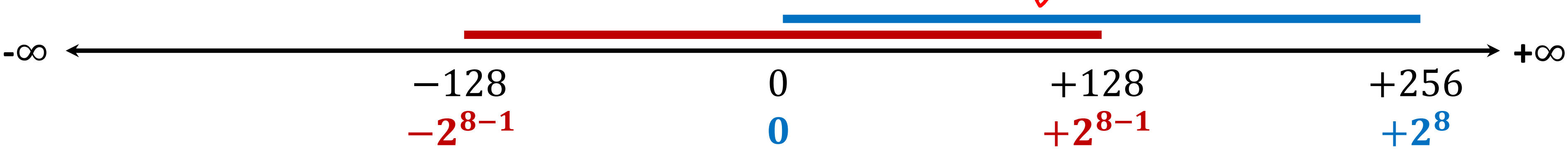
- **Binary representation of integers**
  - **Unsigned and signed**
- Shifting and arithmetic operations – useful for Lab 1a
- In C: Signed, Unsigned and Casting
- Consequences of finite width representations
  - Overflow, sign extension

Encoding Integers

- The hardware (and C) supports two flavors of integers
  - *unsigned* – only the non-negatives
  - *signed* – both negatives and non-negatives
- Cannot represent all integers with w bits
  - Only 2^w distinct bit patterns
  - Unsigned values: $0 \ldots 2^w-1$
  - Signed values: $-2^{w-1} \ldots 2^{w-1}-1$

same widths, just shifted

- **Example:** 8-bit integers (*e.g.* `char`)



Unsigned Integers

- Unsigned values follow the standard base 2 system
  - $b_7b_6b_5b_4b_3b_2b_1b_0 = b_7 2^7 + b_6 2^6 + \cdots + b_1 2^1 + b_0 2^0$
- Add and subtract using the normal "carry" and "borrow" rules, just in binary

$$\begin{array}{r} 63 \\ +\ \ 8 \\ \hline 71 \end{array} \qquad \begin{array}{r} 00111111 \\ +00001000 \\ \hline 01000111 \end{array}$$

- Useful formula: $2^{N-1} + 2^{N-2} + \ldots + 2 + 1 = 2^N - 1$
  - *i.e.* N ones in a row = $2^N - 1$
- How would you make *signed* integers?

