# Caches IV

CSE 351 Spring 2019

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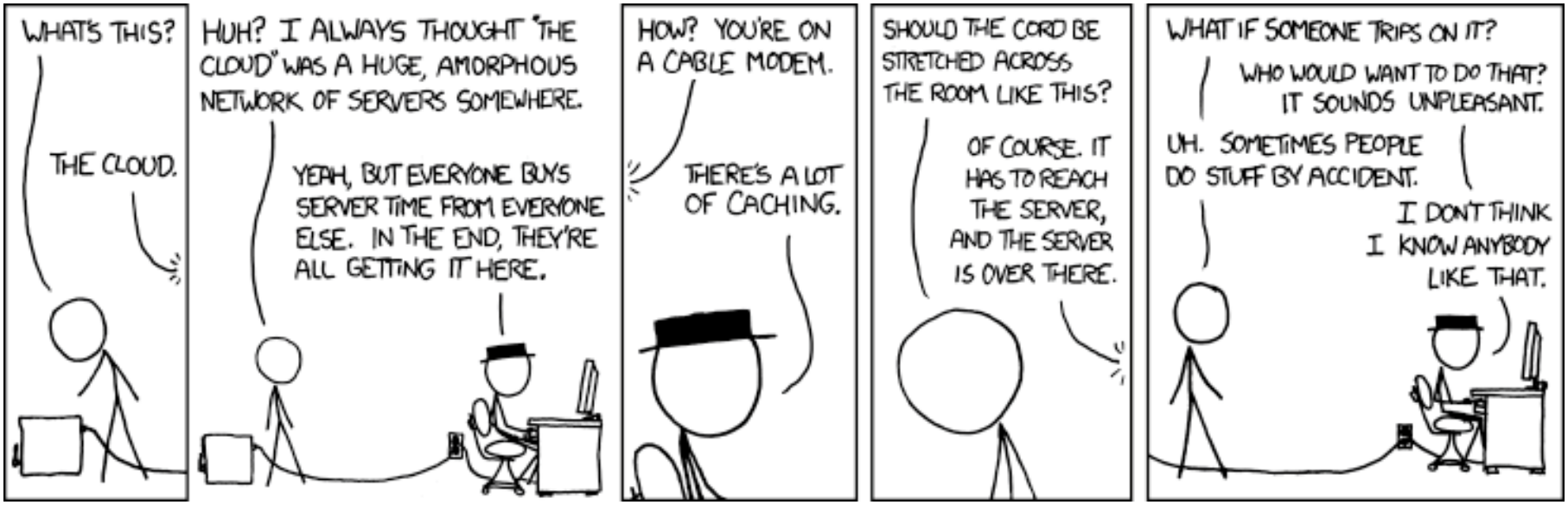
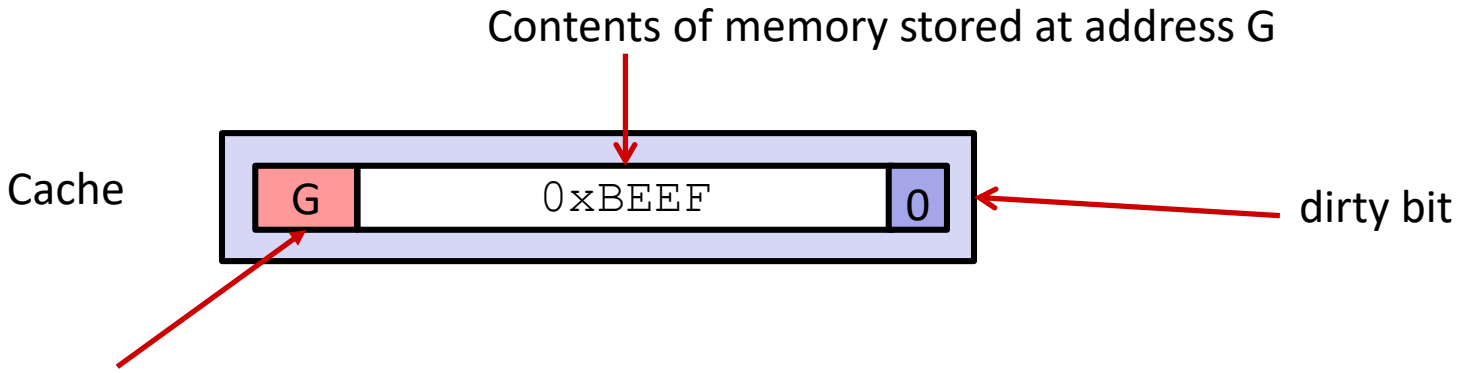


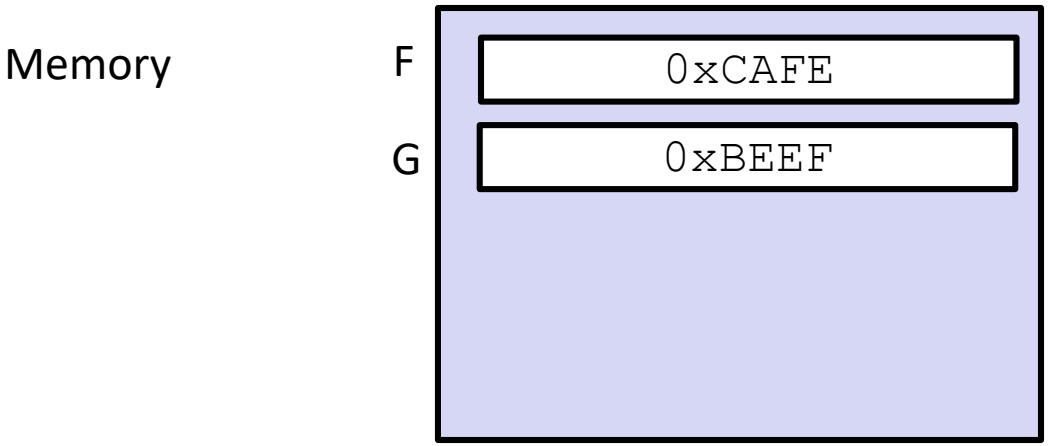

http://xkcd.com/908/

# Administrivia

- Lab 3, due TONIGHT, Wednesday (5/15)
- Homework 4 , due Wed (5/22) (Structs, Caches)
- Lab 4, Coming soon!
  - Cache parameter puzzles and code optimizations

