Integers I

CSE 351 Summer 2020

Instructor:
Porter Jones

Teaching Assistants:
Amy Xu
Callum Walker
Sam Wolfson
Tim Mandzyuk

http://xkcd.com/257/

Administrivia

- No lecture on Friday 7/3 (campus holiday)
- hw3 due Wednesday 7/1 – 10:30am
- hw4 due Monday 7/6 – 10:30am
  - As a heads up, hw5 released 7/1, also due 7/6
- Lab 1a released
  - Workflow:
    1) Edit `pointer.c`
    2) Run the Makefile (`make`) and check for compiler errors & warnings
    3) Run ptest (`./ptest`) and check for correct behavior
    4) Run rule/syntax checker (`python dlc.py`) and check output
  - Due Monday 7/6 at 11:59pm **recommended to finish by 7/3** (to give time to complete lab 1b).
    - Lab 1b will be released later this week, due 7/10

Lab Reflections

- All subsequent labs (after Lab 0) have a "reflection" portion
  - The Reflection questions can be found on the lab specs and are intended to be done *after* you finish the lab
  - You will type up your responses in a `.txt` file for submission on Gradescope
  - These will be graded "by hand" (read by TAs)

- Intended to check your understand of what you should have learned from the lab

Poll Everywhere For Credit

- Poll everywhere counts for credit starting today!
  - Remember that you get credit for any answer, not just correct answers.
  - Make sure you enter a poll response before I close the poll.
- Makeup quizzes released after every lecture on Canvas.
  - Only submit this if you did not answer in lecture.
  - Must provide explanation for your answer to receive full credit.
  - Due before the next lecture at 10:30am.

Memory, Data, and Addressing

- Representing information as bits and bytes
  - Binary, hexadecimal, fixed-widths
- Organizing and addressing data in memory
  - Memory is a byte-addressable array
  - Machine "word" size = address size = register size
  - Endianness – ordering bytes in memory
- Manipulating data in memory using C
  - Assignment
  - Pointers, pointer arithmetic, and arrays
- **Boolean algebra and bit-level manipulations**

Boolean Algebra

Two values like binary

- Developed by George Boole in 19th Century
  - Algebraic representation of logic (True → 1, False → 0)
  - AND: `A&B=1` when both A is 1 and B is 1
  - OR: `A|B=1` when either A is 1 or B is 1
  - XOR: `A^B=1` when either A is 1 or B is 1, but not both
  - NOT: `~A=1` when A is 0 and vice-versa
  - DeMorgan's Law:

```
~(A|B) = ~A & ~B
~(A&B) = ~A | ~B
```

AND

&	0	1
0	**0**	**0**
1	**0**	**1**

OR

|	0	1
0	**0**	**1**
1	**1**	**1**

XOR

^	0	1
0	**0**	**1**
1	**1**	**0**

NOT

~	
0	**1**
1	**0**

General Boolean Algebras

- Operate on bit vectors
  - Operations applied bitwise
  - All of the properties of Boolean algebra apply

```
  01101001      01101001      01101001
& 01010101    | 01010101    ^ 01010101    ~ 01010101
  01000001      01111101      00111100      10101010
```

- Examples of useful operations:

$x \,\hat{}\, x = 0$

"sets to 1" "leaves as is"

$x \mid 1 = 1,$ $x \mid 0 = x$

$0 \mid 1 = 1$ $0 \mid 0 = 0$

$1 \mid 1 = 1$ $1 \mid 0 = 1$

```
  01010101
^ 01010101
  00000000
```

```
  01010101
| 11110000
  11110101
```

input

bit mask

results

Bit-Level Operations in C

- `&` (AND), `|` (OR), `^` (XOR), `~` (NOT)
  - View arguments as bit vectors, apply operations bitwise
  - Apply to any "integral" data type
    - `long`, `int`, `short`, `char`, `unsigned`
- Examples with `char a, b, c;`
  - ```
    a = (char) 0x41;    // 0x41->0b 0100 0001
    b = ~a;             //       0b 1011 1110->0xBE
    ```
  - ```
    a = (char) 0x69;    // 0x69->0b 0110 1001
    b = (char) 0x55;    // 0x55->0b 0101 0101
    c = a & b;          //       0b 0100 0001->0x41
    ```
  - ```
    a = (char) 0x41;    // 0x41->0b 0100 0001
    b = a;              //       0b 0100 0001
    c = a ^ b;          //       0b 0000 0000->0x00
    ```
    ```

Contrast: Logic Operations

- Logical operators in C: `&&` (AND), `||` (OR), `!` (NOT)
  - **0** is False, **anything nonzero** is True
  - **Always** return 0 or 1
  - Early termination (a.k.a. short-circuit evaluation) of `&&`, `||`
- Examples (`char` data type)
  - `!0x41 -> 0x00` (True → False)
  - `!0x00 -> 0x01` (False → True)
  - `!!0x41 -> 0x01` (True → True)
  - `p && *p` (① → ②)
    - If `p` is the null pointer (0x0), then `p` is never dereferenced!
  - `0xCC && 0x33 -> 0x01` (True → True → True) logical!
  - `0x00 || 0x33 -> 0x01` (False → True → True)

0xCC & 0x33
0b1100 1100 & 0b 0011 0011 → 0x00
bitwise!

if ① determines result of logical operator then ② doesn't need to be evaluated

Roadmap

C:

```
car *c = malloc(sizeof(car));
c->miles = 100;
c->gals = 17;
float mpg = get_mpg(c);
free(c);
```

Java:

```
Car c = new Car();
c.setMiles(100);
c.setGals(17);
float mpg =
    c.getMPG();
```

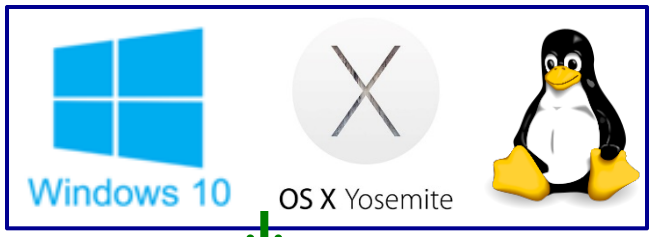
Assembly language:

