Integers II

CSE 351 Autumn 2020

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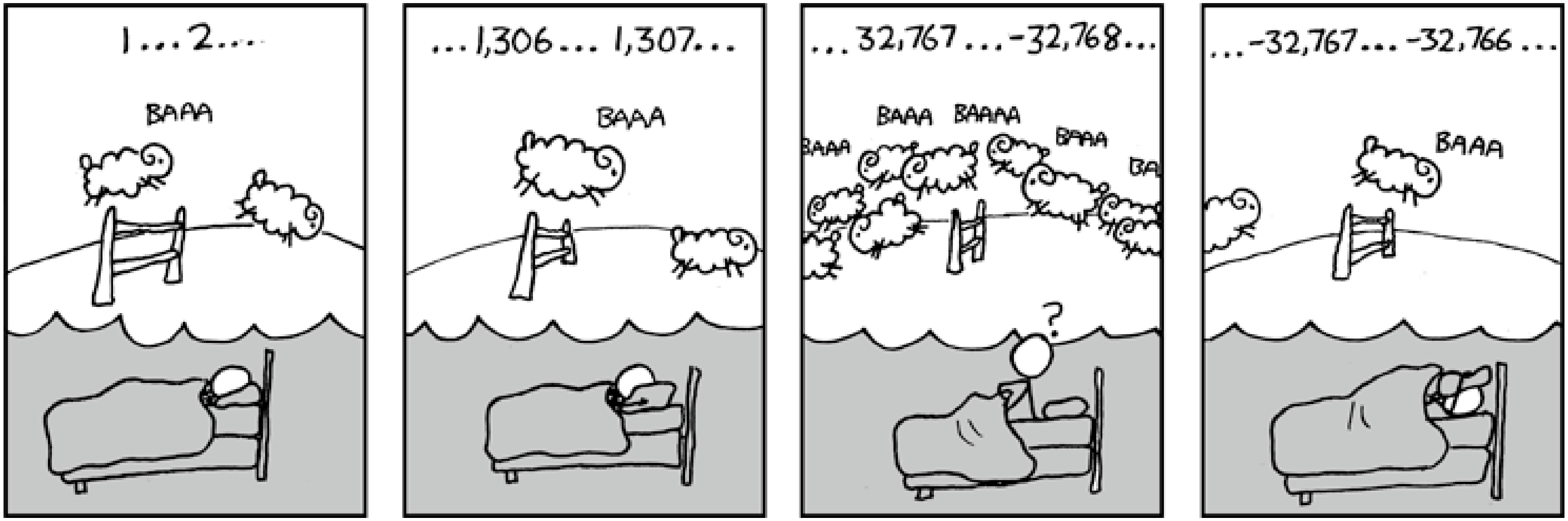


http://xkcd.com/571/

Administrivia

- hw4 due 10/12, hw5 due 10/14
- Lab 1a due Monday (10/12)
  - Submit `pointer.c` and `lab1Areflect.txt` to Gradescope
- Lab 1b released tomorrow, due 10/19
  - Bit manipulation on a custom number representation
  - Bonus slides at the end of today's lecture have relevant examples

Runnable Code Snippets on Ed

- Ed allows you to embed runnable code snippets (*e.g.*, readings, homework, discussion)
  - These are *editable* and *rerunnable*!
  - Hide compiler warnings, but will show compiler errors and runtime errors

- Suggested use
  - Good for experimental questions about basic behaviors in C
  - *NOT* entirely consistent with the CSE Linux environment, so should not be used for any lab-related work

Reading Review

- Terminology:
  - `UMin`, `UMax`, `TMin`, `TMax`
  - Type casting: implicit vs. explicit
  - Integer extension: zero extension vs. sign extension
  - Modular arithmetic and arithmetic overflow
  - Bit shifting: left shift, logical right shift, arithmetic right shift

- Questions from the Reading?

Review Questions

- What is the value (and encoding) of `TMin` for a fictional 6-bit wide integer data type?

- For `unsigned char uc = 0xA1;`, what are the produced data for the cast `(short)uc`?

- What is the result of the following expressions?
  - `(signed char)uc >> 2`
  - `(unsigned char)uc >> 3`

Why Does Two’s Complement Work?