# Write-back, write-allocate example



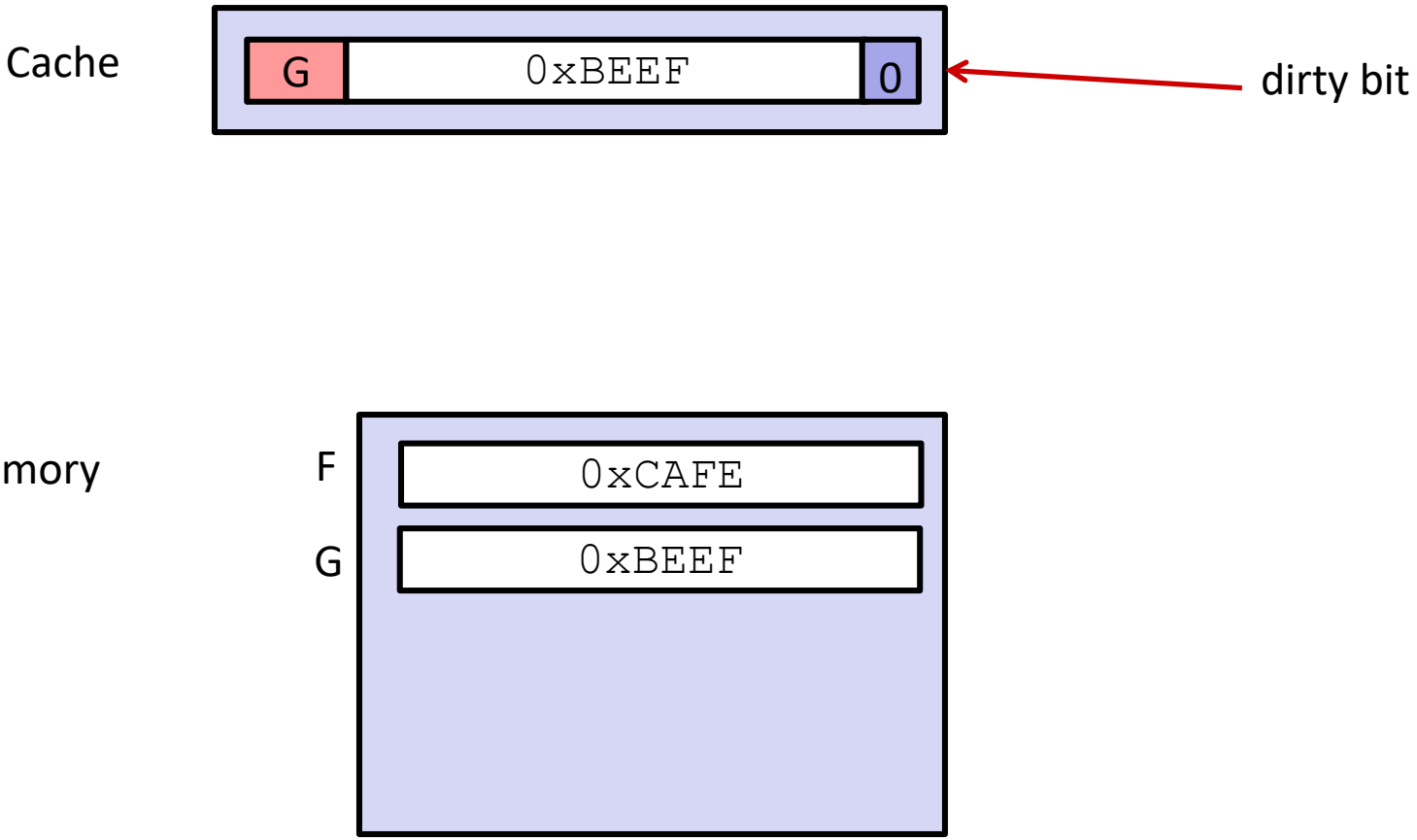




In this example we are sort of ignoring block offsets. Here a block holds 2 bytes (16 bits, 4 hex digits).

Normally a block would be much bigger and thus there would be multiple items per block. While only one item in that block would be written at a time, the entire line would be brought into cache.

# Write-back, write-allocate example

```
mov 0xFACE, F
```



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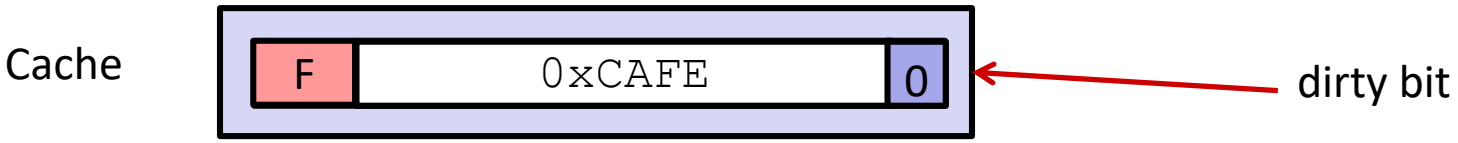
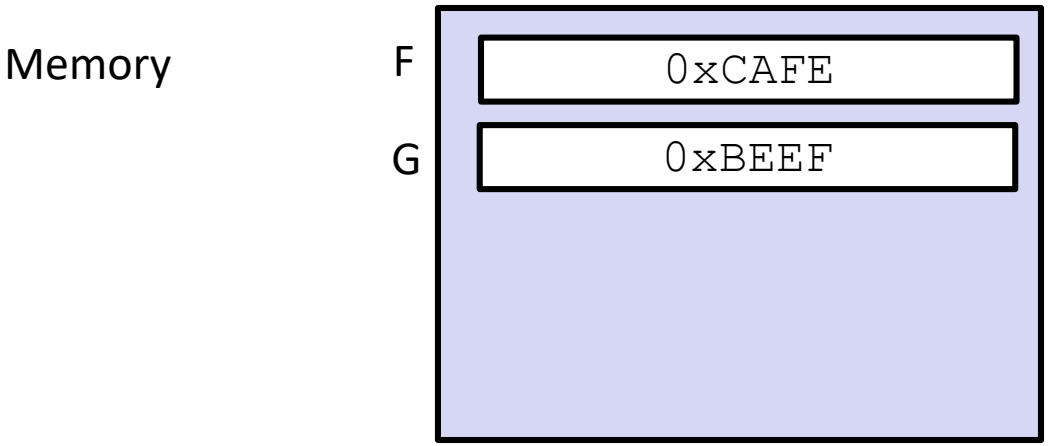