```
get_mpg:
    pushq   %rbp
    movq    %rsp, %rbp
    ...
    popq    %rbp
    ret
```

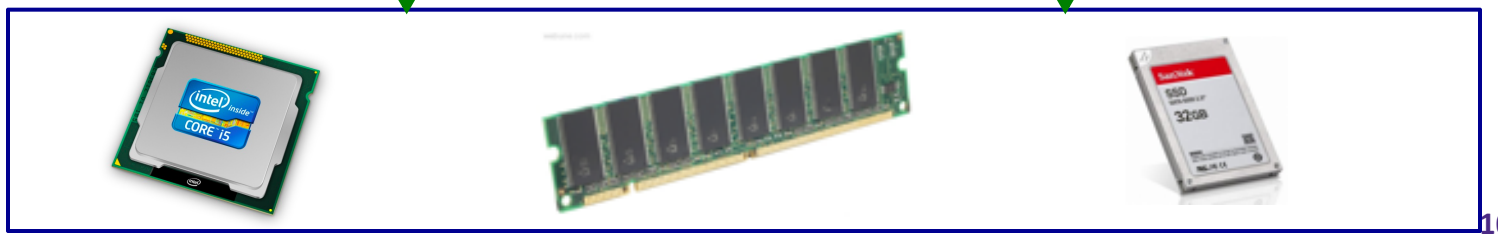
Machine code:

```
0111010000011000
100011010000010000000010
1000100111000010
110000011111101000011111
```

OS:



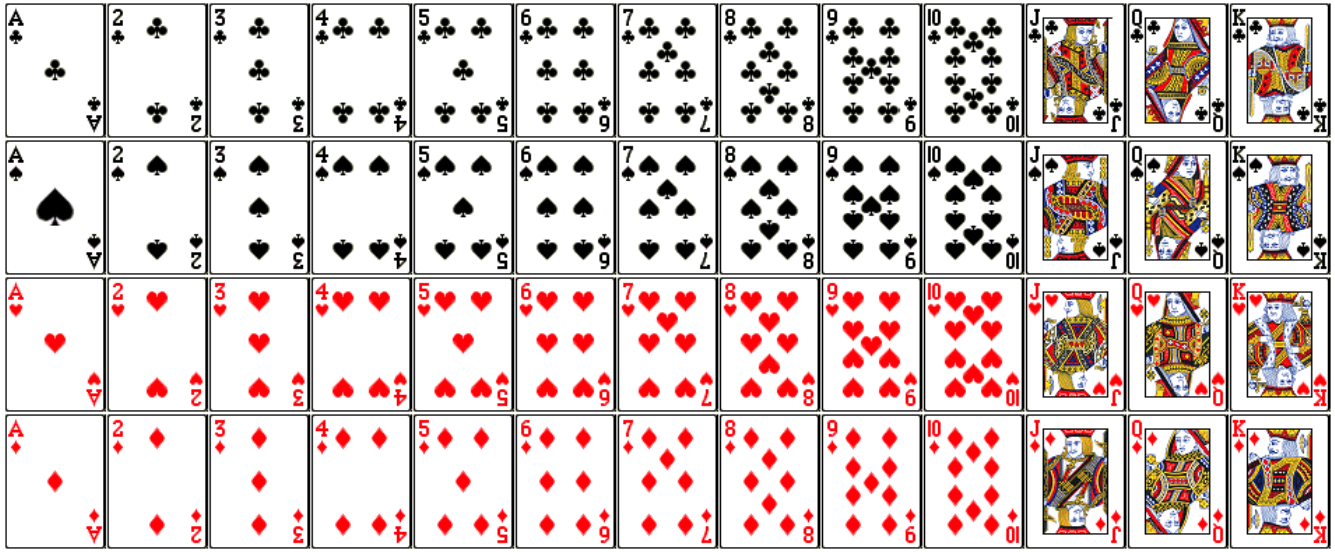
Computer system:



Memory & data
Integers & floats
x86 assembly
Procedures & stacks
Executables
Arrays & structs
Memory & caches
Processes
Virtual memory
Memory allocation
Java vs. C

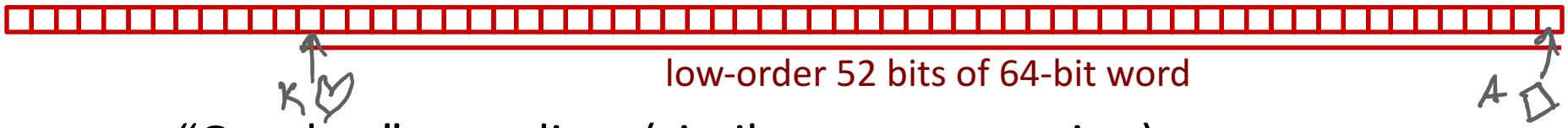
But before we get to integers….

- Encode a standard deck of playing cards
- 52 cards in 4 suits
  - How do we encode suits, face cards?
- What operations do we want to make easy to implement?
  - Which is the higher value card?
  - Are they the same suit?



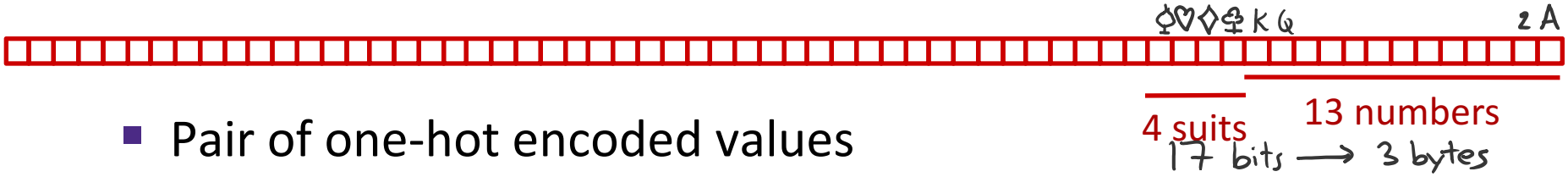
Two possible representations

1) 1 bit per card (52): bit corresponding to card set to 1



- "One-hot" encoding (similar to set notation)
- Drawbacks:
  - Hard to compare values and suits
  - Large number of bits required 52 bits $\xrightarrow{\text{fits in}}$ 7 bytes (56 bits)

2) 1 bit per suit (4), 1 bit per number (13): 2 bits set



- Pair of one-hot encoded values
- Easier to compare suits and values, but still lots of bits used

Two better representations

3) Binary encoding of all 52 cards – only 6 bits needed

- $2^6 = 64 \geq 52$

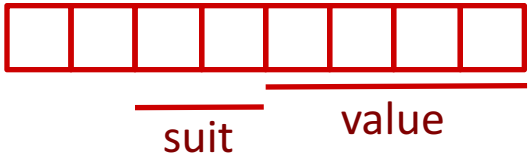
  ~~$2^5 = 32 < 52$~~

- Fits in one byte (smaller than one-hot encodings)
- How can we make value and suit comparisons easier?

need 4 $2^2 = 4$ need 13

4) Separate binary encodings of suit (2 bits) and value (4 bits)

$2^4 = 16$



- Also fits in one byte, and easy to do comparisons

K	Q	J	. . .	3	2	A
1101	1100	1011	. . .	0011	0010	0001

♣	00
♦	01
♥	10
♠	11

Compare Card Suits

mask: a bit vector designed to achieve a desired behavior when used with a bitwise operator on another bit vector *v*.
Here we turn all *but* the bits of interest in *v* to 0.