- For all representable positive integers x, we want:

$$\begin{array}{r} \textit{bit representation of } x \\ + \textit{ bit representation of } -x \\ \hline 0 \end{array}$$

(ignoring the carry-out bit)

  - What are the 8-bit negative encodings for the following?

$$\begin{array}{rr} & 00000001 \\ + & ???????? \\ \hline & 00000000 \end{array} \qquad \begin{array}{rr} & 00000010 \\ + & ???????? \\ \hline & 00000000 \end{array} \qquad \begin{array}{rr} & 11000011 \\ + & ???????? \\ \hline & 00000000 \end{array}$$

Why Does Two's Complement Work?

- For all representable positive integers x, we want:

$$\begin{array}{r} \textit{bit representation of } \; x \\ + \; \textit{bit representation of } -x \\ \hline 0 \end{array}$$

(ignoring the carry-out bit)

  - What are the 8-bit negative encodings for the following?

$$\begin{array}{rcrcr} 00000001 & \quad & 00000010 & \quad & 11000011 \\ +\;\; 11111111 & & +\;\; 11111110 & & +\;\; 00111101 \\ \hline 100000000 & & 100000000 & & 100000000 \end{array}$$

These are the bitwise complement plus 1!

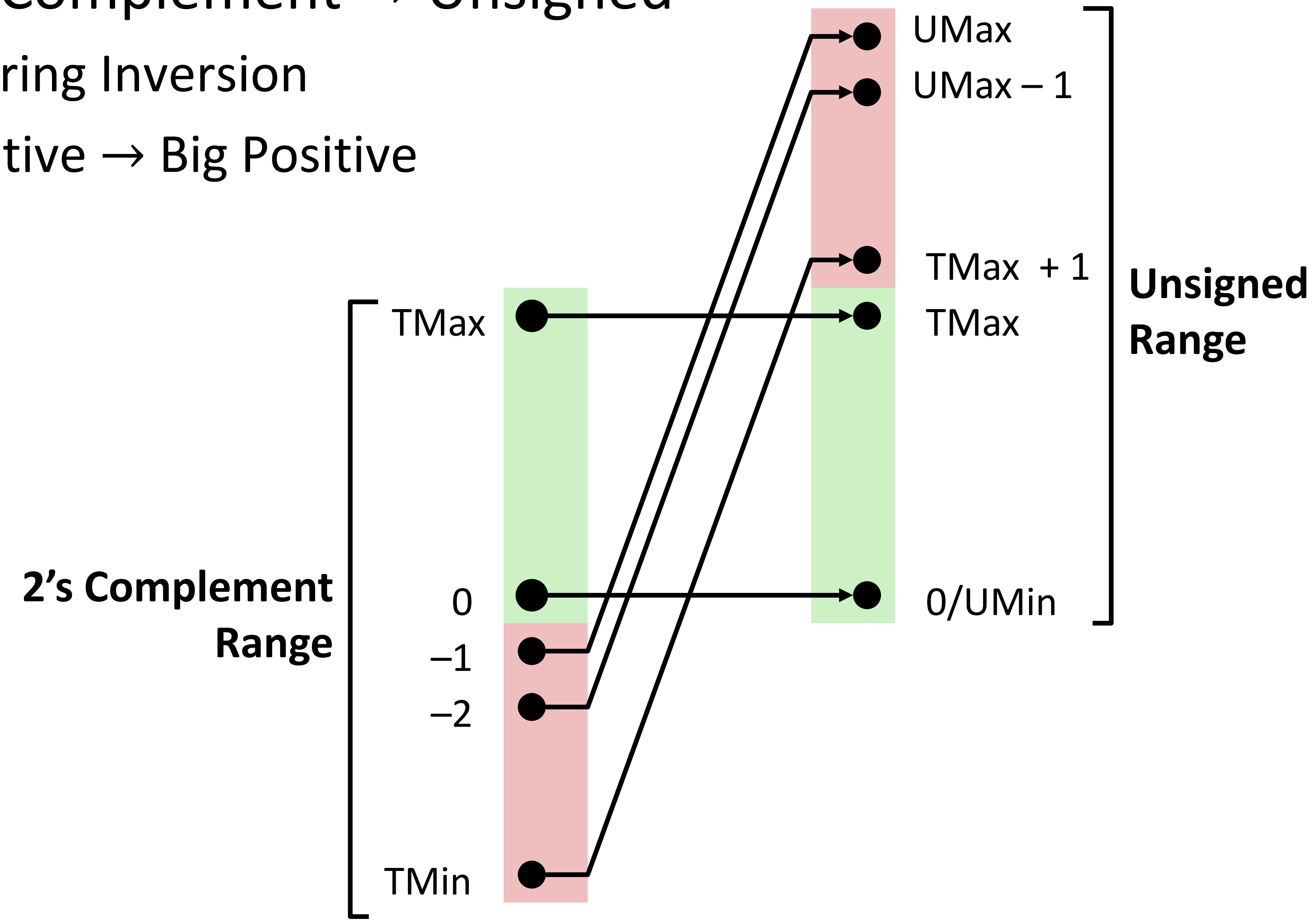
`-x == ~x + 1`

Integers

- **Binary representation of integers**
  - **Unsigned and signed**
  - **Casting in C**
- Consequences of finite width representations
  - Sign extension, overflow
- Shifting and arithmetic operations

Signed/Unsigned Conversion Visualized

- Two’s Complement → Unsigned
  - Ordering Inversion
  - Negative → Big Positive



Values To Remember