Step 1: Bring **F** into cache



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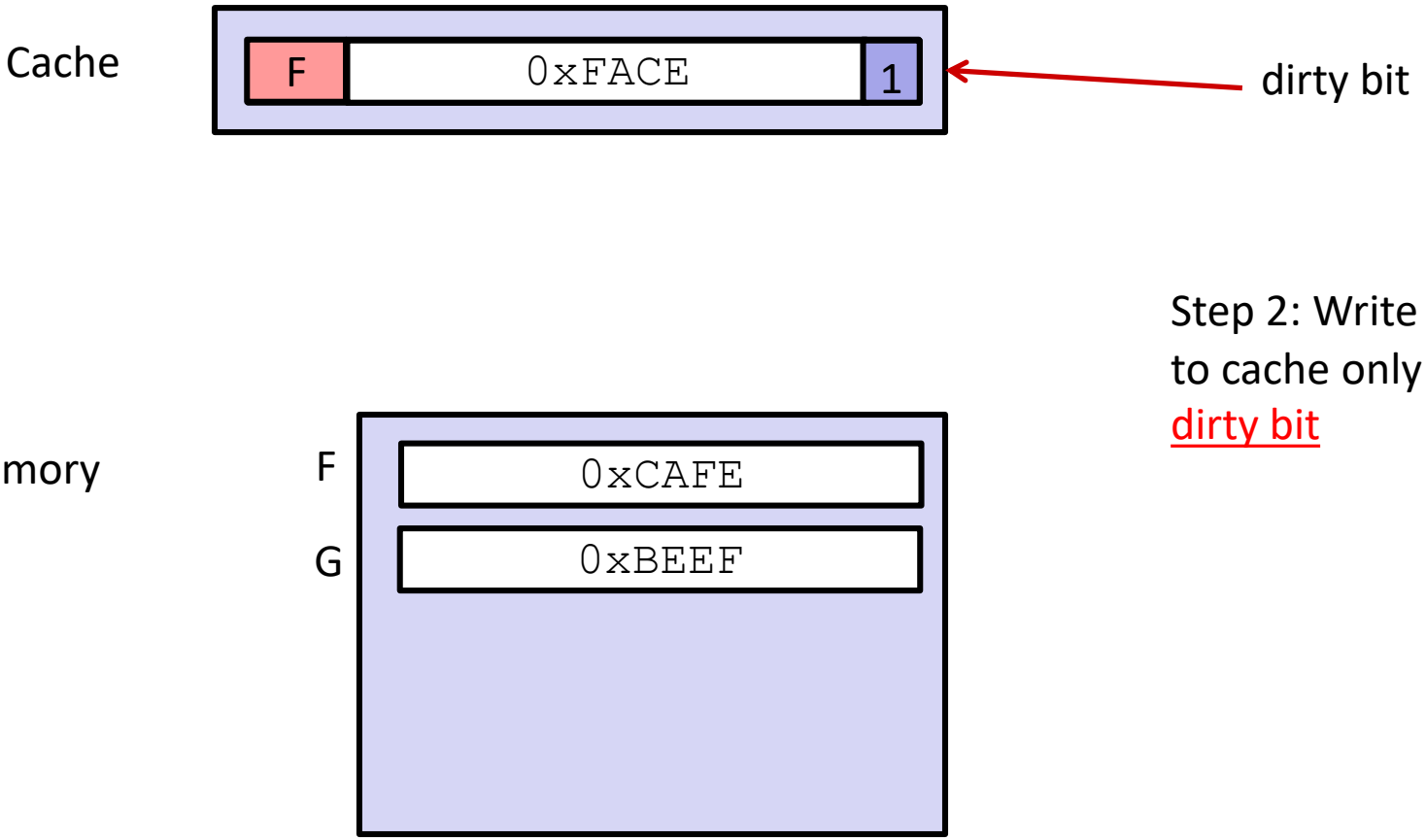
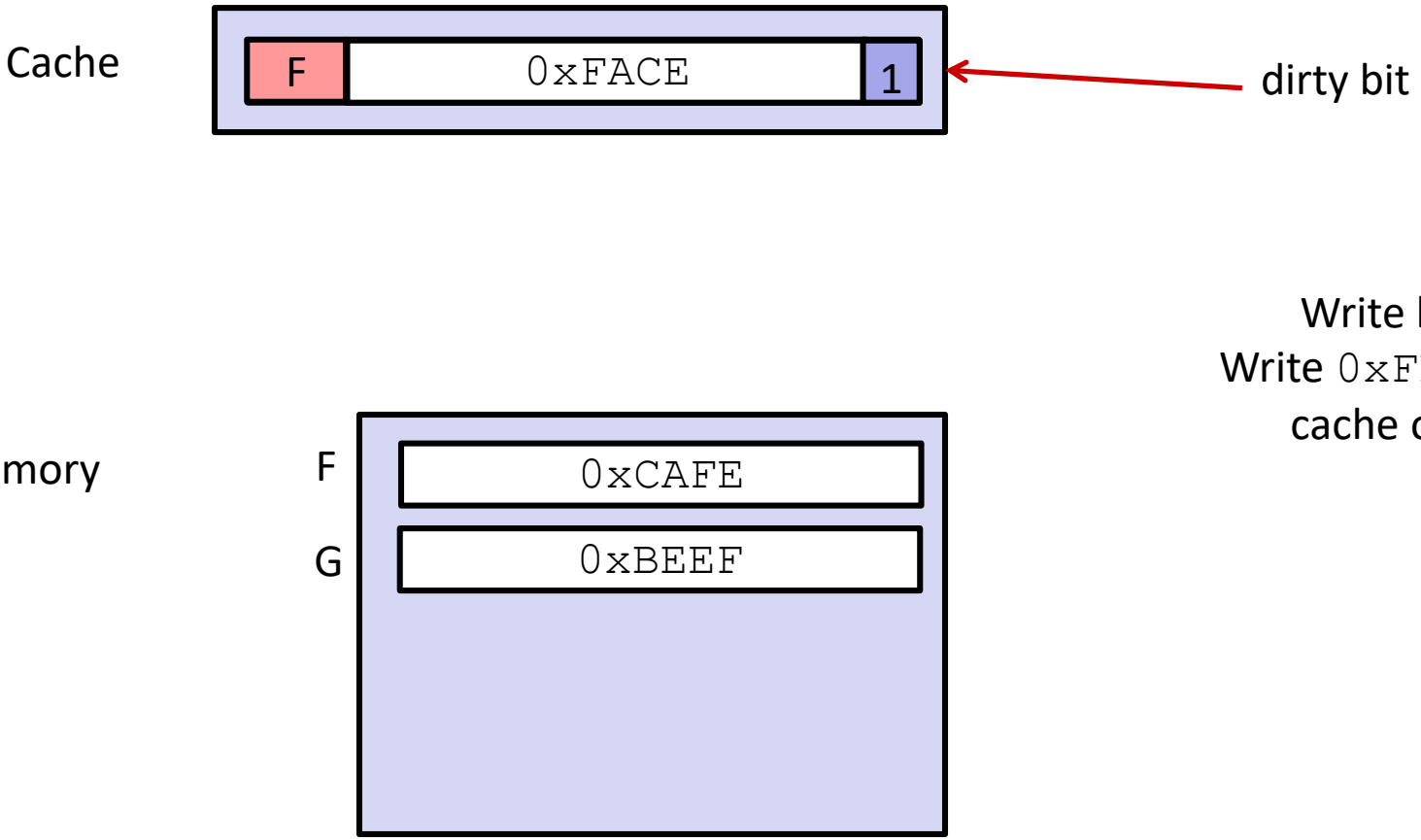