```
char hand[5];          // represents a 5-card hand
char card1, card2;     // two cards to compare
card1 = hand[0];
card2 = hand[1];
...
if ( sameSuitP(card1, card2) ) { ... }
```

x & 1 = x "leave as is"
x & 0 = 0 "set to 0"

```
#define SUIT_MASK  0x30

int sameSuitP(char card1, char card2) {
  return (!((card1 & SUIT_MASK) ^ (card2 & SUIT_MASK)));
  //return (card1 & SUIT_MASK) == (card2 & SUIT_MASK);
}
```

extract only the suit bits

returns **int**

equivalent

SUIT_MASK = 0x30 =

0	0	1	1	0	0	0	0

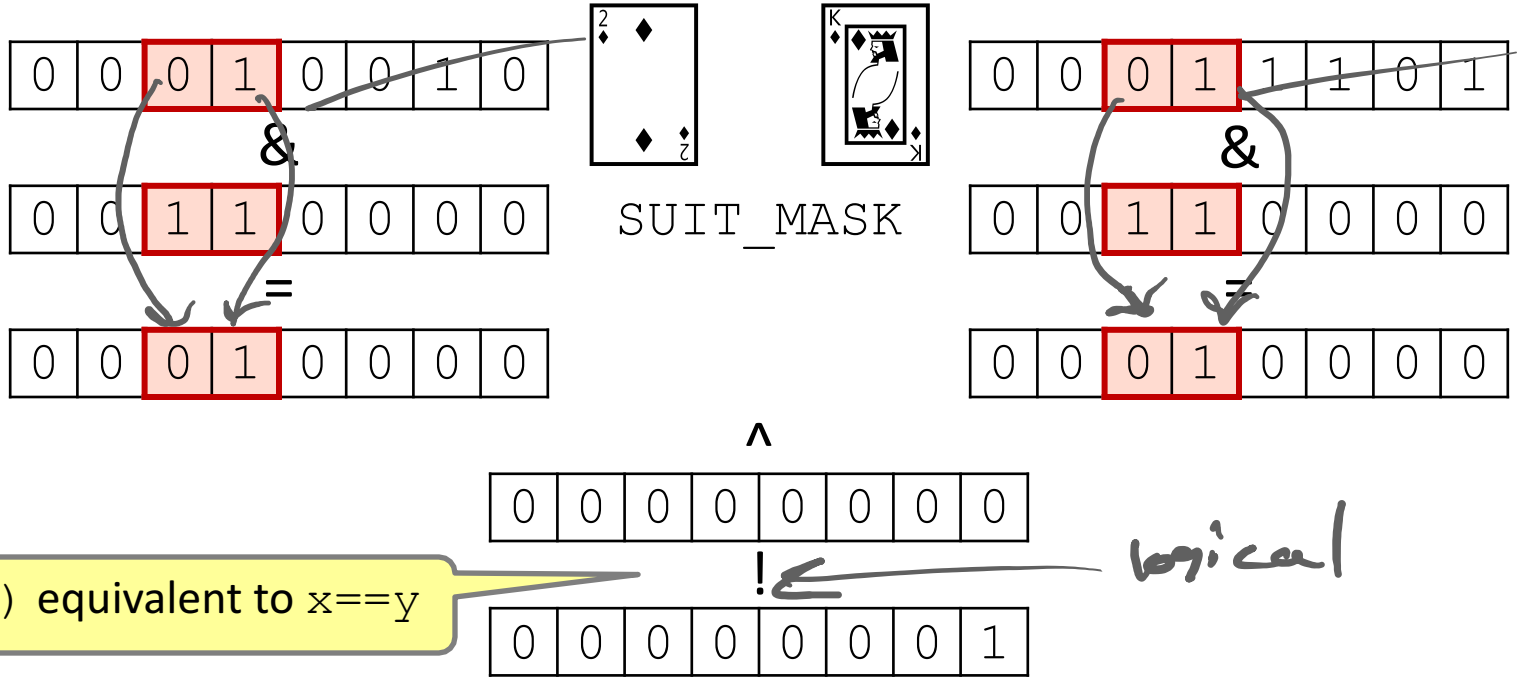
suit (keep) value (discard)

Compare Card Suits

mask: a bit vector designed to achieve a desired behavior when used with a bitwise operator on another bit vector *v*.
Here we turn all *but* the bits of interest in *v* to 0.

```
#define SUIT_MASK  0x30

int sameSuitP(char card1, char card2) {
  return (!((card1 & SUIT_MASK) ^ (card2 & SUIT_MASK)));
  //return (card1 & SUIT_MASK) == (card2 & SUIT_MASK);
}
```



! (x^y) equivalent to x==y

Compare Card Values

mask: a bit vector designed to achieve a desired behavior when used with a bitwise operator on another bit vector *v*.

