

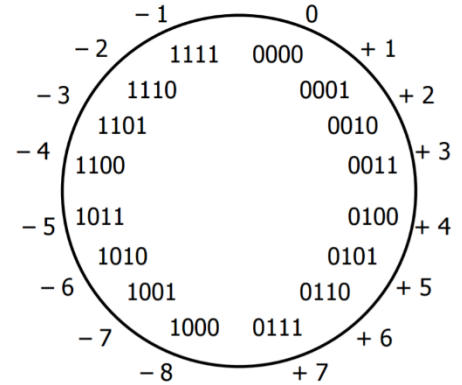
# CSE 351 Section 3 – Integers and Floating Point

Welcome back to section, we're happy that you're here ☺ .....

## Signed Integers with Two's Complement

Two's complement is the standard for representing signed integers:

- The most significant bit (MSB) has a negative value; all others have positive values (same as unsigned)
- Binary addition is performed the same way for signed and unsigned
- The bit representation for the negative value (additive inverse) of a Two's Complement number can be found by:  
flipping all the bits and adding 1 (i.e.  $-x = \sim x + 1$ ).



The “number wheel” showing the relationship between 4-bit numerals and their Two's Complement interpretations is shown on the right:

- The largest number is 7 whereas the smallest number is -8
- There is a nice symmetry between numbers and their negative counterparts except for -8

### Exercises: (assume 8-bit integers)

1) What is the **largest integer**? The **largest integer + 1**?

<u>Unsigned:</u>	<u>Two's Complement:</u>
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2) How do you represent (if possible) the following numbers: **39, -39, 127**?

<u>Unsigned:</u>	<u>Two's Complement:</u>
39:	39:
-39:	-39:
127:	127:

3) Compute the following sums in binary using your Two's Complement answers from above. *Answer in hex.*

<b>a.</b> 39 -> 0b _____ + (-39) -> 0b _____ 0x __ <- 0b _____	<b>b.</b> 127 -> 0b _____ + (-39) -> 0b _____ 0x __ <- 0b _____
<b>c.</b> 39 -> 0b _____ - 127 -> 0b _____ 0x __ <- 0b _____	<b>d.</b> 127 -> 0b _____ + 39 -> 0b _____ 0x __ <- 0b _____

4) Interpret each of your answers above and indicate whether-or-not overflow has occurred.

<b>a.</b> 39+(-39) Unsigned: Two's Complement:	<b>b.</b> 127+(-39) Unsigned: Two's Complement:
<b>c.</b> 39-127 Unsigned: Two's Complement:	<b>d.</b> 127+39 Unsigned: Two's Complement:

## Goals of Floating Point

Representation should include: [1] a large range of values (both very small and very large numbers), [2] a high amount of precision, and [3] real arithmetic results (e.g.  $\infty$  and NaN).

## IEEE 754 Floating Point Standard

The value of a real number can be represented in scientific binary notation as:

$$\text{Value} = (-1)^{\text{sign}} \times \text{Mantissa}_2 \times 2^{\text{Exponent}} = (-1)^S \times 1.M_2 \times 2^{E-\text{bias}}$$

The binary representation for floating point values uses three fields:

- **S**: encodes the *sign* of the number (0 for positive, 1 for negative)
- **E**: encodes the *exponent* in **biased notation** with a bias of  $2^{w-1}-1$
- **M**: encodes the *mantissa* (or *significand*, or *fraction*) – stores the fractional portion, but does not include the implicit leading 1.

	S	E	M
float	1 bit	8 bits	23 bits
double	1 bit	11 bits	52 bits

How a float is interpreted depends on the values in the exponent and mantissa fields:

E	M	Meaning
0	anything	denormalized number (denorm)
1-254	anything	normalized number
255	zero	infinity ( $\infty$ )
255	nonzero	not-a-number (NaN)

### Exercises:

5) What is the largest, finite, positive value that can be stored using a float?

6) What is the smallest, positive, normalized value that can be stored using float?

7) Convert the decimal number 1.25 into single precision floating point representation:

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8) What are the decimal values of the following floats?

0x80000000

0xFF94BEEF

0x41180000

## Floating Point Mathematical Properties

- Not associative:  $(2 + 2^{50}) - 2^{50} \neq 2 + (2^{50} - 2^{50})$
- Not distributive:  $100 \times (0.1 + 0.2) \neq 100 \times 0.1 + 100 \times 0.2$
- Not cumulative:  $2^{25} + 1 + 1 + 1 + 1 \neq 2^{25} + 4$

### Exercises:

9) Based on floating point representation, explain why each of the three statements above occurs.

10) If `x` and `y` are variable type `float`, give two *different* reasons why `(x+2*y) - y == x+y` might evaluate to false.

# 1EEE 754 Float (32 bit) Flowchart