Step 2: Write `0xFACE` to cache only and set dirty bit

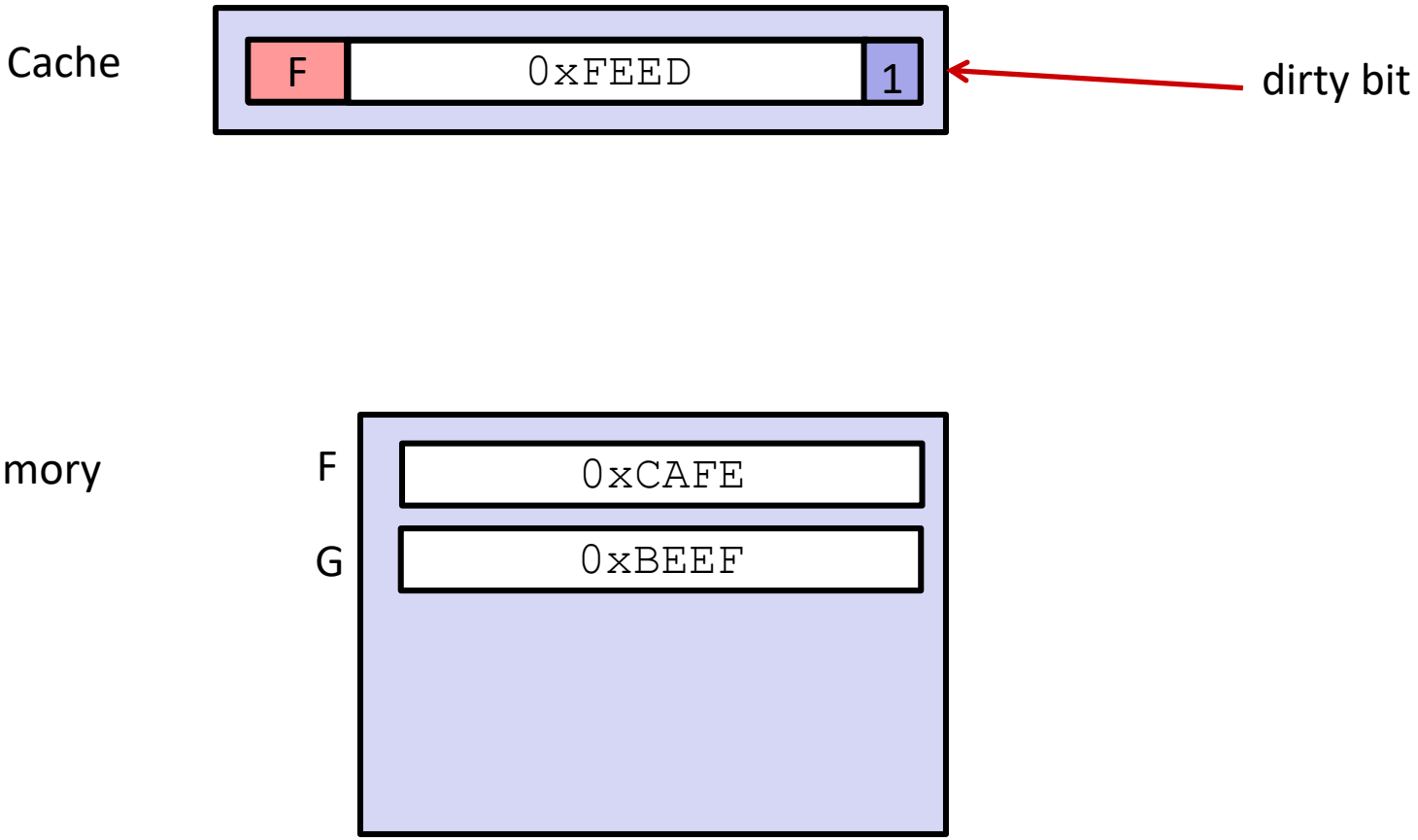
# Write-back, write-allocate example

```
mov 0xFACE, F      mov 0xFEED, F
```



# Write-back, write-allocate example

```
mov 0xFACE, F      mov 0xFEED, F          mov G, %rax
```



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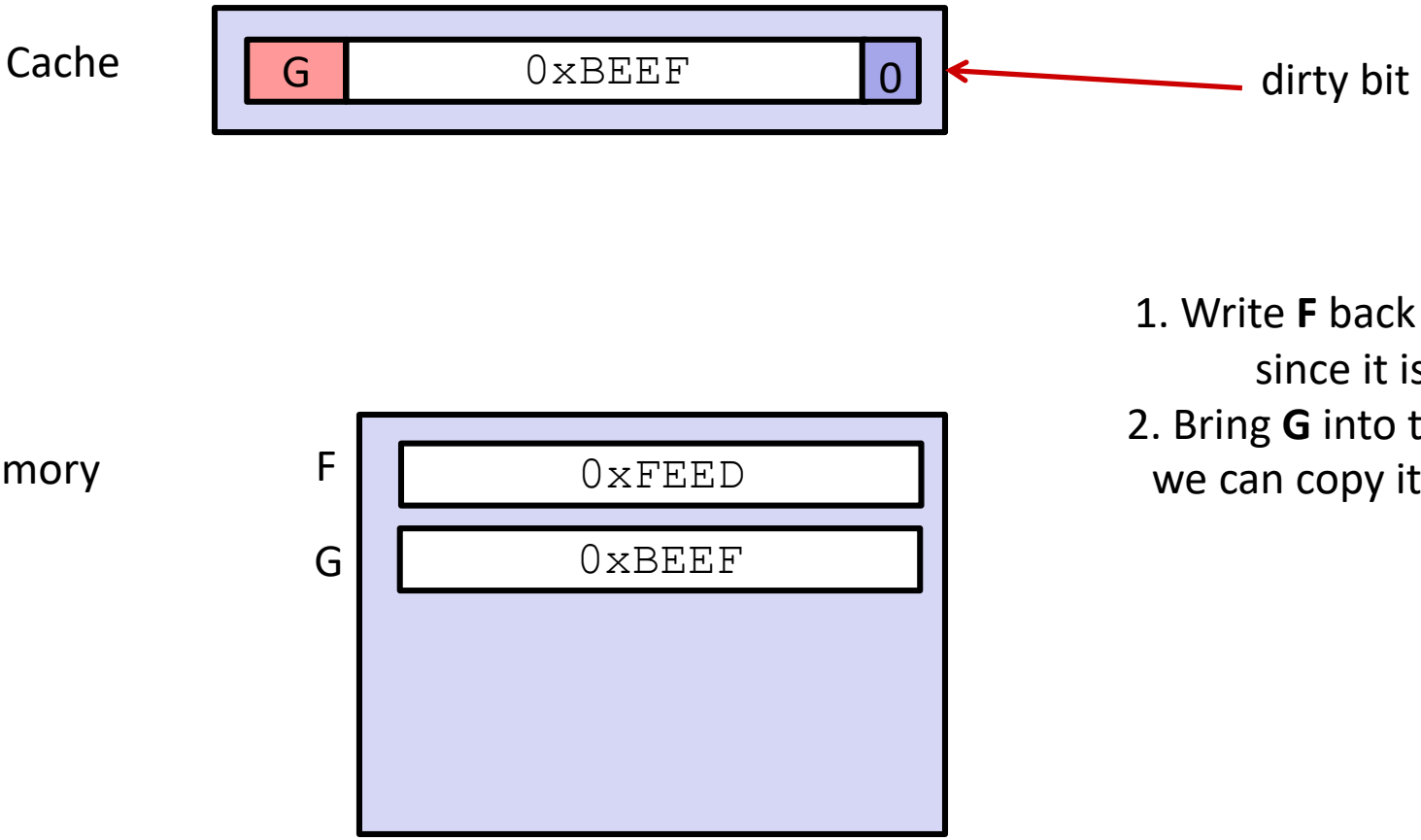



1. Write **F** back to memory since it is dirty
2. Bring **G** into the cache so we can copy it into `%rax`

# Peer Instruction Question

- Which of the following cache statements is FALSE?
  - Vote at http://pollev.com/rea