```
char hand[5];          // represents a 5-card hand
char card1, card2;     // two cards to compare
card1 = hand[0];
card2 = hand[1];
...
if ( greaterValue(card1, card2) ) { ... }
```

```
#define VALUE_MASK  0x0F

int greaterValue(char card1, char card2) {
  return ((unsigned int)(card1 & VALUE_MASK) >
          (unsigned int)(card2 & VALUE_MASK));
}
```

VALUE_MASK = 0x0F =

0	0	0	0	1	1	1	1

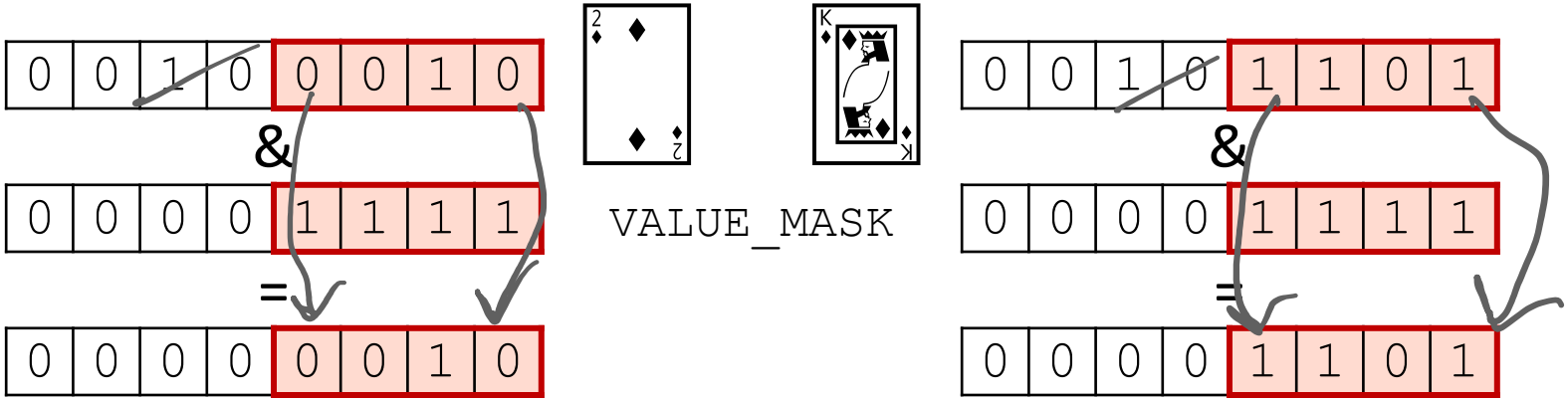
suit (discard) value (keep)

Compare Card Values

mask: a bit vector designed to achieve a desired behavior when used with a bitwise operator on another bit vector *v*.

```
#define VALUE_MASK  0x0F

int greaterValue(char card1, char card2) {
  return ((unsigned int)(card1 & VALUE_MASK) >
          (unsigned int)(card2 & VALUE_MASK));
}
```



$2_{10} > 13_{10}$

0 (false)

only works based on our values representation

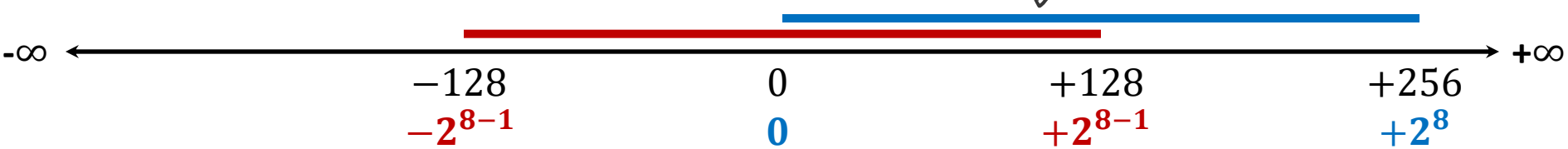
Integers

- **Binary representation of integers**
  - **Unsigned and signed**
- Shifting and arithmetic operations – useful for Lab 1a
- In C: Signed, Unsigned and Casting
- Consequences of finite width representations
  - Overflow, sign extension

Encoding Integers

- The hardware (and C) supports two flavors of integers
  - *unsigned* – only the non-negatives
  - *signed* – both negatives and non-negatives
- Cannot represent all integers with w bits
  - Only 2^w distinct bit patterns
  - Unsigned values: $0 \ldots 2^w - 1$
  - Signed values: $-2^{w-1} \ldots 2^{w-1} - 1$
- **Example:** 8-bit integers (*e.g.* `char`)

same widths, just shifted



Unsigned Integers

- Unsigned values follow the standard base 2 system
  - $b_7b_6b_5b_4b_3b_2b_1b_0 = b_7 2^7 + b_6 2^6 + \cdots + b_1 2^1 + b_0 2^0$
- Add and subtract using the normal "carry" and "borrow" rules, just in binary