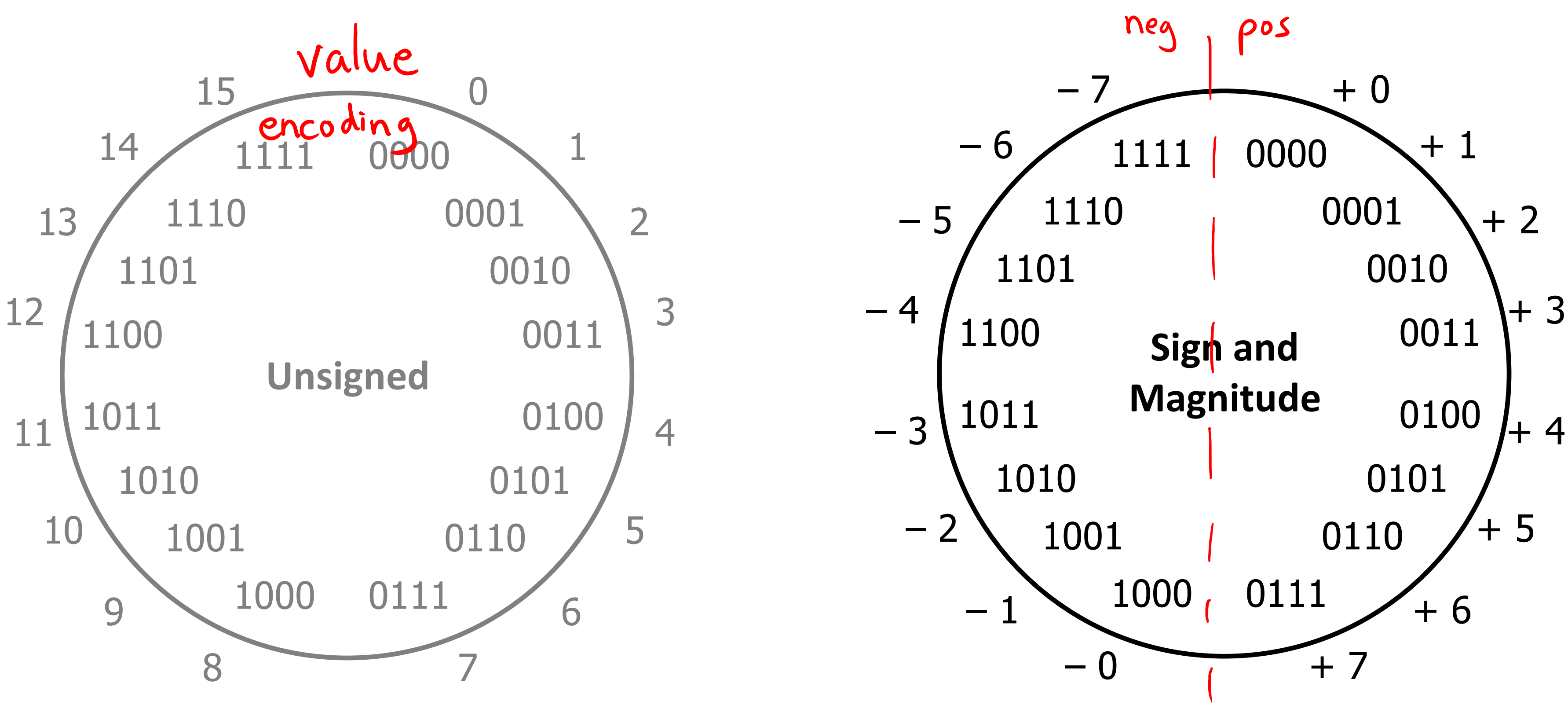
Sign and Magnitude

Most Significant Bit

- Designate the high-order bit (MSB) as the "sign bit"
  - `sign=0`: positive numbers; `sign=1`: negative numbers
- Benefits:
  - Using MSB as sign bit matches positive numbers with unsigned unsigned: 0b0010 = 2^1 = 2 ; sign+mag: 0b0010 = $+2^1$ = 2 ☑
  - All zeros encoding is still = 0
- Examples (8 bits):
  - 0x00 = 00000000_2 is non-negative, because the sign bit is 0
  - 0x7F = 01111111_2 is non-negative ($+127_{10}$)
  - 0x85 = 10000101_2 is negative (-5_{10})
  - 0x80 = 10000000_2 is negative... zero???

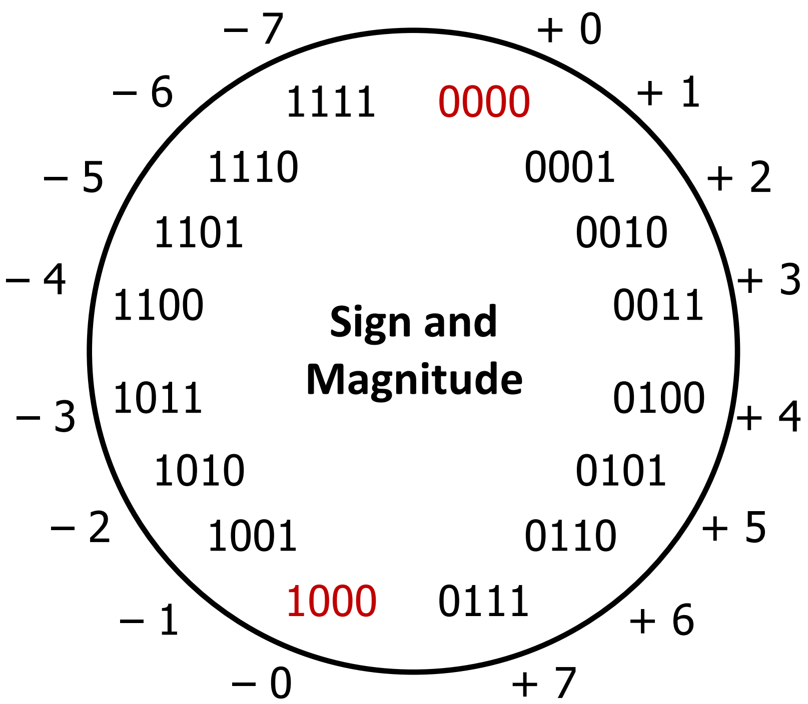
Sign and Magnitude

- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks?



