Caches

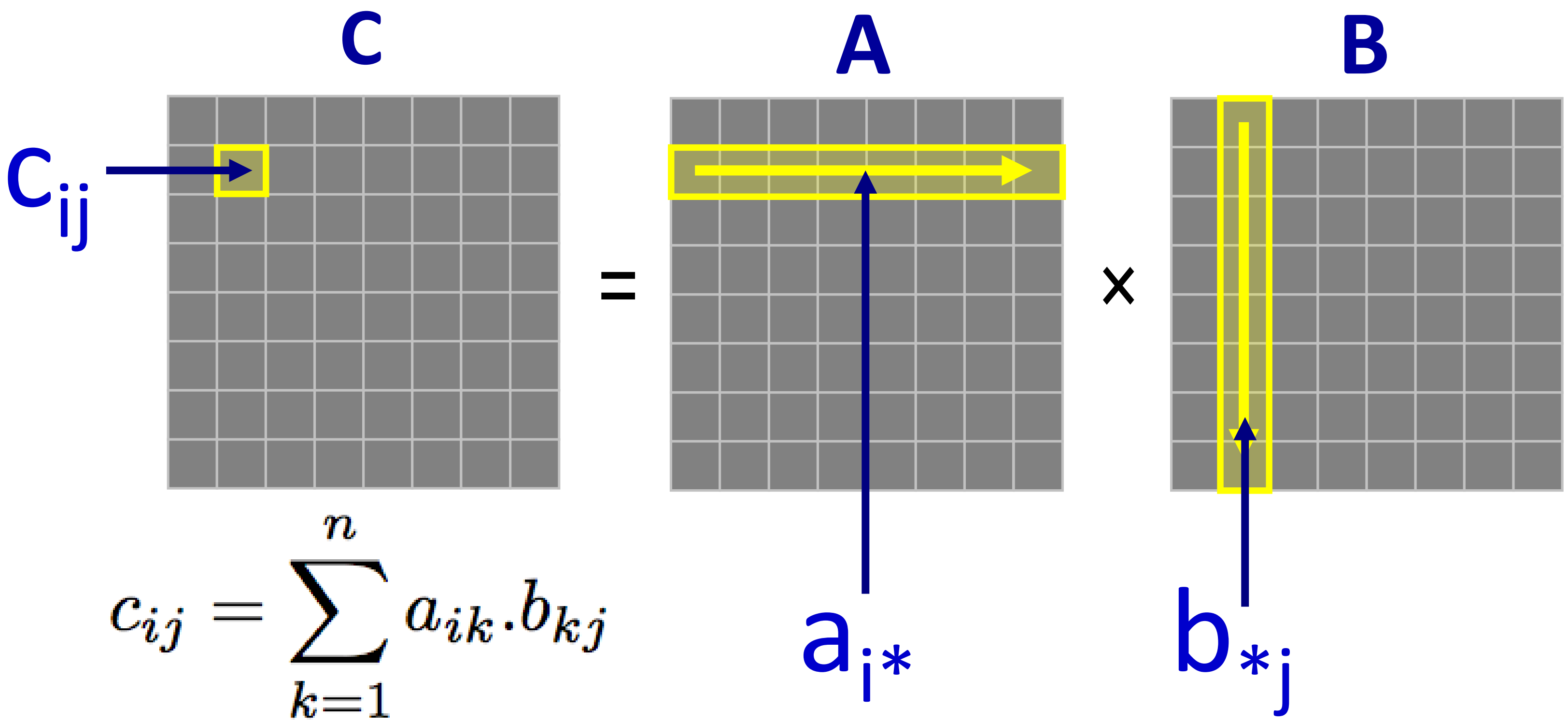
Making them work for you

Optimizations for the Memory Hierarchy

- Write code that has locality!
  - Spatial: access data contiguously
  - Temporal: make sure access to the same data is not too far apart in time

- How can you achieve locality?
  - Adjust memory accesses in *code* (software) to improve miss rate (MR)
    - Requires knowledge of *both* how caches work as well as your system's parameters
  - Proper choice of algorithm
  - Loop transformations

