Floating Point II, x86-64 Intro

CSE 351 Winter 2019

Instructors:

Max Willsey

Luis Ceze

Teaching Assistants:

Britt Henderson

Lukas Joswiak

Josie Lee

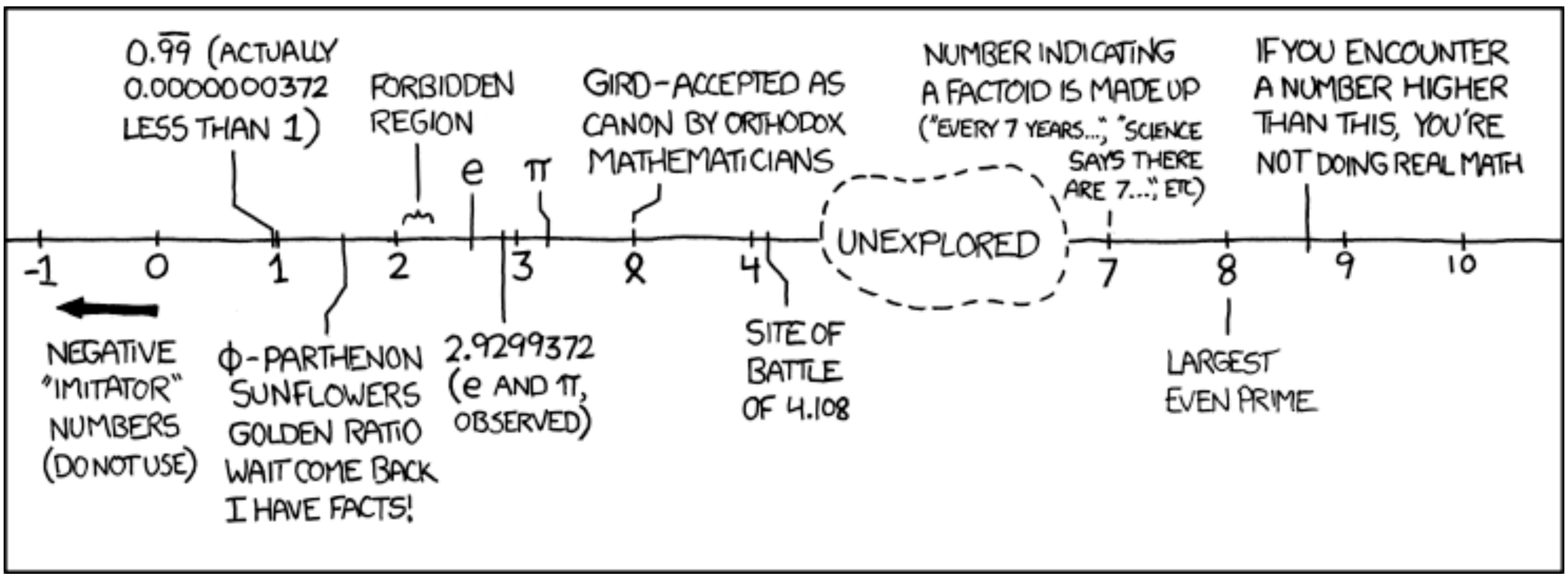
Wei Lin

Daniel Snitkovsky

Luis Vega

Kory Watson

Ivy Yu



http://xkcd.com/899/

Administrivia

- Lab 1b due Friday (1/25)
  - Submit `bits.c` and `lab1Breflect.txt`
- Homework 2 due next Friday (2/1)
  - On Integers, Floating Point, and x86-64
- Section tomorrow on Integers and Floating Point

Denorm Numbers

This is extra (non-testable) material

- Denormalized numbers (E = 0x00)
  - No leading 1
  - Uses implicit exponent of –126
- Denormalized numbers close the gap between zero and the smallest normalized number
  - Smallest norm: $\pm\ 1.0\ldots0_{two}\times 2^{-126} = \pm\ 2^{-126}$
  - Smallest denorm: $\pm\ 0.0\ldots01_{two}\times 2^{-126} = \pm\ 2^{-149}$
    - There is still a gap between zero and the smallest denormalized number