A. **We can reduce compulsory misses by decreasing our block size**

B. **We can reduce conflict misses by increasing associativity**

C. **A write-back cache will save time for code with good temporal locality on writes**

D. **A write-through cache will always match data with the memory hierarchy level below it**

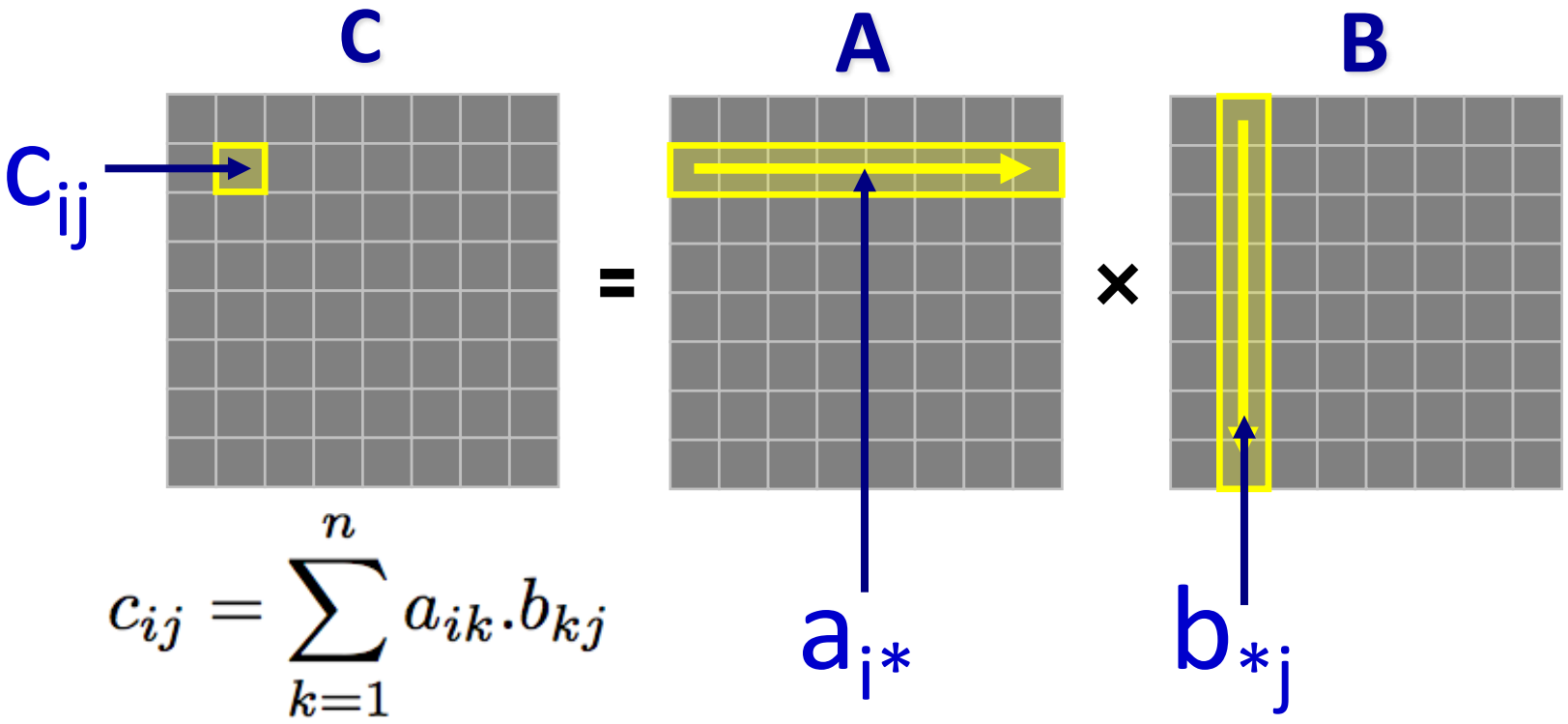
E. **We're lost…**

# Optimizations for the Memory Hierarchy

- Write code that has locality!
  - Spatial: access data contiguously
  - Temporal: make sure access to the same data is not too far apart in time

- How can you achieve locality?
  - Adjust memory accesses in *code* (software) to improve miss rate (MR)
    - Requires knowledge of *both* how caches work as well as your system's parameters
  - Proper choice of algorithm
  - Loop transformations

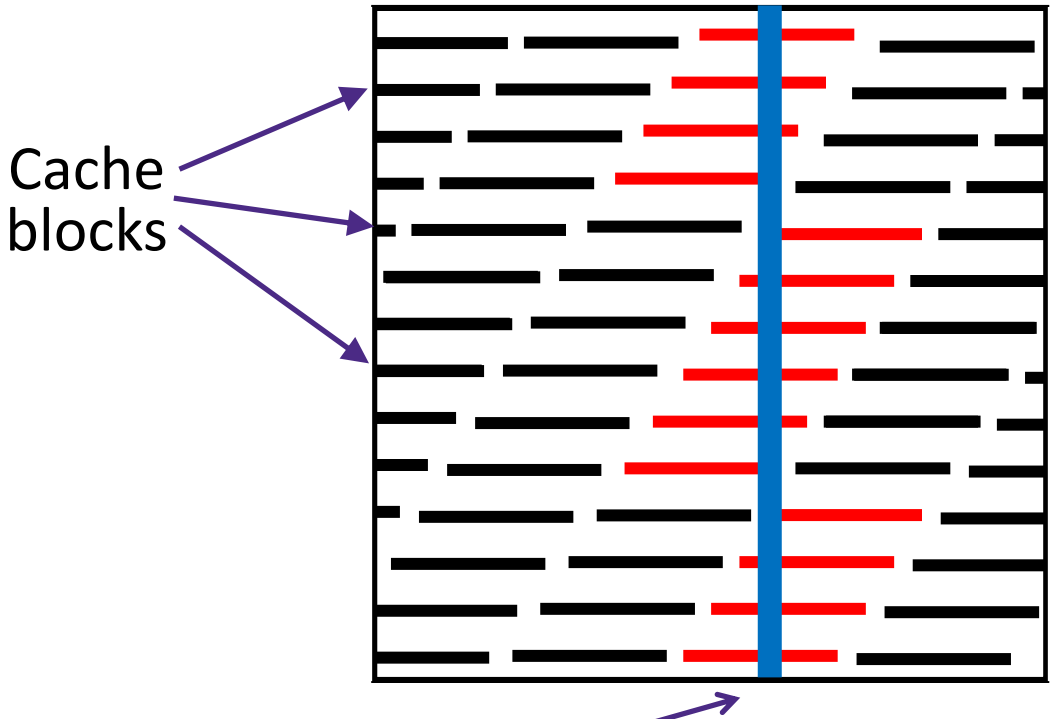
# Example: Matrix Multiplication




$$c_{ij} = \sum_{k=1}^{n} a_{ik}.b_{kj}$$

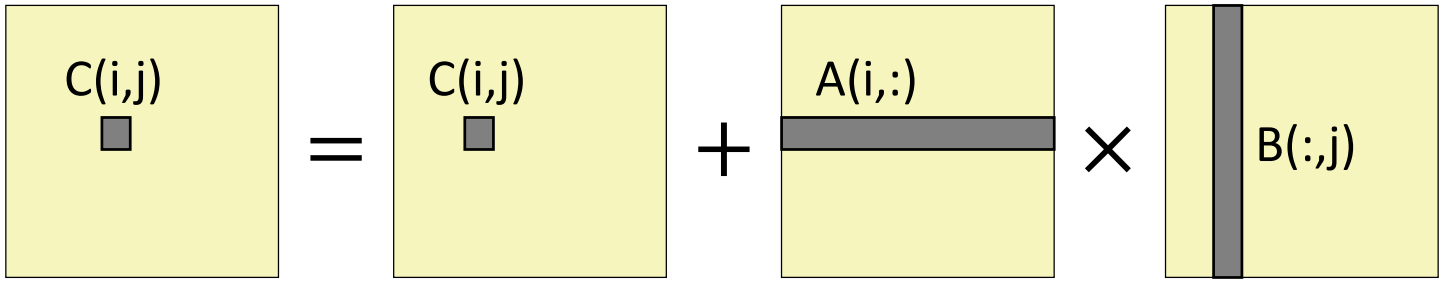
# Matrices in Memory

- How do cache blocks fit into this scheme?
  - Row major matrix in memory:



# Naïve Matrix Multiply

```
# move along rows of A
for (i = 0; i < n; i++)
  # move along columns of B
  for (j = 0; j < n; j++)
    # EACH k loop reads row of A, col of B
    # Also read & write c(i,j) n times
    for (k = 0; k < n; k++)
      c[i*n+j] += a[i*n+k] * b[k*n+j];
```

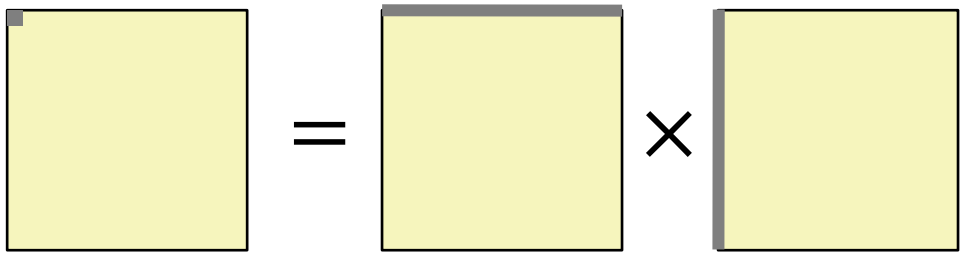


# Cache Miss Analysis (Naïve)

Ignoring matrix `c`

- Scenario Parameters:
  - Square matrix ($n \times n$), elements are `doubles`
  - Cache block size $K$ = 64 B = 8 doubles
  - Cache size $C \ll n$ (much smaller than $n$)

- Each iteration:
  - $\frac{n}{8} + n = \frac{9n}{8}$ misses



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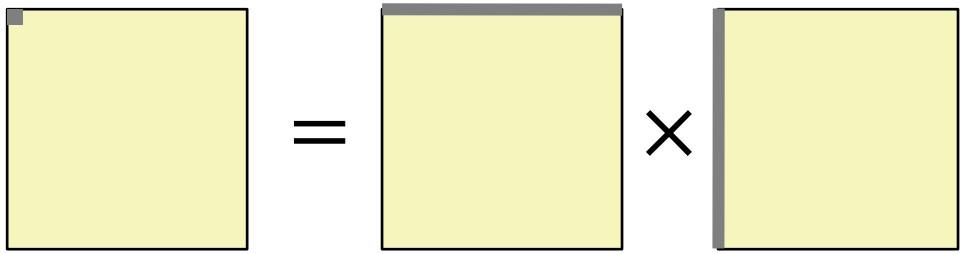
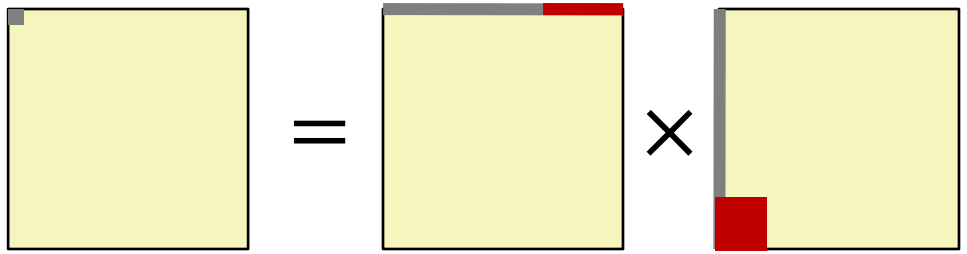


- Afterwards in cache: (schematic)



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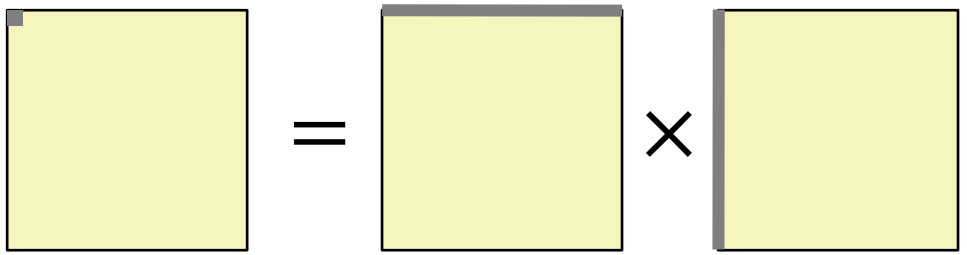

- Total misses: $\frac{9n}{8} \times n^2 = \frac{9}{8}n^3$

once per element

# Linear Algebra to the Rescue (1)

This is extra (non-testable) material

- Can get the same result of a matrix multiplication by splitting the matrices into smaller submatrices (matrix "blocks")
- For example, multiply two 4×4 matrices:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \text{ with } B \text{ defined similarly.}$$

$$AB = \begin{bmatrix} (A_{11}B_{11} + A_{12}B_{21}) & (A_{11}B_{12} + A_{12}B_{22}) \\ (A_{21}B_{11} + A_{22}B_{21}) & (A_{21}B_{12} + A_{22}B_{22}) \end{bmatrix}$$

# Linear Algebra to the Rescue (2)

This is extra (non-testable) material

| | | | |
|---|---|---|---|
| $C_{11}$ | $C_{12}$ | $C_{13}$ | $C_{14}$ |
| $C_{21}$ | $C_{22}$ | $C_{23}$ | $C_{24}$ |
| $C_{31}$ | $C_{32}$ | $C_{43}$ | $C_{34}$ |
| $C_{41}$ | $C_{42}$ | $C_{43}$ | $C_{44}$ |

| | | | |
|---|---|---|---|
| $A_{11}$ | $A_{12}$ | $A_{13}$ | $A_{14}$ |
| $A_{21}$ | $A_{22}$ | $A_{23}$ | $A_{24}$ |
| $A_{31}$ | $A_{32}$ | $A_{33}$ | $A_{34}$ |
| $A_{41}$ | $A_{42}$ | $A_{43}$ | $A_{144}$ |

| | | | |
|---|---|---|---|
| $B_{11}$ | $B_{12}$ | $B_{13}$ | $B_{14}$ |
| $B_{21}$ | $B_{22}$ | $B_{23}$ | $B_{24}$ |
| $B_{32}$ | $B_{32}$ | $B_{33}$ | $B_{34}$ |
| $B_{41}$ | $B_{42}$ | $B_{43}$ | $B_{44}$ |