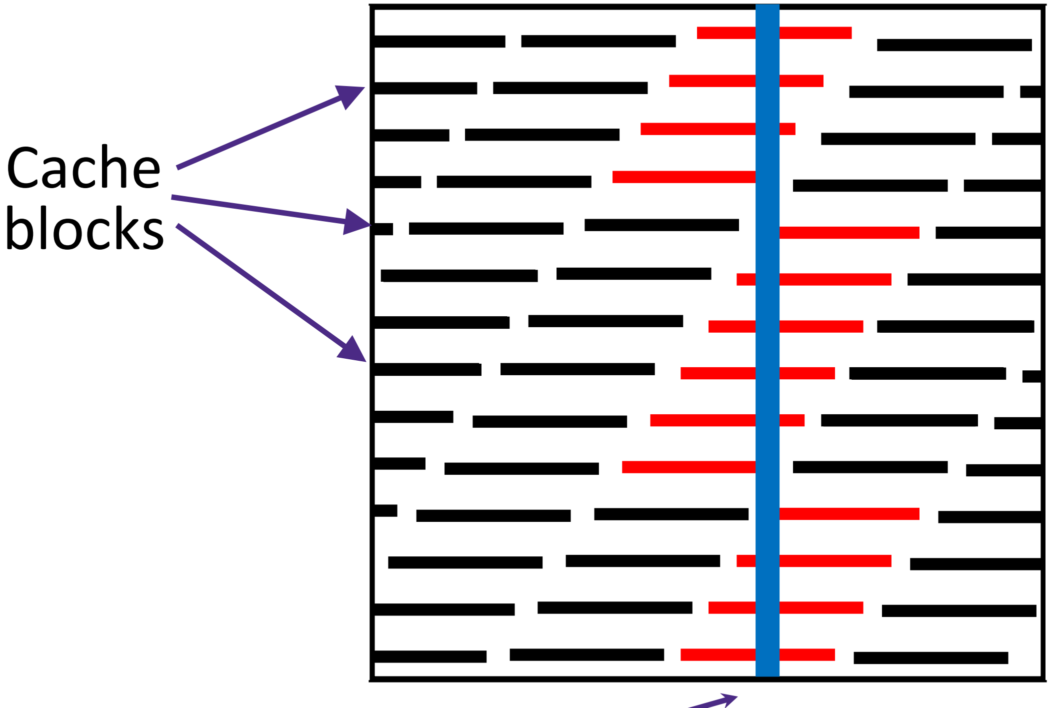
Example: Matrix Multiplication



$$c_{ij} = \sum_{k=1}^{n} a_{ik} . b_{kj}$$

Matrices in Memory

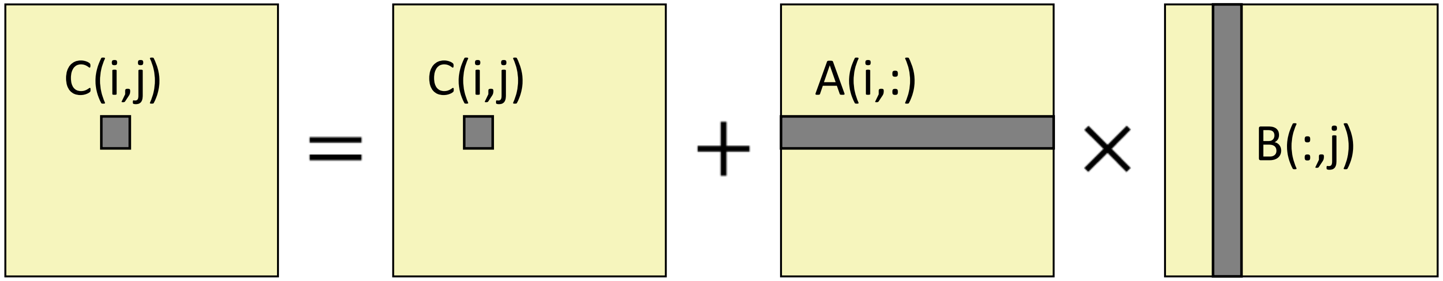
- How do cache blocks fit into this scheme?
  - Row major matrix in memory:



COLUMN of matrix (blue) is spread among cache blocks shown in red

Naïve Matrix Multiply

```
# move along rows of A
for (i = 0; i < n; i++)
  # move along columns of B
  for (j = 0; j < n; j++)
    # EACH k loop reads row of A, col of B
    # Also read & write c(i,j) n times
    for (k = 0; k < n; k++)
      c[i*n+j] += a[i*n+k] * b[k*n+j];
```

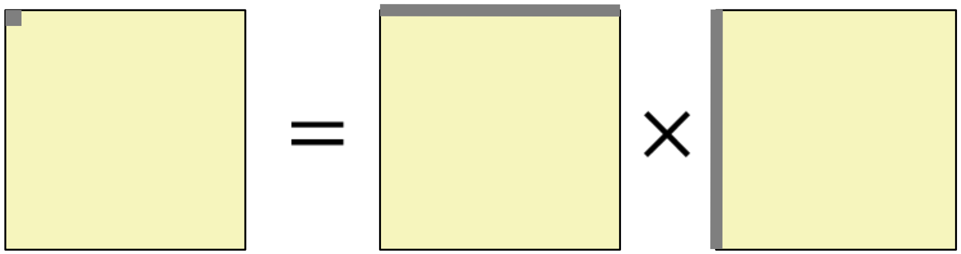


Cache Miss Analysis (Naïve)

Ignoring matrix c

- Scenario Parameters:
  - Square matrix ($n \times n$), elements are `doubles`
  - Cache block size K = 64 B = 8 doubles
  - Cache size $C \ll n$ (much smaller than n)

- Each iteration:
  - $\frac{n}{8} + n = \frac{9n}{8}$ misses



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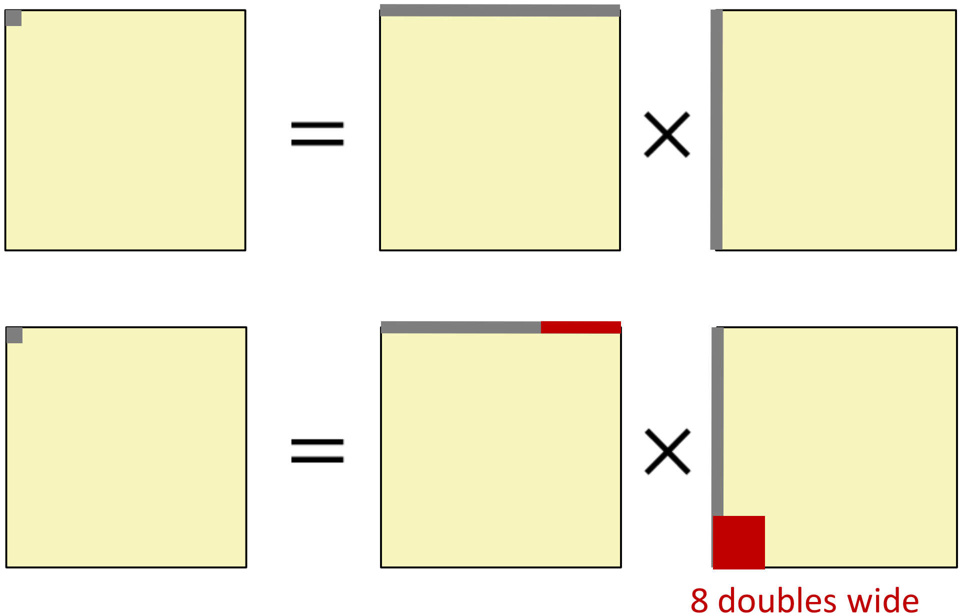
  - Afterwards in cache: (schematic)