- Unsigned Values
  - UMin = `0b00…0`
    = 0
  - UMax = `0b11…1`
    = $2^w - 1$
- Two's Complement Values
  - TMin = `0b10…0`
    = -2^{w-1}
  - TMax = `0b01…1`
    = $2^{w-1} - 1$
  - -1 = `0b11…1`
- **Example:** Values for $w = 64$

	Decimal	Hex
UMax	18,446,744,073,709,551,615	`FF FF FF FF  FF FF FF FF`
TMax	9,223,372,036,854,775,807	`7F FF FF FF  FF FF FF FF`
TMin	-9,223,372,036,854,775,808	`80 00 00 00  00 00 00 00`
-1	-1	`FF FF FF FF  FF FF FF FF`
0	0	`00 00 00 00  00 00 00 00`

In C: Signed vs. Unsigned

- Casting
  - Bits are unchanged, just interpreted differently!
    - **int** tx, ty;
    - **unsigned int** ux, uy;
  - *Explicit* casting
    - tx = (**int**) ux;
    - uy = (**unsigned int**) ty;
  - *Implicit* casting can occur during assignments or function calls
    - tx = ux;
    - uy = ty;

Casting Surprises

!!!

- Integer literals (constants)
  - By default, integer constants are considered *signed* integers
    - Hex constants already have an explicit binary representation
  - Use “U” (or “u”) suffix to explicitly force *unsigned*
    - **Examples:** 0U, 4294967259u

- Expression Evaluation
  - When you mixed unsigned and signed in a single expression, then **signed values are implicitly cast to <u>unsigned</u>**
  - Including comparison operators <, >, ==, <=, >=

Practice Question 1

- Assuming 8-bit data (*i.e.*, bit position 7 is the MSB), what will the following expression evaluate to?
    - UMin = 0, UMax = 255, TMin = -128, TMax = 127