Sign and Magnitude

- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks:
  - Two representations of 0 (bad for checking equality)



Sign and Magnitude

- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks:
  - Two representations of 0 (bad for checking equality)
  - Arithmetic is cumbersome
    - Example: `4-3 != 4+(-3)`

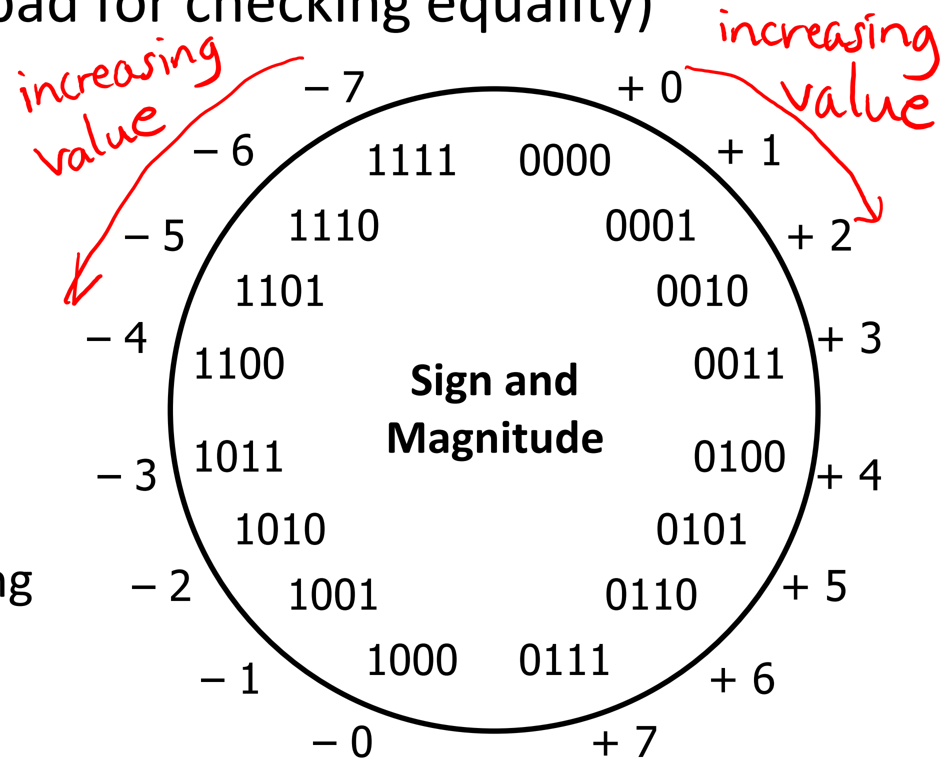
4	0100
– 3	– 0011
1	0001

✓

4	0100
+ -3	+ 1011
-7	1111

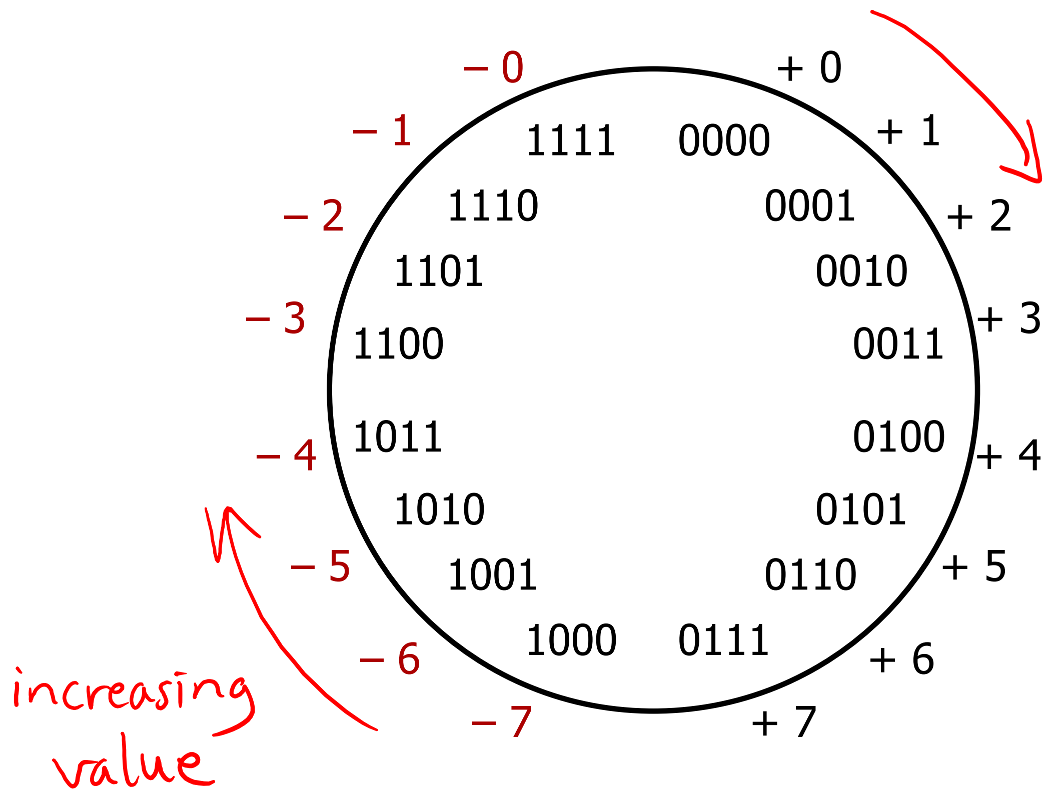
✗

    - Negatives "increment" in wrong direction!