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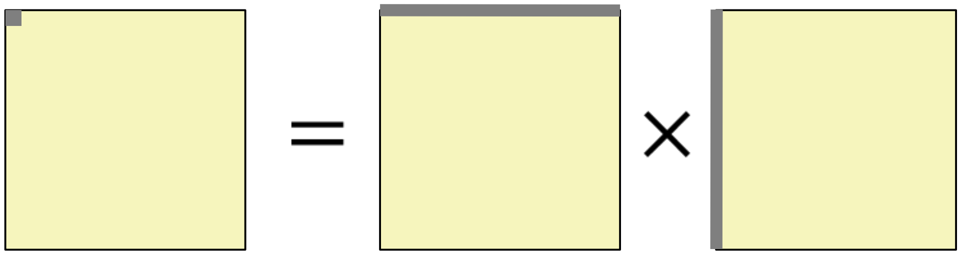

- Total misses: $\frac{9n}{8} \times n^2 = \frac{9}{8}n^3$

once per element

Linear Algebra to the Rescue (1)

This is extra (non-testable) material

- Can get the same result of a matrix multiplication by splitting the matrices into smaller submatrices (matrix "blocks")

- For example, multiply two 4×4 matrices:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \text{ with } B \text{ defined similarly.}$$

$$AB = \begin{bmatrix} (A_{11}B_{11} + A_{12}B_{21}) & (A_{11}B_{12} + A_{12}B_{22}) \\ (A_{21}B_{11} + A_{22}B_{21}) & (A_{21}B_{12} + A_{22}B_{22}) \end{bmatrix}$$

Linear Algebra to the Rescue (2)

This is extra (non-testable) material

C_{11}	C_{12}	C_{13}	C_{14}
C_{21}	C_{22}	C_{23}	C_{24}
C_{31}	C_{32}	C_{43}	C_{34}
C_{41}	C_{42}	C_{43}	C_{44}

A_{11}	A_{12}	A_{13}	A_{14}
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B_{11}	B_{12}	B_{13}	B_{14}
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Matrices of size $n \times n$, split into 4 blocks of size r (n=4r)

$$C_{22} = A_{21}B_{12} + A_{22}B_{22} + A_{23}B_{32} + A_{24}B_{42} = \sum_k A_{2k}*B_{k2}$$

- Multiplication operates on small "block" matrices
  - Choose size so that they fit in the cache!
  - This technique called "*cache blocking*"

Blocked Matrix Multiply

- Blocked version of the naïve algorithm:

```
# move by rxr BLOCKS now
for (i = 0; i < n; i += r)
  for (j = 0; j < n; j += r)
    for (k = 0; k < n; k += r)
      # block matrix multiplication
      for (ib = i; ib < i+r; ib++)
        for (jb = j; jb < j+r; jb++)
          for (kb = k; kb < k+r; kb++)
            c[ib*n+jb] += a[ib*n+kb]*b[kb*n+jb];
```

  - r = block matrix size (assume r divides n evenly)

Cache Miss Analysis (Blocked)

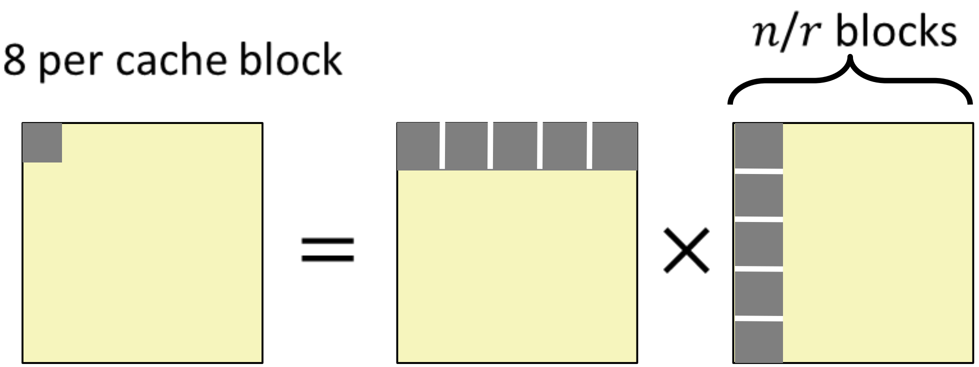
Ignoring matrix c

- Scenario Parameters:
  - Cache block size K = 64 B = 8 doubles
  - Cache size $C \ll n$ (much smaller than n)
  - Three blocks ■ $(r \times r)$ fit into cache: $3r^2 < C$