Matrices of size $n \times n$, split into 4 blocks of size $r$ ($n$=$4r$)

$$C_{22} = A_{21}B_{12} + A_{22}B_{22} + A_{23}B_{32} + A_{24}B_{42} = \sum_k A_{2k} * B_{k2}$$

- Multiplication operates on small "block" matrices
  - Choose size so that they fit in the cache!
  - This technique called "*cache blocking*"

# Blocked Matrix Multiply

- Blocked version of the naïve algorithm:

```
# move by rxr BLOCKS now
for (i = 0; i < n; i += r)
  for (j = 0; j < n; j += r)
    for (k = 0; k < n; k += r)
      # block matrix multiplication
      for (ib = i; ib < i+r; ib++)
        for (jb = j; jb < j+r; jb++)
          for (kb = k; kb < k+r; kb++)
            c[ib*n+jb] += a[ib*n+kb]*b[kb*n+jb];
```

  - $r$ = block matrix size (assume $r$ divides $n$ evenly)

# Cache Miss Analysis (Blocked)

Ignoring matrix C

- Scenario Parameters:
  - Cache block size $K$ = 64 B = 8 doubles
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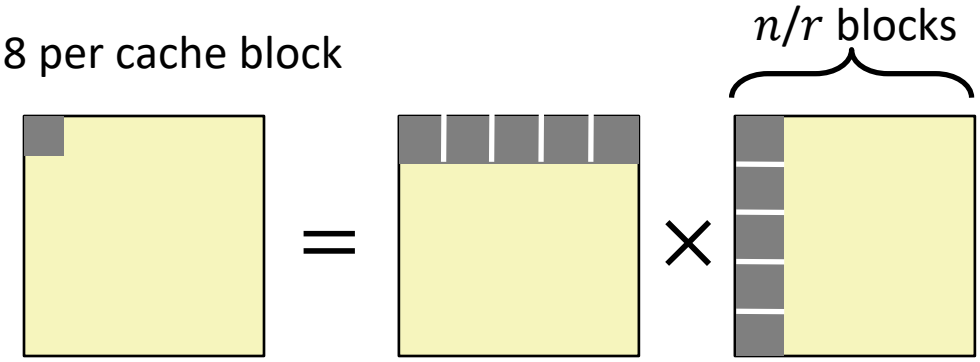
  - Three blocks ■ ($r \times r$) fit into cache: $3r^2 < C$

$r^2$ elements per block, 8 per cache block

- Each block iteration:
  - $r^2/8$ misses per block
  - $2n/r \times r^2/8 = nr/4$

$n/r$ blocks in row and column



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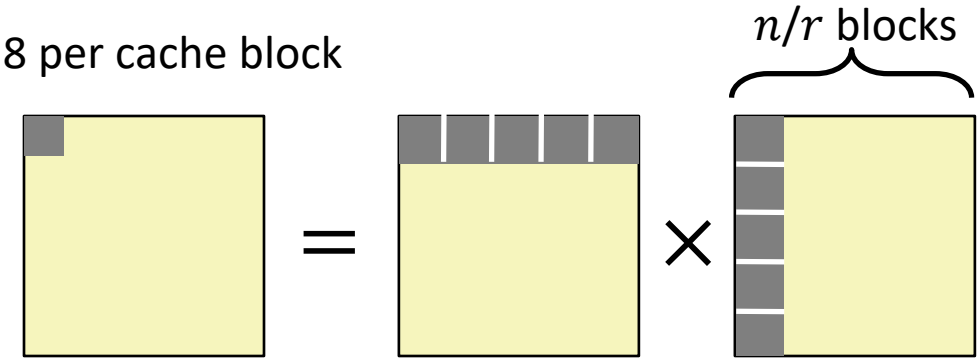
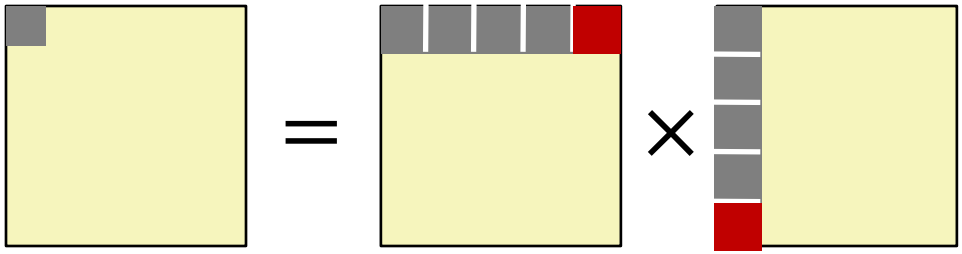



  - Afterwards in cache (schematic)



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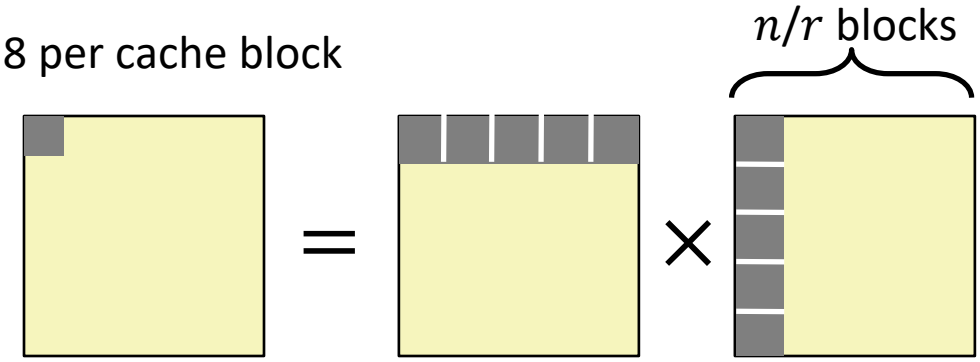
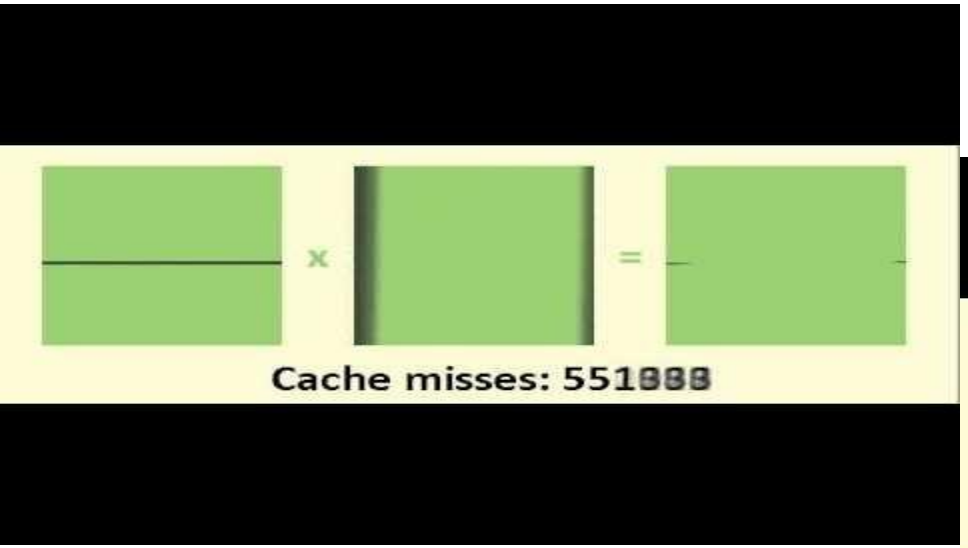