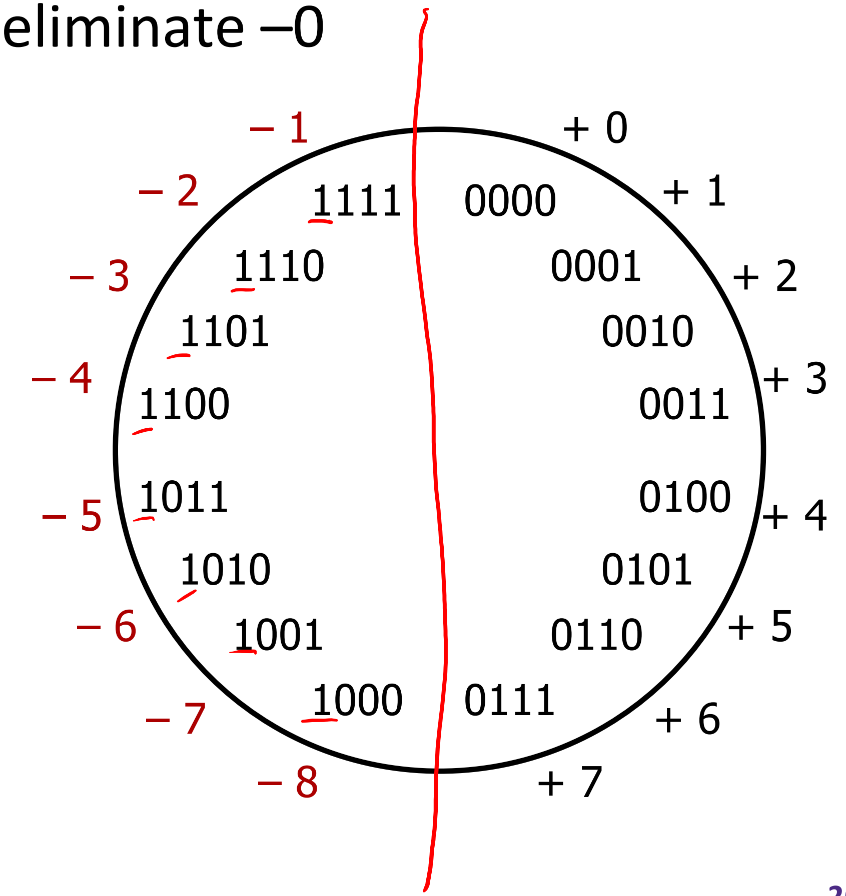
Two's Complement

- Let's fix these problems:
  1) "Flip" negative encodings so incrementing works



Two's Complement

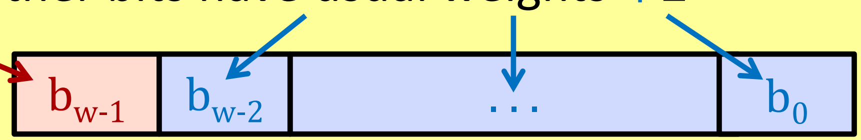
- Let's fix these problems:
  1) "Flip" negative encodings so incrementing works
  2) "Shift" negative numbers to eliminate –0
- MSB *still* indicates sign!
  - This is why we represent one more negative than positive number (-2^{N-1} to 2^{N-1} -1)



Two's Complement Negatives

- Accomplished with one neat mathematical trick!