So much closer to 0

Other Special Cases

all ones

- E = 0xFF, M = 0: $\pm\infty$
  - *e.g.* division by 0
  - Still work in comparisons!
- E = 0xFF, M ≠ 0: Not a Number (NaN)
  - *e.g.* square root of negative number, 0/0, $\infty - \infty$
  - NaN propagates through computations
  - Value of M can be useful in debugging (tells you cause of NaN)
- New largest value (besides ∞)?
  - E = 0xFF has now been taken!
  - E = 0xFE has largest: $1.\overbrace{1\ldots1}^{23\text{ ones}}{}_2\times2^{127} = 2^{128} - 2^{104}$

    254 - bias

Floating Point Encoding Summary

	E	M	Meaning
smallest E (all 0's)	0x00	0	± 0
	0x00	non-zero	± denorm num
everything else	0x01 – 0xFE	anything	± norm num
largest E (all 1's)	0xFF	0	± ∞
	0xFF	non-zero	NaN

Floating point topics

- Fractional binary numbers
- IEEE floating-point standard
- **Floating-point operations and rounding**
- Floating-point in C

- There are many more details that we won't cover
  - It's a 58-page standard…

Tiny Floating Point Representation

- We will use the following **8-bit** floating point representation to illustrate some key points:

S	E	M
1	4	3

- Assume that it has the same properties as IEEE floating point:
  - bias = $2^{w-1}-1 = 2^{4-1}-1 = 7$
  - encoding of -0 = 0b 1 0000 000 = 0x80
  - encoding of $+\infty$ = 0b 0 1111 000 = 0x78
  - encoding of the largest (+) normalized # = 0b 0 1110 111 = 0x77
  - encoding of the smallest (+) normalized # = 0b 0 0001 000 = 0x08

Peer Instruction Question

- Using our **8-bit** representation, what value gets stored when we try to encode **2.625** = $2^1 + 2^{-1} + 2^{-3}$?

S	E	M
1	4	3

$= 2^1 (1 + 2^{-2} + 2^{-4})$

$= 2^1 \times 1.0101_2$

$S = 0$

$E = \text{Exp} + \text{bias}$
$= 1 + 7 = 8$
$= 0b\ 1000$

$M = 0b\ 010/1$

can only store 3 bits!

stored as: $\boxed{0b\ 0\ 1000\ 010 = 2.5}$

A. $\boxed{+2.5}$

B. + 2.625

C. + 2.75

D. + 3.25

E. We're lost…

Peer Instruction Question

- Using our **8-bit** representation, what value gets stored when we try to encode **384** = $2^8 + 2^7$? $= 2^8(1+2^{-1})$

$= 2^8 \times 1.1_2$

S	E	M
1	4	3

S=0

E = Exp + bias
= 8 + 7 = 15
= 0b1111

this falls outside of the normalized exponent range!

A. + 256

B. + 384

C. $+\infty$

D. NaN

E. We're lost…

this number is too large, so we store

$+\infty \longleftrightarrow$ 0b0 1111 000

instead

Distribution of Values

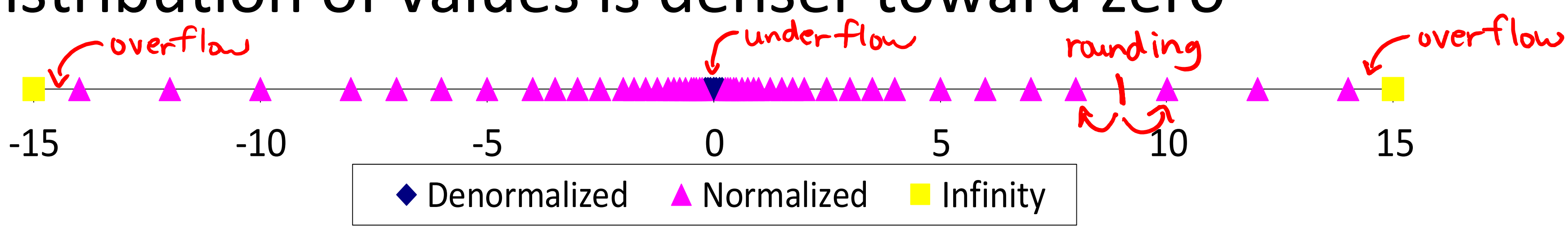
- What ranges are NOT representable?
  - Between largest norm and infinity **Overflow** (Exp too large)
  - Between zero and smallest denorm **Underflow** (Exp too small)
  - Between norm numbers? **Rounding**
- Given a FP number, what's the bit pattern of the next largest representable number?