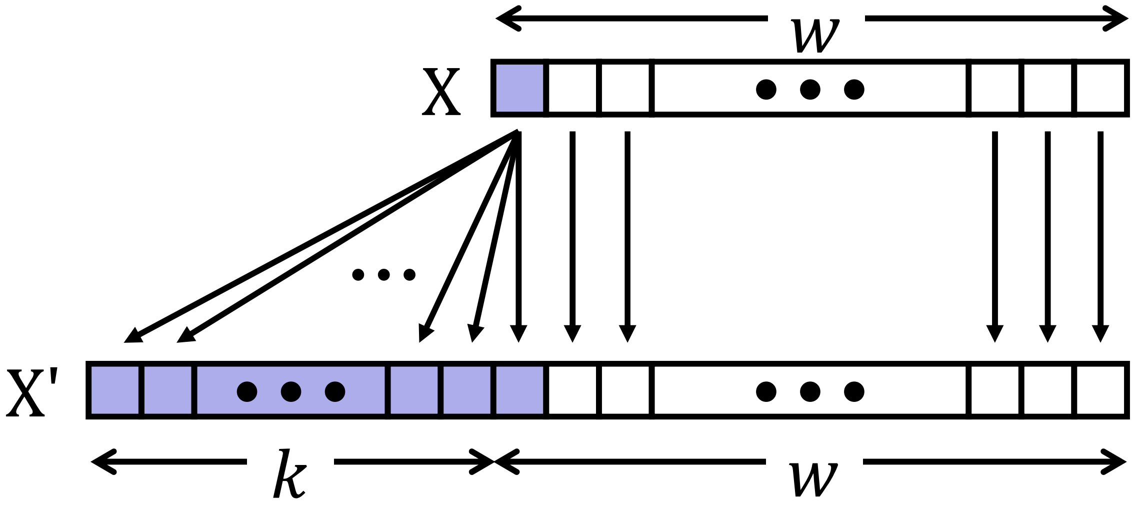
- `127 < (signed char) 128u`

Integers

- Binary representation of integers
  - Unsigned and signed
  - Casting in C
- **Consequences of finite width representations**
  - **Sign extension, overflow**
- Shifting and arithmetic operations

Sign Extension

- **Task:** Given a w-bit signed integer X, convert it to w+k-bit signed integer X' *with the same value*
- **Rule:** Add k copies of sign bit
  - Let x_i be the i-th digit of X in binary
  - $\mathrm{X}' = \underbrace{x_{w-1}, \ldots, x_{w-1}}_{k \textbf{ copies of MSB}}, \underbrace{x_{w-1}, x_{w-2}, \ldots, x_1, x_0}_{\textbf{original } \mathrm{X}}$



Two's Complement Arithmetic

- The same addition procedure works for both unsigned and two's complement integers
  - **Simplifies hardware:** only one algorithm for addition
  - **Algorithm:** simple addition, discard the highest carry bit
    - Called modular addition: result is sum *modulo* 2^w

Arithmetic Overflow

Bits	Unsigned	Signed
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	-8
1001	9	-7
1010	10	-6
1011	11	-5
1100	12	-4
1101	13	-3
1110	14	-2
1111	15	-1

- When a calculation produces a result that can’t be represented in the current encoding scheme
  - Integer range limited by fixed width
  - Can occur in both the positive and negative directions
- C and Java ignore overflow exceptions
  - You end up with a bad value in your program and no warning/indication… oops!

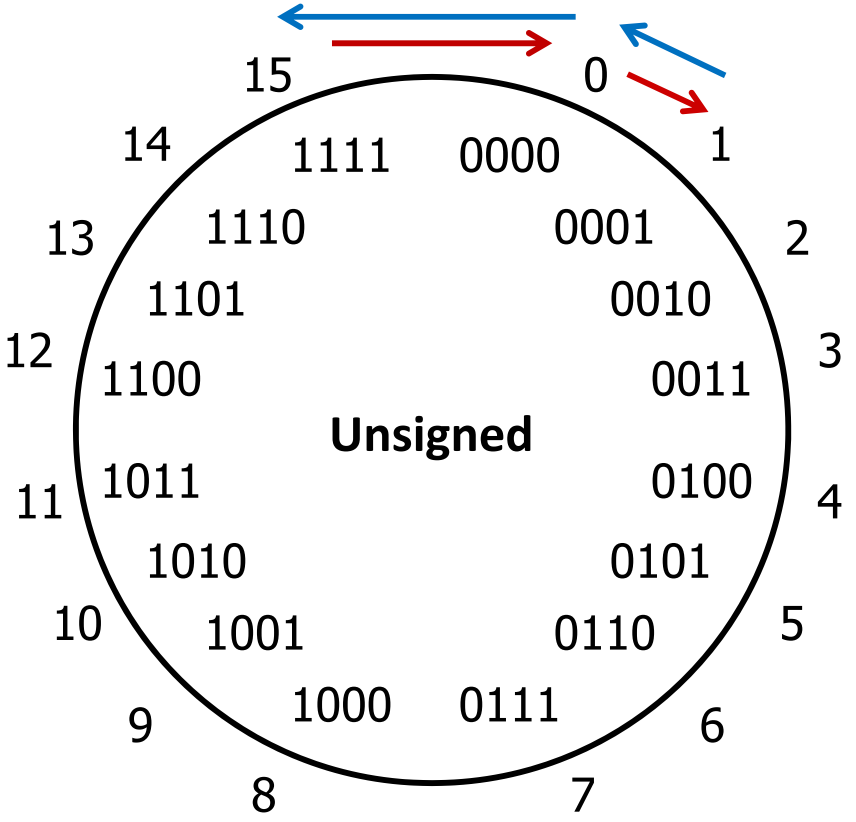
Overflow: Unsigned

- **Addition:** drop carry bit (-2^N)

$$\begin{array}{r} 15 \\ +\ \ 2 \\ \hline \cancel{17} \\ 1 \end{array} \qquad \begin{array}{r} 1111 \\ +\ \ 0010 \\ \hline \cancel{1}0001 \end{array}$$