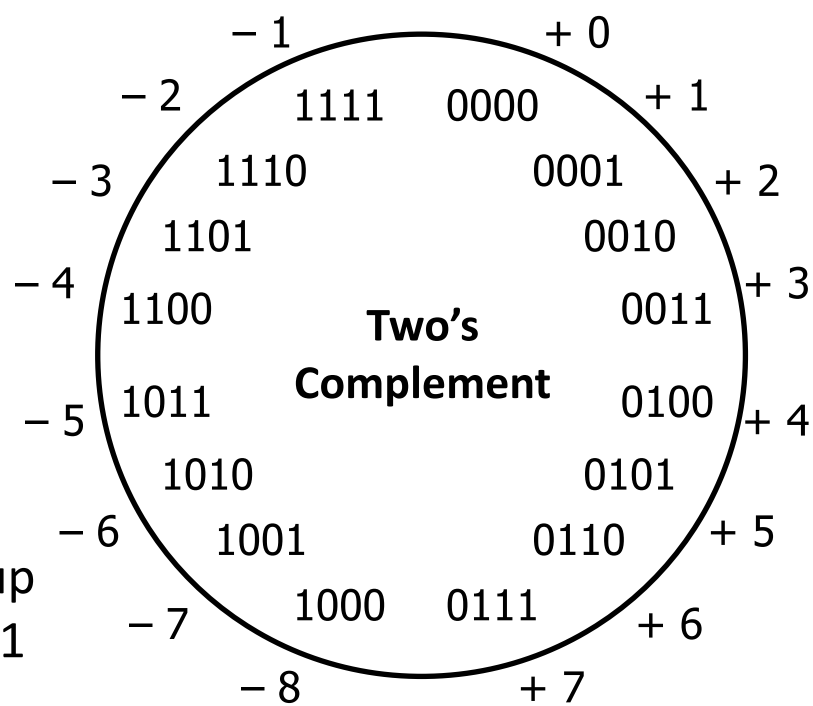
b_{w-1} has weight -2^{w-1}, other bits have usual weights $+2^i$



- 4-bit Examples:
  - 1010_2 unsigned:
    $1*2^3+0*2^2+1*2^1+0*2^0$ = **10**
  - 1010_2 two's complement:
    $-1*2^3+0*2^2+1*2^1+0*2^0$ = **–6**
- -1 represented as:

  $1111_2 = -2^3+(2^3-1)$

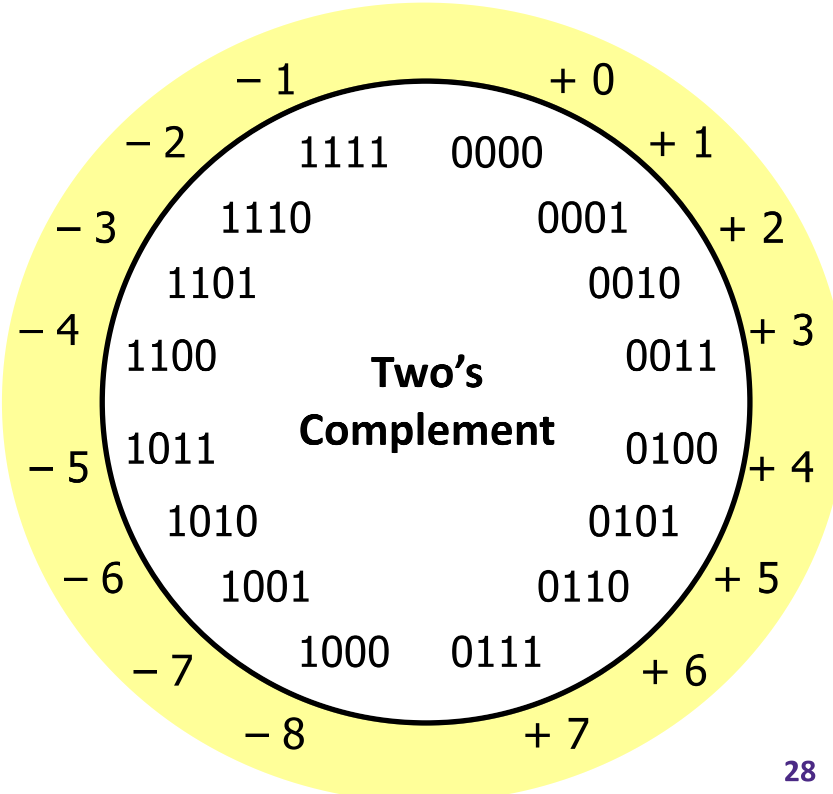
  3 one's in a row
  - MSB makes it super negative, add up all the other bits to get back up to -1



Why Two's Complement is So Great

- Roughly same number of (+) and (–) numbers
- Positive number encodings match unsigned
- Single zero
- All zeros encoding = 0