  if M=0b0...00, then $2^{Exp} \times 1.0$

  if M=0b0...01, then $2^{Exp} \times (1+2^{-23})$

  diff = 2^{Exp-23}

  - What is this "step" when Exp = 0? 2^{-23}
  - What is this "step" when Exp = 100? 2^{77}
- Distribution of values is denser toward zero



Floating Point Rounding

This is extra (non-testable) material

- The IEEE 754 standard actually specifies different rounding modes:
  - Round to nearest, ties to nearest even digit
  - Round toward $+\infty$ (round up)
  - Round toward $-\infty$ (round down)
  - Round toward 0 (truncation)

- In our tiny example:

S	E	M
1	4	3

  - < half
  - Man = 1.001/01 rounded to M = 0b001
  - > half
  - Man = 1.001/11 rounded to M = 0b010
  - == half
  - Man = 1.001/10 rounded to M = 0b010

  Man = 1.000/10 rounded to M = 0b000

  even digit

Floating Point Operations: Basic Idea

$$\text{Value} = (-1)^{S} \times \text{Mantissa} \times 2^{\text{Exponent}}$$

S	E	M

- $x +_{f} y = \mathrm{Round}(x + y)$
- $x *_{f} y = \mathrm{Round}(x * y)$

- Basic idea for floating point operations:
  - First, compute the exact result
  - Then *round* the result to make it fit into the specificed precision (width of M)
    - Possibly over/underflow if exponent outside of range

Mathematical Properties of FP Operations

- Overflow yields $\pm\infty$ and underflow yields 0
- Floats with value $\pm\infty$ and NaN can be used in operations
  - Result usually still $\pm\infty$ or NaN, but not always intuitive
- Floating point operations do not work like real math, due to rounding
  - Not associative: `(3.14+1e100)-1e100 != 3.14+(1e100-1e100)`

    10^{100} 0

    0 **3.14**
  - Not distributive: `100*(0.1+0.2) != 100*0.1+100*0.2`

    30.000000000000003553 **30**
  - Not cumulative
    - Repeatedly adding a very small number to a large one may do nothing

Floating point topics

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- IEEE floating-point standard
- Floating-point operations and rounding
- **Floating-point in C**

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  - It's a 58-page standard…

Floating Point in C

!!!

- Two common levels of precision:

float	1.0f	single precision (32-bit)
double	1.0	double precision (64-bit)

- #include <math.h> to get INFINITY and NAN constants <float.h> for additional constants

- Equality (==) comparisons between floating point numbers are tricky, and often return unexpected results, so just avoid them!

instead use $abs(f1 - f2) < 2^{-20}$

some arbitrary threshold

Floating Point Conversions in C

!!!

- Casting between `int`, `float`, and `double` **changes** the bit representation
  - `int` → `float`
    - May be rounded (not enough bits in mantissa: 23)
    - Overflow impossible
  - `int` or `float` → `double`
    - Exact conversion (all 32-bit `int`s representable)
  - `long` → `double`
    - Depends on word size (32-bit is exact, 64-bit may be rounded)
  - `double` or `float` → `int`
    - Truncates fractional part (rounded toward zero)
    - "Not defined" when out of range or NaN: generally sets to `Tmin` (even if the value is a very big positive)

Peer Instruction Question

- We execute the following code in C. How many bytes are the same (value and position) between `i` and `f`?

```
int i = 384;   // 2^8 + 2^7
float f = (float) i;
```

$$= 0b\ 1\,1000\,0000$$

(bit positions: 8 7 6 5 4 3 2 1 0)