r^2 elements per block, 8 per cache block

- Each block iteration:
  - $r^2/8$ misses per block
  - $2n/r \times r^2/8 = nr/4$

n/r blocks in row and column



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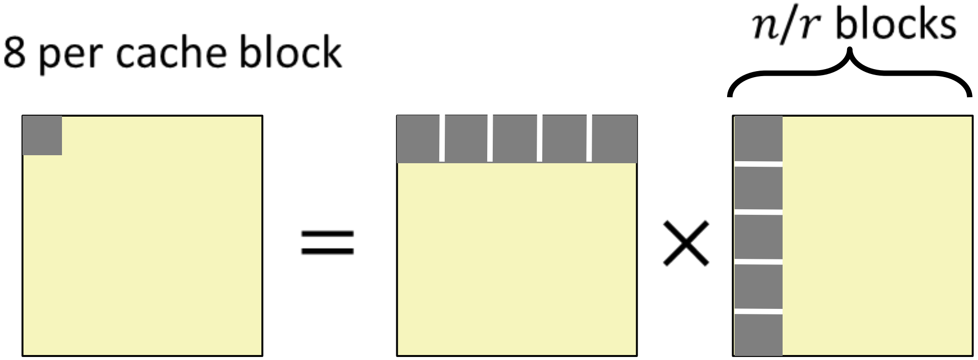
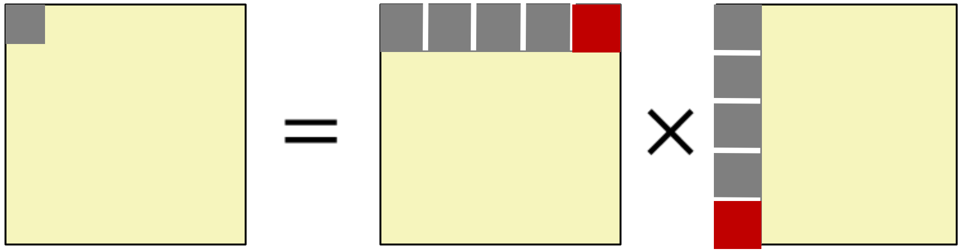


n/r blocks in row and column

  - Afterwards in cache (schematic)



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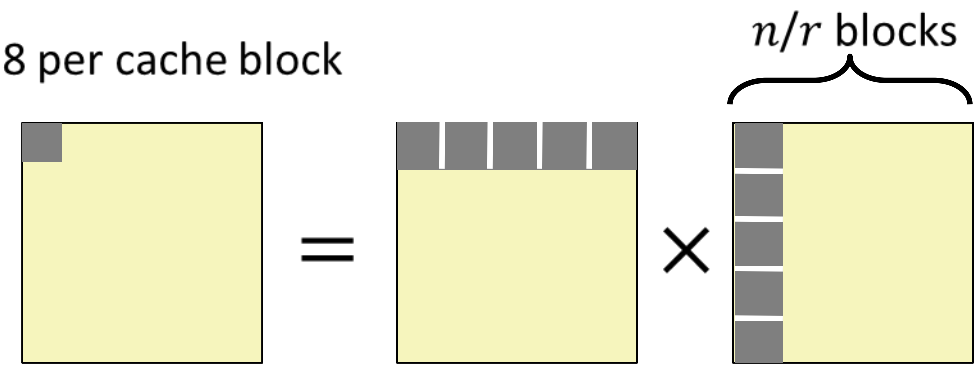

- Total misses: $nr/4 \times (n/r)^2 = n^3/(4r)$

Cache-Friendly Code

- Programmer can optimize for cache performance
  - How data structures are organized
  - How data are accessed
    - Nested loop structure
    - Blocking is a general technique
- All systems favor "cache-friendly code"
  - Getting absolute optimum performance is very platform specific
    - Cache size, cache block size, associativity, etc.
  - Can get most of the advantage with generic code
    - Keep working set reasonably small (temporal locality)
    - Use small strides (spatial locality)
    - Focus on inner loop code