- Simple negation procedure:
  - Get negative representation of any integer by taking bitwise complement and then adding one!
    `( ~x + 1 == -x )`



Polling Question [Int I - b]

- Take the 4-bit number encoding **`x = 0b1011`**
- Which of the following numbers is NOT a valid interpretation of `x` using any of the number representation schemes discussed today?
  - Unsigned, Sign and Magnitude, Two's Complement
  - Vote at http://pollev.com/pbjones

A. **-4**

B. **-5**

C. **11**

D. **-3**

E. **We're lost...**

Integers

- Binary representation of integers
  - Unsigned and signed
- **Shifting and arithmetic operations** – useful for Lab 1a
- In C: Signed, Unsigned and Casting
- Consequences of finite width representations
  - Overflow, sign extension

Shift Operations

- Left shift (`x<<n`) bit vector `x` by `n` positions
  - Throw away (drop) extra bits on left
  - Fill with `0`s on right
- Right shift (`x>>n`) bit-vector `x` by `n` positions
  - Throw away (drop) extra bits on right
  - Logical shift (for unsigned values)
    - Fill with `0`s on left
  - Arithmetic shift (for signed values)
    - Replicate most significant bit on left
    - Maintains sign of `x`

Shift Operations

- Left shift (`x<<n`)
  - Fill with `0`s on right
- Right shift (`x>>n`)
  - Logical shift (for unsigned values)
    - Fill with 0s on left
  - Arithmetic shift (for signed values)
    - Replicate most significant bit on left

	`x`	`0010 0010`
	`x<<3`	`0001 0000`
logical:	`x>>2`	`0000 1000`
arithmetic:	`x>>2`	`0000 1000`

	`x`	`1010 0010`
	`x<<3`	`0001 0000`
logical:	`x>>2`	`0010 1000`
arithmetic:	`x>>2`	`1110 1000`

- Notes:
  - Shifts by `n<0` or `n≥w` (`w` is bit width of `x`) are *undefined*
  - **In C:** behavior of `>>` is determined by compiler
    - In gcc / C lang, depends on data type of `x` (signed/unsigned)
  - **In Java:** logical shift is `>>>` and arithmetic shift is `>>`

Shifting Arithmetic?

- What are the following computing?
  - `x>>n`
    - `0b 0100 >> 1 = 0b 0010`
    - `0b 0100 >> 2 = 0b 0001`
    - Divide by 2^n
  - `x<<n`
    - `0b 0001 << 1 = 0b 0010`
    - `0b 0001 << 2 = 0b 0100`
    - Multiply by 2^n
- Shifting is faster than general multiply and divide operations

Left Shifting Arithmetic 8-bit Example

- No difference in left shift operation for unsigned and signed numbers (just manipulates bits)
  - Difference comes during interpretation: `x*2`^{n}?

			Signed	Unsigned
`x = 25;`	`00011001`	=	25	25
`L1=x<<2;`	`0001100100`	=	100	100
`L2=x<<3;`	`00011001000`	=	-56	200
`L3=x<<4;`	`000110010000`	=	-112	144

signed overflow

unsigned overflow

Right Shifting Arithmetic 8-bit Examples

- **Reminder:** C operator >> does *logical* shift on unsigned values and *arithmetic* shift on signed values
  - Logical Shift: `x/2`^{n}?

```
xu = 240u;      11110000         = 240
R1u=xu>>3;   00011110000         =  30
R2u=xu>>5; 0000011110000         =   7
```

rounding (down)

Right Shifting Arithmetic 8-bit Examples

- **Reminder:** C operator >> does *logical* shift on unsigned values and *arithmetic* shift on signed values
  - Arithmetic Shift: `x/2`^{n}?

```
xs = -16;   11110000        = -16
R1s=xu>>3;  11111110000     =  -2
R2s=xu>>5;  1111111110000   =  -1
```

rounding (down)

Summary

- Bit-level operators allow for fine-grained manipulations of data
  - Bitwise AND (`&`), OR (`|`), and NOT (`~`) different than logical AND (`&&`), OR (`||`), and NOT (`!`)
  - Especially useful with bit masks
- Choice of *encoding scheme* is important
  - Tradeoffs based on size requirements and desired operations
- Integers represented using unsigned and two's complement representations
  - Limited by fixed bit width
  - We'll examine arithmetic operations next lecture
- Shifting is a useful bitwise operator
  - Right shifting can be arithmetic (sign) or logical (0)
  - Can be used in multiplication with constant or bit masking