```
  63        1 1 1
+  8        00111111
----       +00001000
  71       --------
           01000111
```

x, 6 1's in a row

$x + 1 = 0b100\,0000$

$= 2^6$

$\boxed{x = 2^6 - 1}$

- Useful formula: $2^{N-1} + 2^{N-2} + \ldots + 2 + 1 = 2^N - 1$
  - *i.e.* N ones in a row = $2^N - 1$
- How would you make *signed* integers?

Sign and Magnitude

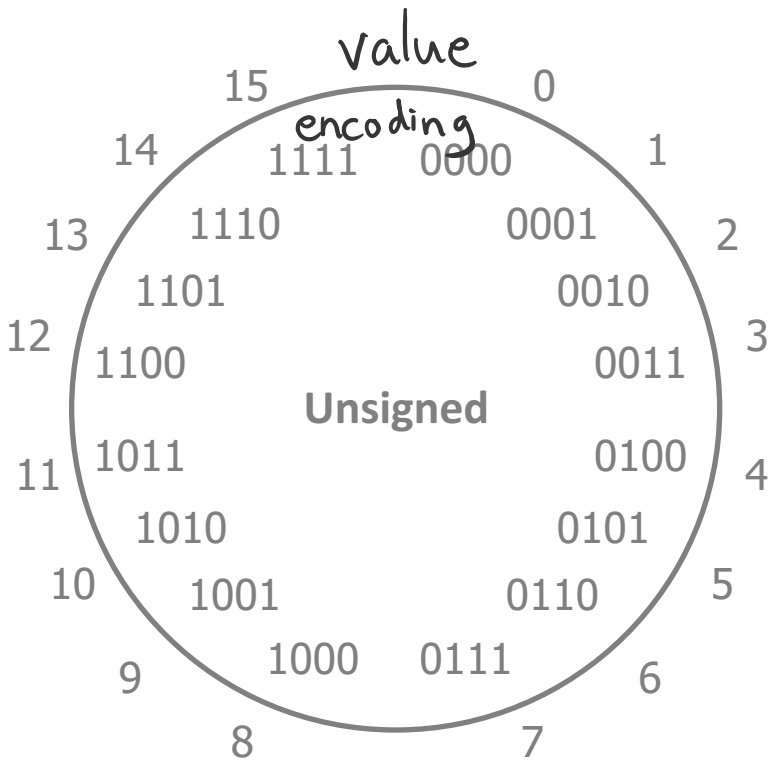
Most Significant Bit

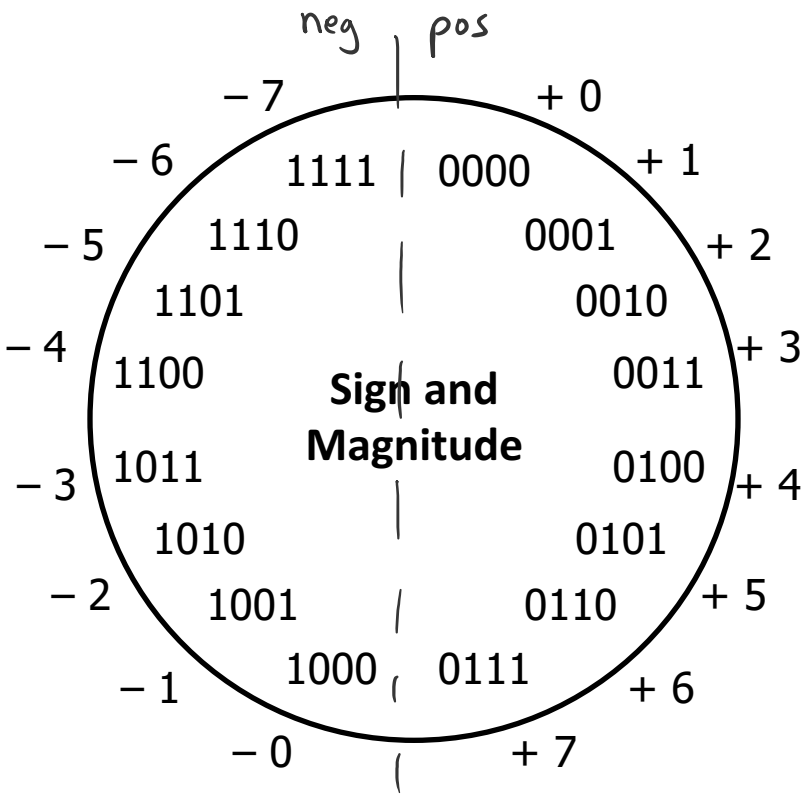
- Designate the high-order bit (MSB) as the "sign bit"
  - `sign=0`: positive numbers; `sign=1`: negative numbers
- Benefits:
  - Using MSB as sign bit matches positive numbers with unsigned unsigned: $0b\,0010 = 2^1 = 2$; sign + mag: $0b\,0010 = +2^1 = 2$ ☑
  - All zeros encoding is still = 0
- <u>Examples</u> (8 bits):
  - 0x00 = 00000000_2 is non-negative, because the sign bit is 0
  - 0x7F = 01111111_2 is non-negative ($+127_{10}$)
  - 0x85 = 10000101_2 is negative (-5_{10})
  - 0x80 = 10000000_2 is negative… zero???

sign

Sign and Magnitude

- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks?

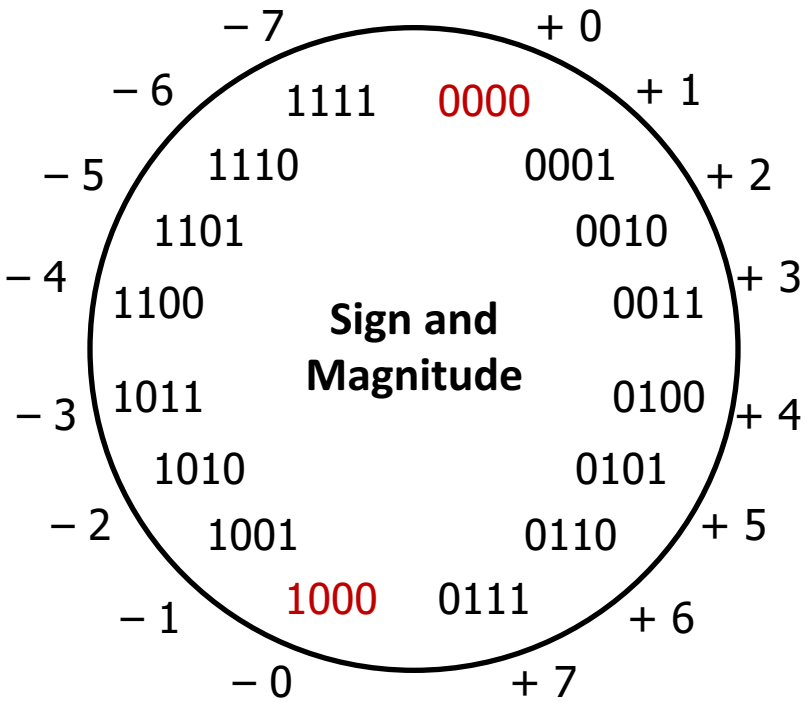




Sign and Magnitude

- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks:
  - Two representations of 0 (bad for checking equality)

Can't just compare bits



Sign and Magnitude

- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks:
  - Two representations of 0 (bad for checking equality)
  - Arithmetic is cumbersome
    - Example: `4-3 != 4+(-3)`

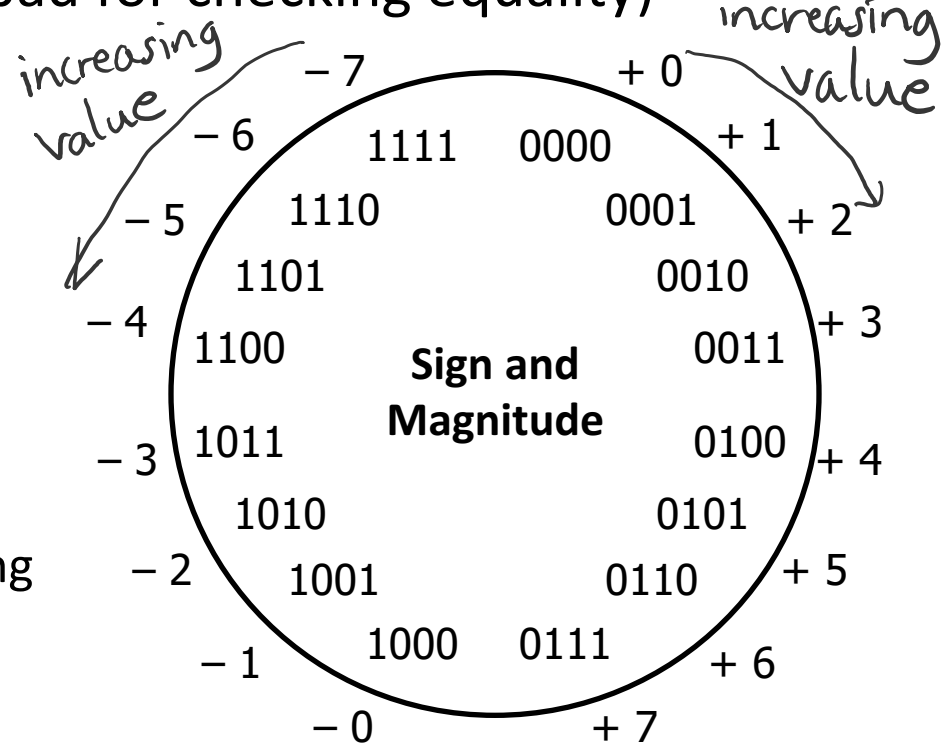
4	0100
− 3	− 0011
1	0001

✓

4	0100
+ −3	+ 1011
−7	1111

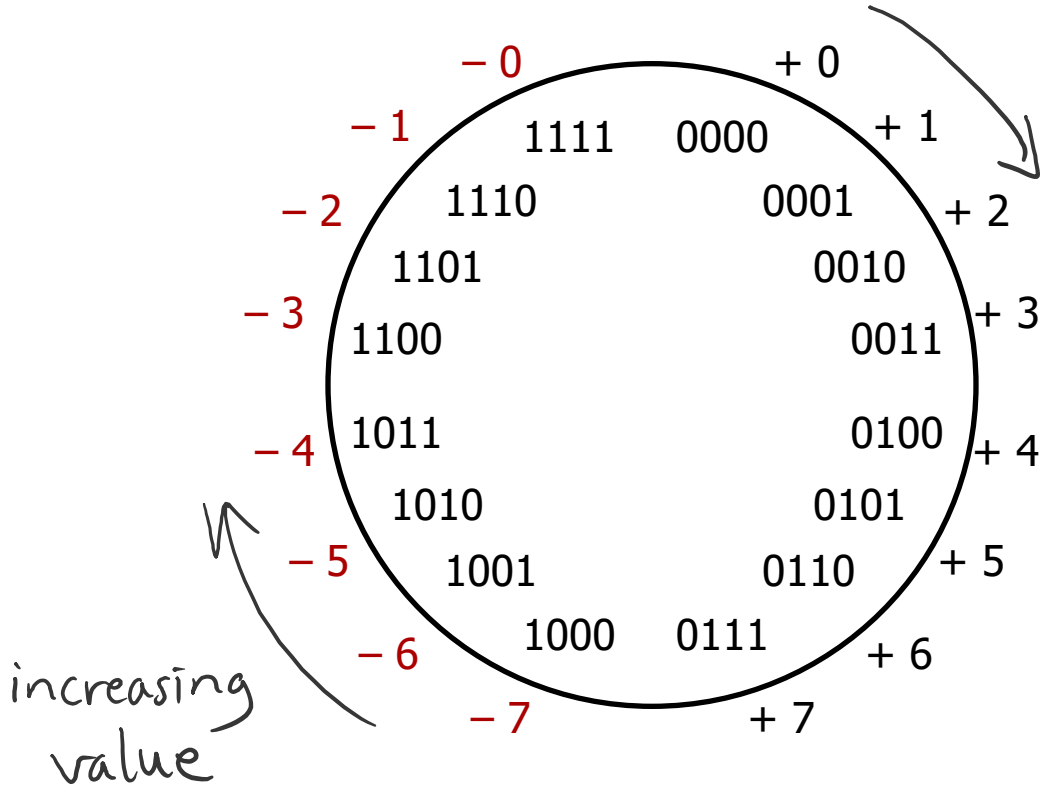
✗

    - Negatives "increment" in wrong direction!



Two's Complement

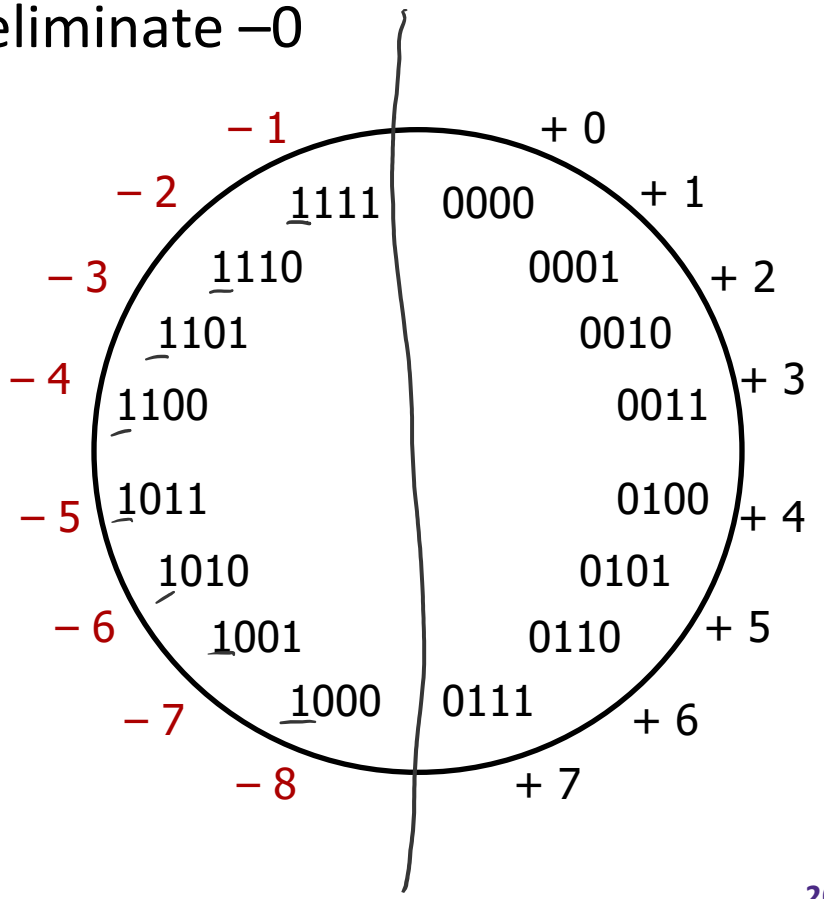