$$= 1.1_2 \times 2^8$$

$S = 0$

$E = 8 + 127 = 135$

$= 0b\,1000\ 0111$

$M = 0b\,10\ldots0$

$0b\ 0\ 1000\ 0111\ 100\ldots0$

A. 0 bytes

B. 1 byte

C. 2 bytes

D. 3 bytes

E. We're lost…

i stored as 0x 00 00 01 80

f stored as 0x 43 C0 00 00

Floating Point and the Programmer

$1.0 \times 2^0 \rightarrow S=0, E=0111\ 1111, M=0\ldots0$

f1 = 0b 0/011 1111 1/000 0000 0000 0000 0000 0000 = 0x3F800000

```
#include <stdio.h>

int main(int argc, char* argv[]) {
  float f1 = 1.0;   <- specify float constant
  float f2 = 0.0;
  int i;
  for (i = 0; i < 10; i++)
    f2 += 1.0/10.0;
    f2 should == 10×1/10 = 1
  printf("0x%08x  0x%08x\n", *(int*)&f1, *(int*)&f2);
  printf("f1 = %10.9f\n", f1);
  printf("f2 = %10.9f\n\n", f2);

  f1 = 1E30;   10^30
  f2 = 1E-30;  10^-30
  float f3 = f1 + f2;
  printf("f1 == f3? %s\n", f1 == f3 ? "yes" : "no" );
             10^30 == 10^30 + 10^-30
  return 0;
}
```

```
$ ./a.out
0x3f800000  0x3f800001   f2
f1 = 1.000000000
f2 = 1.000000119

f1 == f3? yes
```

see float.c

Floating Point Summary

- Floats also suffer from the fixed number of bits available to represent them
  - Can get overflow/underflow
  - "Gaps" produced in representable numbers means we can lose precision, unlike `int`s
    - Some "simple fractions" have no exact representation (*e.g.* 0.2)
    - "Every operation gets a slightly wrong result"
- Floating point arithmetic not associative or distributive
  - Mathematically equivalent ways of writing an expression may compute different results
- Never test floating point values for equality!
- Careful when converting between `int`s and `float`s!

Number Representation Really Matters

- **1991:** Patriot missile targeting error
  - clock skew due to conversion from integer to floating point
- **1996:** Ariane 5 rocket exploded ($1 billion)
  - overflow converting 64-bit floating point to 16-bit integer
- **2000:** Y2K problem
  - limited (decimal) representation: overflow, wrap-around
- **2038:** Unix epoch rollover
  - Unix epoch = seconds since 12am, January 1, 1970
  - signed 32-bit integer representation rolls over to TMin in 2038
- **Other related bugs:**
  - 1982: Vancouver Stock Exchange 10% error in less than 2 years
  - 1994: Intel Pentium FDIV (floating point division) HW bug ($475 million)
  - 1997: USS Yorktown “smart” warship stranded: divide by zero
  - 1998: Mars Climate Orbiter crashed: unit mismatch ($193 million)

Roadmap

C:

```
car *c = malloc(sizeof(car));
c->miles = 100;
c->gals = 17;
float mpg = get_mpg(c);
free(c);
```

Java:

```
Car c = new Car();
c.setMiles(100);
c.setGals(17);
float mpg =
    c.getMPG();
```

Assembly language:

```
get_mpg:
    pushq   %rbp
    movq    %rsp, %rbp
    ...
    popq    %rbp
    ret
```

Machine code:

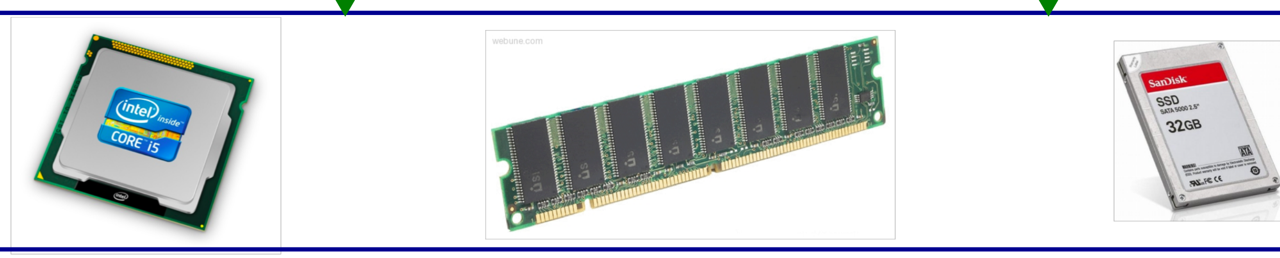
```
0111010000011000
100011010000010000000010
1000100111000010
110000011111101000011111
```

OS:

Windows 10

OS X Yosemite

Computer system:



- Memory & data
- Integers & floats
- **x86 assembly**
- Procedures & stacks
- Executables
- Arrays & structs
- Memory & caches
- Processes
- Virtual memory
- Memory allocation
- Java vs. C