- **Subtraction:** borrow ($+2^N$)

$$\begin{array}{r} 1 \\ -\ \ 2 \\ \hline \cancel{-1} \\ 15 \end{array} \qquad \begin{array}{r} 10001 \\ -\ \ 0010 \\ \hline 1111 \end{array}$$



$\pm 2^N$ because of modular arithmetic

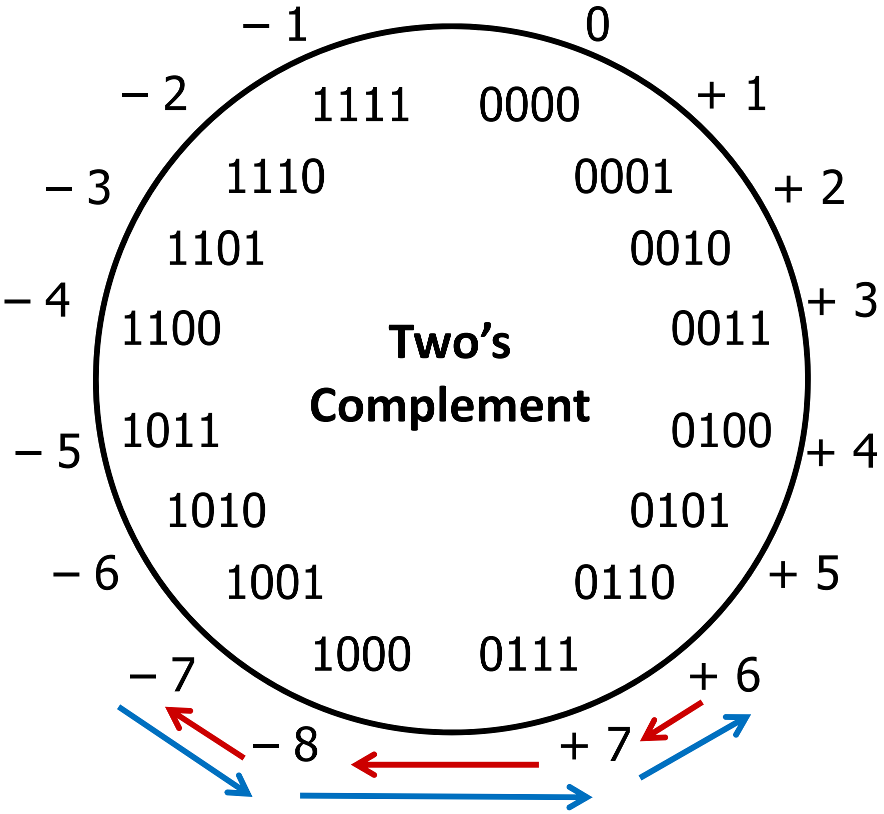
Overflow: Two’s Complement

- **Addition:** (+) + (+) = (–) result?

$$\begin{array}{r} 6 \\ +\ \ 3 \\ \hline \cancel{9} \\ -7 \end{array} \qquad \begin{array}{r} 0110 \\ +\ \ 0011 \\ \hline 1001 \end{array}$$

- **Subtraction:** (–) + (–) = (+)?

$$\begin{array}{r} -7 \\ -\ \ 3 \\ \hline \cancel{-10} \\ 6 \end{array} \qquad \begin{array}{r} 1001 \\ -\ \ 0011 \\ \hline 0110 \end{array}$$



For signed: overflow if operands have same sign and result’s sign is different

Practice Questions 2

- Assuming 8-bit integers:
  - `0x27` = 39 (signed) = 39 (unsigned)
  - `0xD9` = -39 (signed) = 217 (unsigned)
  - `0x7F` = 127 (signed) = 127 (unsigned)
  - `0x81` = -127 (signed) = 129 (unsigned)