- Let's fix these problems:
  1) "Flip" negative encodings so incrementing works



Two's Complement

- Let's fix these problems:
  1) "Flip" negative encodings so incrementing works
  2) "Shift" negative numbers to eliminate –0

- MSB *still* indicates sign!
  - This is why we represent one more negative than positive number (-2^{N-1} to $2^{N-1}-1$)



Two's Complement Negatives

- Accomplished with one neat mathematical trick!

b_{w-1} has weight -2^{w-1}, other bits have usual weights $+2^i$

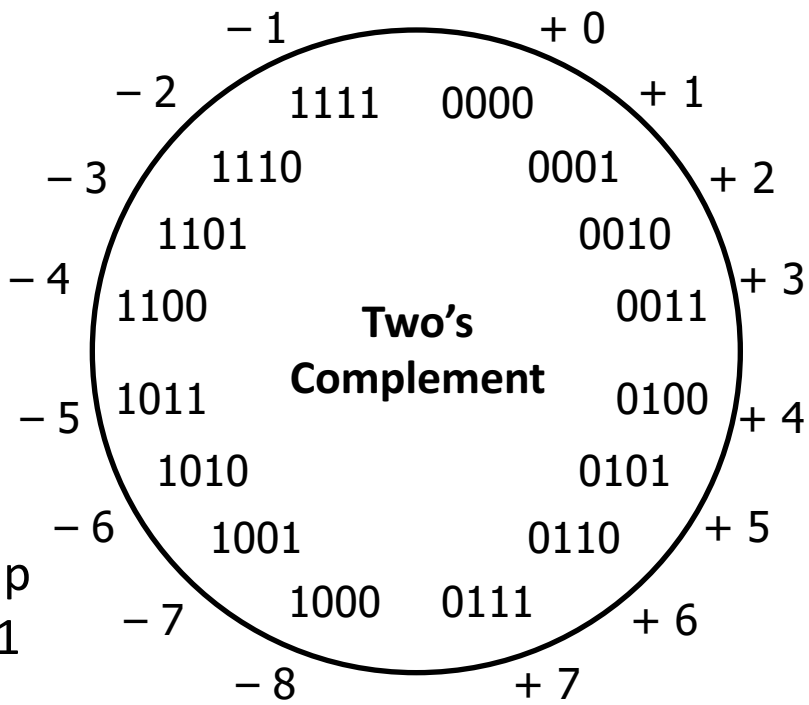
- 4-bit Examples:
  - 1010_2 unsigned:

    pos weight → $1*2^3+0*2^2+1*2^1+0*2^0$ = **10**
  - 1010_2 two's complement:

    neg weight → $-1*2^3+0*2^2+1*2^1+0*2^0$ = **–6**
- -1 represented as:

  $1111_2 = -2^3+(2^3-1)$

  3 one's in a row
  - MSB makes it super negative, add up all the other bits to get back up to -1



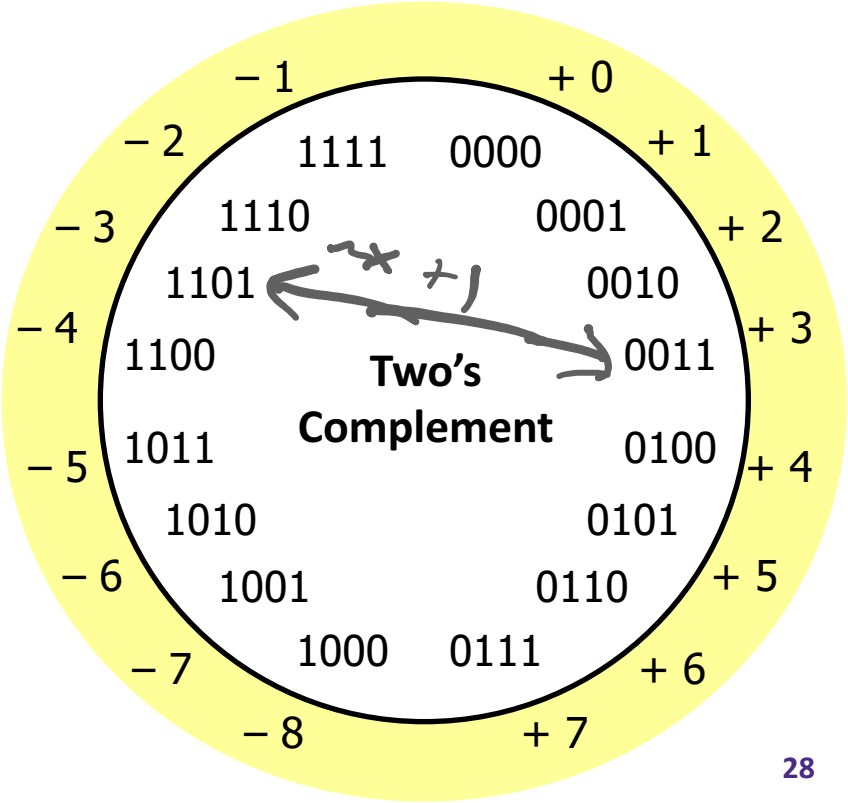
Why Two's Complement is So Great

- Roughly same number of (+) and (–) numbers
- Positive number encodings match unsigned
- Single zero
- All zeros encoding = 0

~ 0b1101 → 0b0010 + 0b0001 → 0b0011

- Simple negation procedure:
  - Get negative representation of any integer by taking bitwise complement and then adding one!

    `( ~x + 1 == -x )`



Polling Question [Int I - b]

- Take the 4-bit number encoding **`x = 0b1011`**

  $2^3\ 2^2\ 2^1\ 2^0$

- Which of the following numbers is NOT a valid interpretation of `x` using any of the number representation schemes discussed today?
  - Unsigned, Sign and Magnitude, Two's Complement
  - Vote at http://pollev.com/pbjones

A. -4

B. -5

C. 11

D. -3

E. We're lost…

unsigned: 8 + 2 + 1 = 11

sign + mag: neg 0b011 = −3

2's comp: −8 + 2 + 1 = 5

Integers