- Total misses:
  - $nr/4 \times (n/r)2 = n^3/(4r)$

# Matrix Multiply Visualization

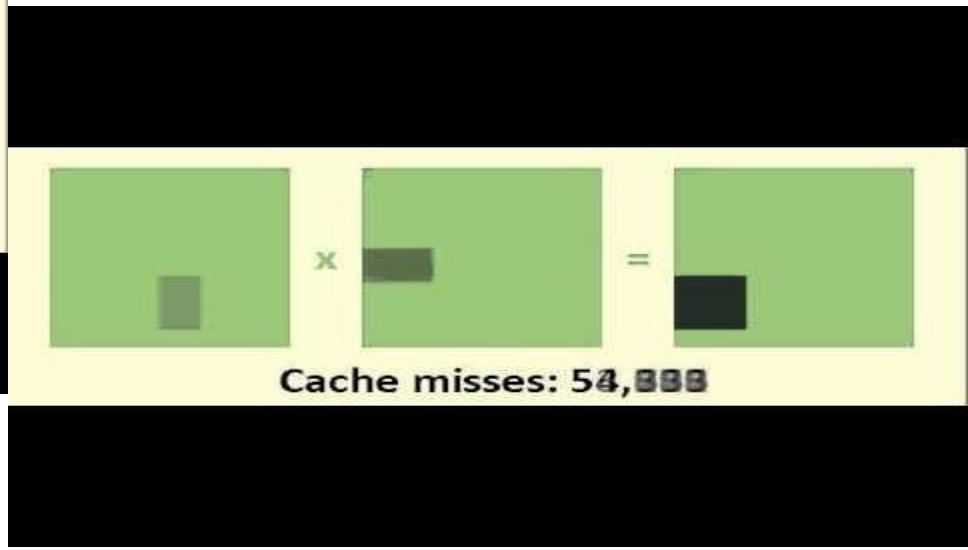
- Here $n$ = 100, $C$ = 32 KiB, $r$ = 30

**Naïve:**




≈ 1,020,000
cache misses

**Blocked:**



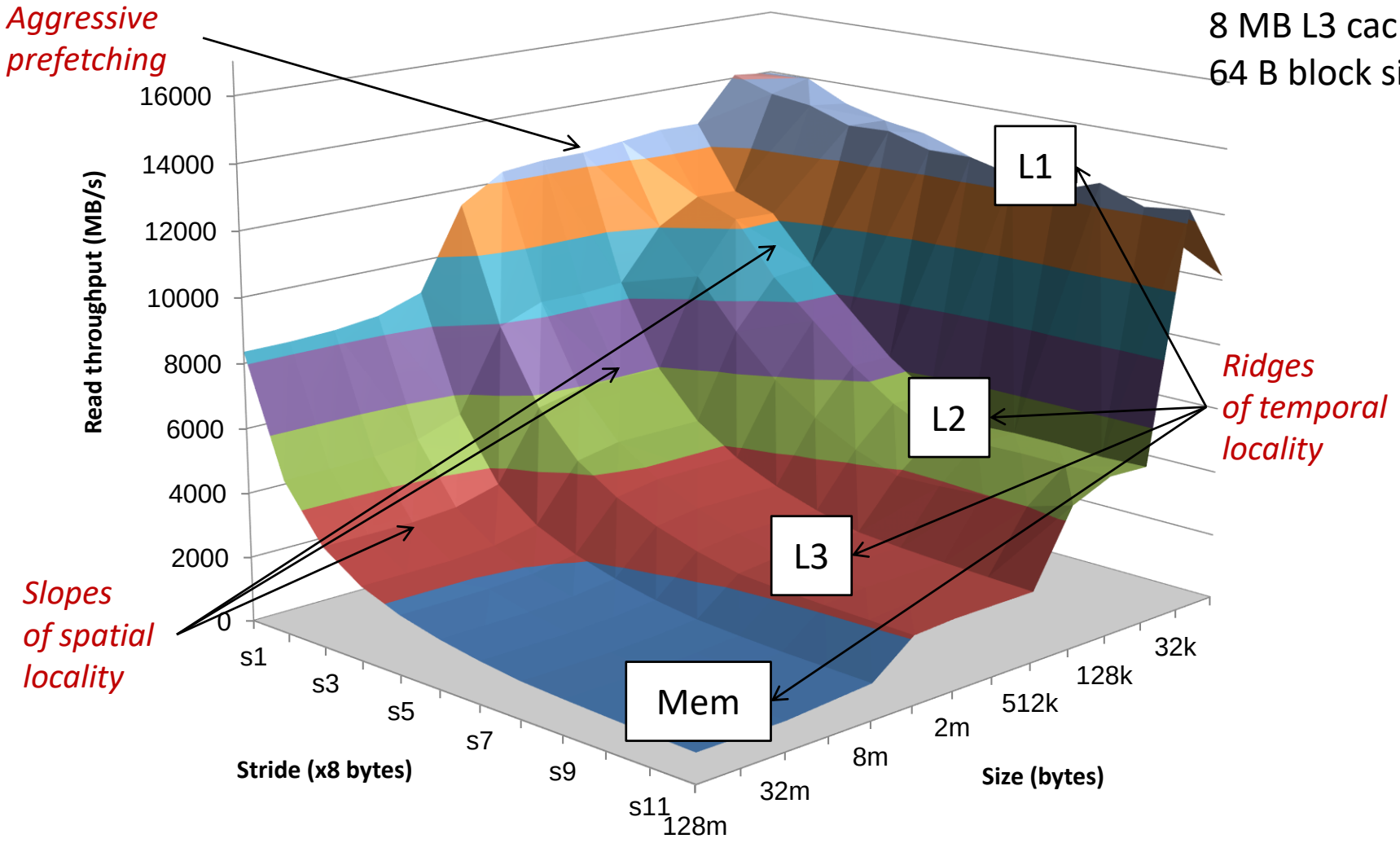

≈ 90,000
cache misses

# Cache-Friendly Code

- Programmer can optimize for cache performance
  - How data structures are organized
  - How data are accessed
    - Nested loop structure
    - Blocking is a general technique
- All systems favor "cache-friendly code"
  - Getting absolute optimum performance is very platform specific
    - Cache size, cache block size, associativity, etc.
  - Can get most of the advantage with generic code
    - Keep working set reasonably small (temporal locality)
    - Use small strides (spatial locality)
    - Focus on inner loop code

# The Memory Mountain

Core i7 Haswell
2.1 GHz
32 KB L1 d-cache
256 KB L2 cache
8 MB L3 cache
64 B block size



# Learning About Your Machine

- **Linux:**
  - `lscpu`
  - ls /sys/devices/system/cpu/cpu0/cache/index0/
    - Ex: cat /sys/devices/system/cpu/cpu0/cache/index*/size
- **Windows:**
  - `wmic memcache get <query>` (all values in KB)
  - Ex: `wmic memcache get MaxCacheSize`

- Modern processor specs: http://www.7-cpu.com/