- For the following additions, did signed and/or unsigned overflow occur?
  - `0x27 + 0x81`
  - `0x7F + 0xD9`

Integers

- Binary representation of integers
  - Unsigned and signed
  - Casting in C
- Consequences of finite width representations
  - Sign extension, overflow
- **Shifting and arithmetic operations**

Shift Operations

- Throw away (drop) extra bits that "fall off" the end
- Left shift (`x<<n`) bit vector `x` by `n` positions
  - Fill with `0`'s on right
- Right shift (`x>>n`) bit-vector `x` by `n` positions
  - Logical shift (for unsigned values)
    - Fill with `0`'s on left
  - Arithmetic shift (for signed values)
    - Replicate most significant bit on left (maintains sign of `x`)

	`x`	`0010 0010`
	`x<<3`	`0001 0000`
logical:	`x>>2`	`0000 1000`
arithmetic:	`x>>2`	`0000 1000`

	`x`	`1010 0010`
	`x<<3`	`0001 0000`
logical:	`x>>2`	`0010 1000`
arithmetic:	`x>>2`	`1110 1000`

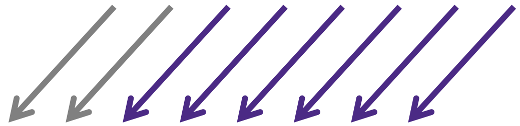
Shift Operations

- Arithmetic:
  - Left shift (`x<<n`) is equivalent to <u>multiply</u> by 2^n
  - Right shift (`x>>n`) is equivalent to <u>divide</u> by 2^n
  - Shifting is faster than general multiply and divide operations!

- Notes:
  - Shifts by `n<0` or `n`≥`w` (`w` is bit width of `x`) are *undefined*
  - **In C:** behavior of `>>` is determined by the compiler
    - In gcc / C lang, depends on data type of `x` (signed/unsigned)
  - **In Java:** logical shift is `>>>` and arithmetic shift is `>>`

Left Shifting Arithmetic 8-bit Example

- No difference in left shift operation for unsigned and signed numbers (just manipulates bits)
    - Difference comes during interpretation: `x*2`^{n}?



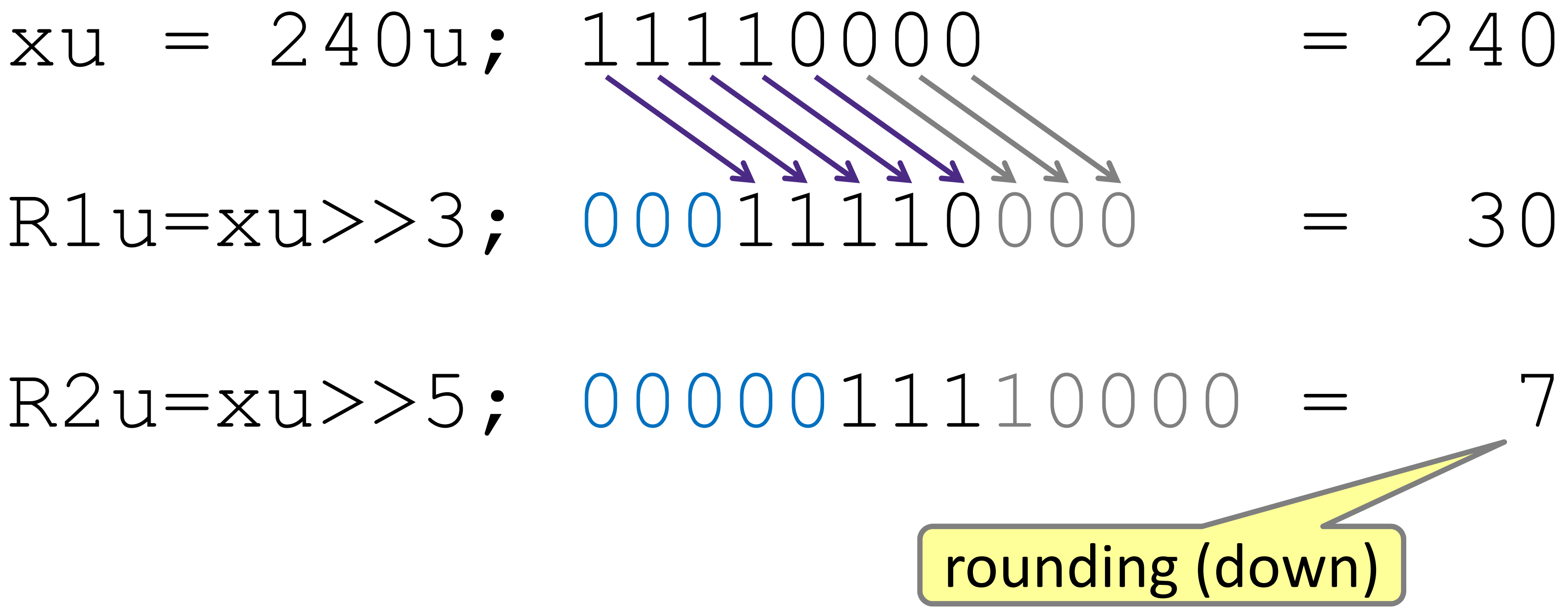
			Signed	Unsigned
`x = 25;`	`00011001`	=	25	25
`L1=x<<2;`	`0001100100`	=	100	100
`L2=x<<3;`	`00011001000`	=	-56	200
`L3=x<<4;`	`000110010000`	=	-112	144