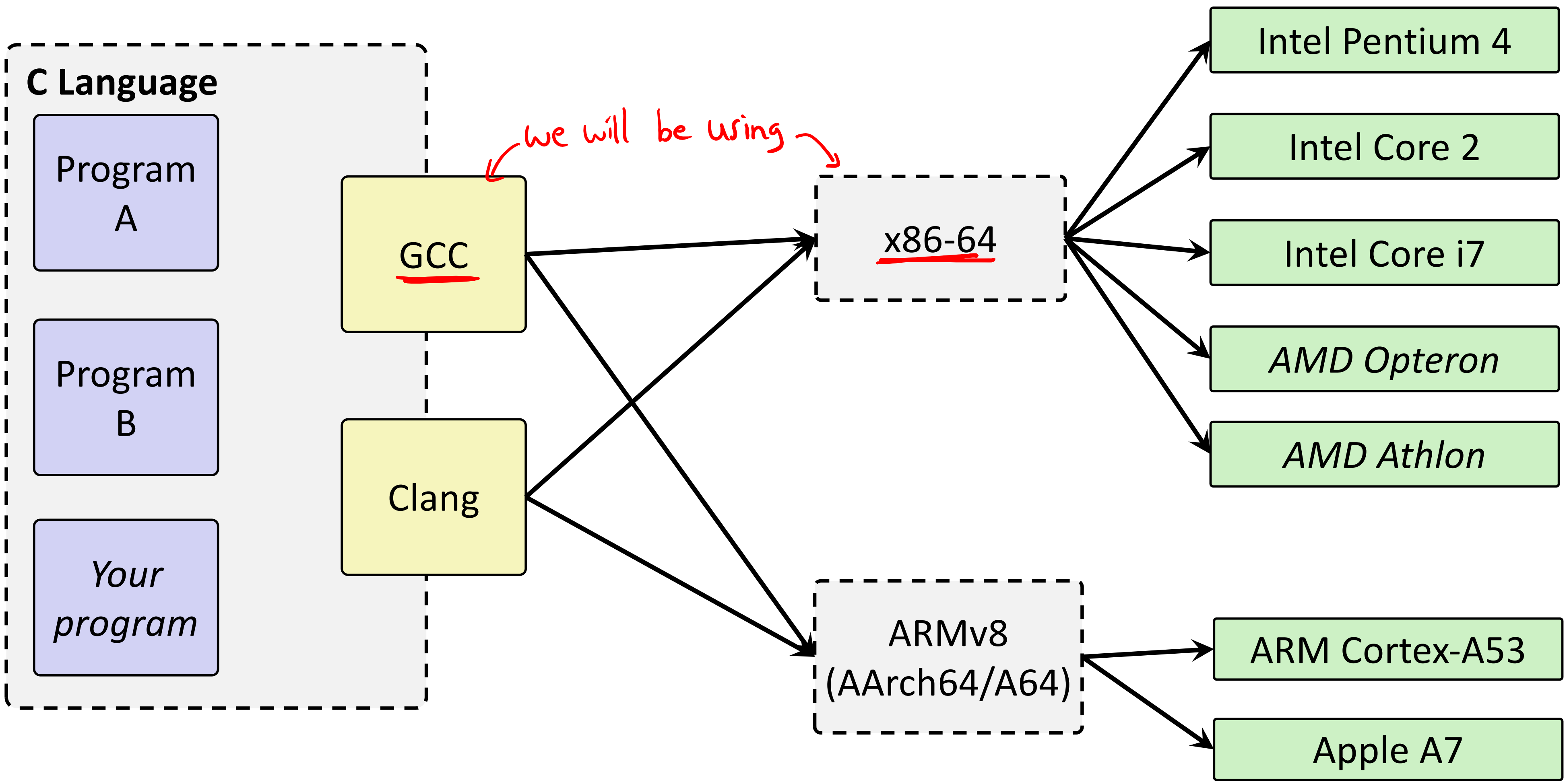
Architecture Sits at the Hardware Interface