- Binary representation of integers
  - Unsigned and signed
- **Shifting and arithmetic operations** – useful for Lab 1a
- In C: Signed, Unsigned and Casting
- Consequences of finite width representations
  - Overflow, sign extension

Shift Operations

- Left shift (`x<<n`) bit vector `x` by `n` positions
  - Throw away (drop) extra bits on left
  - Fill with `0`s on right
- Right shift (`x>>n`) bit-vector `x` by `n` positions
  - Throw away (drop) extra bits on right
  - Logical shift (for unsigned values)
    - Fill with `0`s on left
  - Arithmetic shift (for signed values)
    - Replicate most significant bit on left
    - Maintains sign of `x`

Shift Operations

8 bit width

	x	0010 0010
	x<<3	0001 0000
logical:	x>>2	0000 1000
arithmetic:	x>>2	0000 1000

	x	1010 0010
	x<<3	0001 0000
logical:	x>>2	0010 1000
arithmetic:	x>>2	1110 1000

- Left shift (`x<<n`)
  - Fill with `0`s on right
- Right shift (`x>>n`)
  - Logical shift (for unsigned values)
    - Fill with 0s on left
  - Arithmetic shift (for signed values)
    - Replicate most significant bit on left
- Notes:
  - Shifts by `n<0` or `n≥w` (`w` is bit width of `x`) are *undefined*
  - **In C:** behavior of `>>` is determined by compiler
    - In gcc / C lang, depends on data type of `x` (signed/unsigned) arithmetic/logical
  - **In Java:** logical shift is `>>>` and arithmetic shift is `>>`

Shifting Arithmetic?

- What are the following computing?
  - `x>>n` 4 / 2^1 = 2
    - `0b 0100 >> 1 = 0b 0010`
    - `0b 0100 >> 2 = 0b 0001`
      4 / 2^2 = 1
    - Divide by 2^n
  - `x<<n` 1 * 2^1 = 2
    - `0b 0001 << 1 = 0b 0010`
    - `0b 0001 << 2 = 0b 0100`
      1 * 2^2 = 4
    - Multiply by 2^n
- Shifting is faster than general multiply and divide operations