signed overflow

unsigned overflow

Right Shifting Arithmetic 8-bit Examples

- **Reminder:** C operator >> does *logical* shift on unsigned values and *arithmetic* shift on signed values
  - Logical Shift: $x/2^n$?

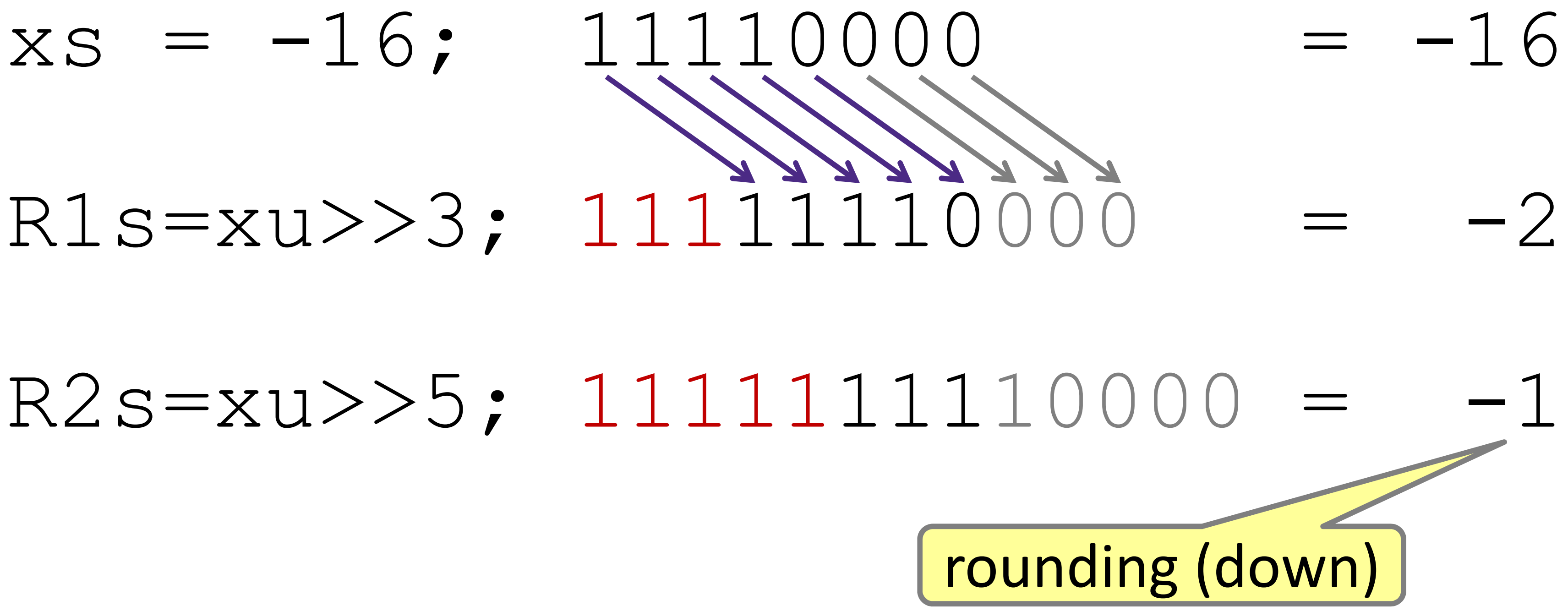


```
xu  = 240u;  11110000          = 240
R1u=xu>>3;  00011110000       =  30
R2u=xu>>5;  0000011110000     =   7
```

rounding (down)