Source code
Different applications or algorithms

Compiler
Perform optimizations, generate instructions

Architecture
Instruction set

Hardware
Different implementations

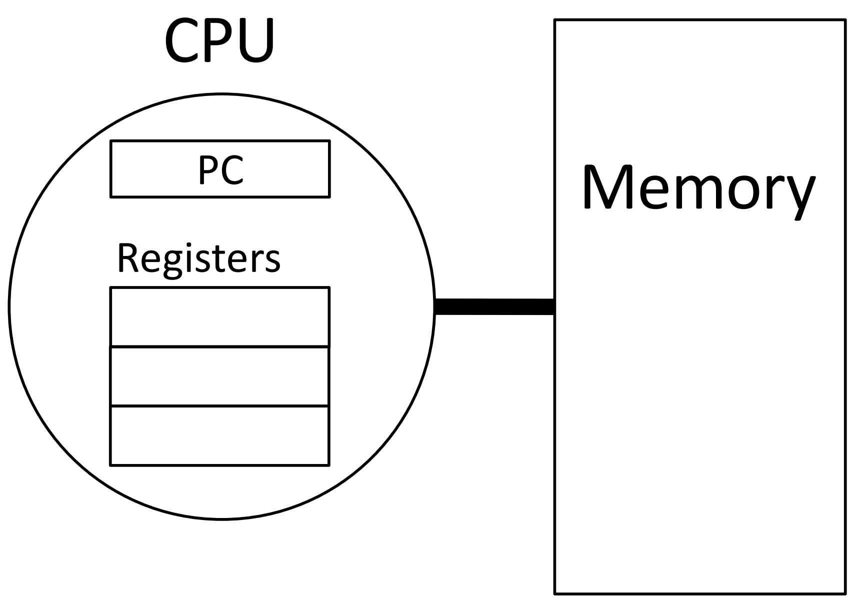


Definitions

- **Architecture (ISA):** The parts of a processor design that one needs to understand to write assembly code
  - "What is directly visible to software"
- **Microarchitecture:** Implementation of the architecture
  - CSE/EE 469

Instruction Set Architectures

- The ISA defines:
  - The system's state (*e.g.* registers, memory, program counter)
  - The instructions the CPU can execute
  - The effect that each of these instructions will have on the system state



Instruction Set Philosophies

- *Complex Instruction Set Computing* (CISC): Add more and more elaborate and specialized instructions as needed
  - Lots of tools for programmers to use, but hardware must be able to handle all instructions
  - x86-64 is CISC, but only a small subset of instructions encountered with Linux programs
- *Reduced Instruction Set Computing* (RISC): Keep instruction set small and regular
  - Easier to build fast hardware
  - Let software do the complicated operations by composing simpler ones

General ISA Design Decisions

- Instructions
  - What instructions are available? What do they do?
  - How are they encoded?
- Registers
  - How many registers are there?
  - How wide are they?
- Memory
  - How do you specify a memory location?

Mainstream ISAs

x86

Designer	Intel, AMD
Bits	16-bit, 32-bit and 64-bit
Introduced	1978 (16-bit), 1985 (32-bit), 2003 (64-bit)
Design	CISC
Type	Register-memory
Encoding	Variable (1 to 15 bytes)
Endianness	Little

Macbooks & PCs
(Core i3, i5, i7, M)
[x86-64 Instruction Set]

ARM architectures

Designer	ARM Holdings
Bits	32-bit, 64-bit
Introduced	1985; 31 years ago
Design	RISC
Type	Register-Register
Encoding	AArch64/A64 and AArch32/A32 use 32-bit instructions, T32 (Thumb-2) uses mixed 16- and 32-bit instructions. ARMv7 user-space compatibility[1]
Endianness	Bi (little as default)

Smartphone-like devices
(iPhone, iPad, Raspberry Pi)
[ARM Instruction Set]

MIPS

Designer	MIPS Technologies, Inc.
Bits	64-bit (32→64)
Introduced	1981; 35 years ago
Design	RISC
Type	Register-Register
Encoding	Fixed
Endianness	Bi

Digital home & networking equipment
(Blu-ray, PlayStation 2)
[MIPS Instruction Set]

Summary

- Floating point encoding has many limitations
  - Overflow, underflow, rounding
  - Rounding is a HUGE issue due to limited mantissa bits and gaps that are scaled by the value of the exponent
  - Floating point arithmetic is NOT associative or distributive
- Converting between integral and floating point data types *does* change the bits
- x86-64 is a complex instruction set computing (CISC) architecture