Left Shifting Arithmetic 8-bit Example

- No difference in left shift operation for unsigned and signed numbers (just manipulates bits)
  - Difference comes during interpretation: $x*2^n$?

			Signed	Unsigned
`x = 25;`	00011001	=	25	25
`L1=x<<2;`	~~00~~011001**00**	=	100	100
`L2=x<<3;`	~~000~~11001**000**	=	-56	200
`L3=x<<4;`	~~0001~~1001**0000**	=	-112	144

signed overflow ← gone past "bounds"

unsigned overflow

Right Shifting Arithmetic 8-bit Examples

- **Reminder:** C operator >> does *logical* shift on unsigned values and *arithmetic* shift on signed values
  - Logical Shift: `x/2`^{n}?

`xu = 240u;`	11110000	=	240 /2 =30
`R1u=xu>>3;` /2^3	00011110 ~~000~~	=	30
`R2u=xu>>5;` /2^5	00000111 ~~10000~~	=	7

rounding (down)

Right Shifting Arithmetic 8-bit Examples

- **Reminder:** C operator >> does *logical* shift on unsigned values and *arithmetic* shift on signed values
  - Arithmetic Shift: `x/2`^{n}?

```
xs = -16;      11110000          = -16
R1s=xu>>3;  11111110000          =  -2
R2s=xu>>5;  1111111110000        =  -1
```

rounding (down)

Summary

- Bit-level operators allow for fine-grained manipulations of data
  - Bitwise AND (&), OR (|), and NOT (~) different than logical AND (&&), OR (||), and NOT (!)
  - Especially useful with bit masks
- Choice of *encoding scheme* is important
  - Tradeoffs based on size requirements and desired operations
- Integers represented using unsigned and two's complement representations
  - Limited by fixed bit width
  - We'll examine arithmetic operations next lecture
- Shifting is a useful bitwise operator
  - Right shifting can be arithmetic (sign) or logical (0)
  - Can be used in multiplication with constant or bit masking