Right Shifting Arithmetic 8-bit Examples

- **Reminder:** C operator >> does *logical* shift on unsigned values and *arithmetic* shift on signed values
  - Arithmetic Shift: `x/2`^{n}?



```
xs = -16;   11110000           = -16
R1s=xu>>3; 11111110000        =  -2
R2s=xu>>5; 1111111110000      =  -1
```

rounding (down)

Challenge Questions

For the following expressions, find a value of `signed char x`, if there exists one, that makes the expression True.

- Assume we are using 8-bit arithmetic:

	Example:	All solutions:
`x == (unsigned char) x`		
`x >= 128U`		
`x != (x>>2)<<2`		
`x == -x` Hint: there are two solutions		
`(x < 128U) && (x > 0x3F)`		

Summary

- Sign and unsigned variables in C
    - Bit pattern remains the same, just *interpreted* differently
    - Strange things can happen with our arithmetic when we convert/cast between sign and unsigned numbers
        - Type of variables affects behavior of operators (shifting, comparison)
- We can only represent so many numbers in *w* bits
    - When we exceed the limits, *arithmetic overflow* occurs
    - *Sign extension* tries to preserve value when expanding
- Shifting is a useful bitwise operator
    - Right shifting can be arithmetic (sign) or logical (0)
    - Can be used in multiplication with constant or bit masking

BONUS SLIDES

Some examples of using shift operators in combination with bitmasks, which you may find helpful for Lab 1b.

- Extract the 2nd most significant byte of an `int`
- Extract the sign bit of a signed `int`
- Conditionals as Boolean expressions

Using Shifts and Masks

- Extract the 2nd most significant *byte* of an `int`:
  - First shift, then mask: `(x>>16) & 0xFF`

`x`	00000001 00000010 00000011 00000100
`x>>16`	00000000 00000000 00000001 00000010
`0xFF`	00000000 00000000 00000000 11111111
`(x>>16) & 0xFF`	00000000 00000000 00000000 00000010

  - Or first mask, then shift: `(x & 0xFF0000)>>16`

`x`	00000001 00000010 00000011 00000100
`0xFF0000`	00000000 11111111 00000000 00000000
`x & 0xFF0000`	00000000 00000010 00000000 00000000
`(x&0xFF0000)>>16`	00000000 00000000 00000000 00000010

Using Shifts and Masks

- Extract the *sign bit* of a signed `int`:
  - First shift, then mask: `(x>>31) & 0x1`
    - Assuming arithmetic shift here, but this works in either case
    - Need mask to clear `1`s possibly shifted in

`x`	`00000001 00000010 00000011 00000100`
`x>>31`	`00000000 00000000 00000000 00000000`
`0x1`	`00000000 00000000 00000000 00000001`
`(x>>31) & 0x1`	`00000000 00000000 00000000 00000000`

`x`	`10000001 00000010 00000011 00000100`
`x>>31`	`11111111 11111111 11111111 11111111`
`0x1`	`00000000 00000000 00000000 00000001`
`(x>>31) & 0x1`	`00000000 00000000 00000000 00000001`

Using Shifts and Masks

- Conditionals as Boolean expressions
  - For **`int`** `x`, what does `(x<<31)>>31` do?

`x=!!123`	`00000000 00000000 00000000 00000001`
`x<<31`	`10000000 00000000 00000000 00000000`
`(x<<31)>>31`	`11111111 11111111 11111111 11111111`
`!x`	`00000000 00000000 00000000 00000000`
`!x<<31`	`00000000 00000000 00000000 00000000`
`(!x<<31)>>31`	`00000000 00000000 00000000 00000000`

  - Can use in place of conditional:
    - In C: `if(x) {a=y;} else {a=z;}` equivalent to `a=x?y:z;`
    - `a=(((x<<31)>>31)&y) | (((!x<<31)>>31)&z);`