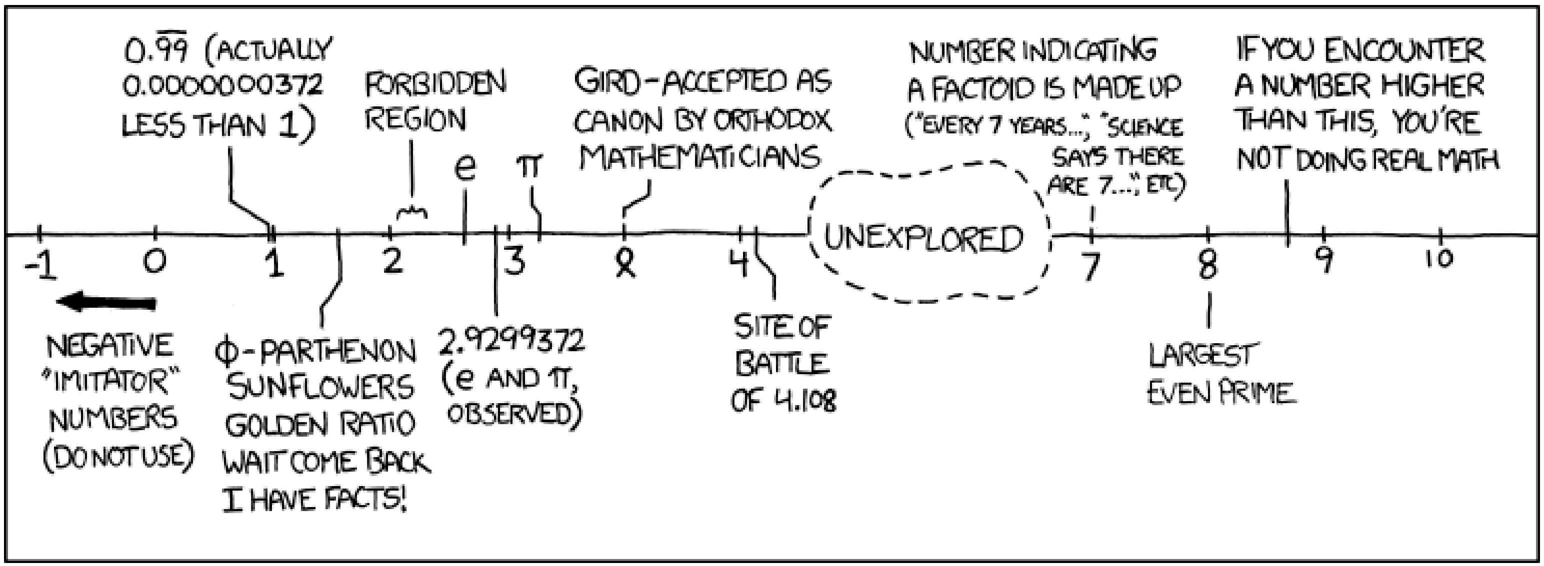
Floating Point II, x86-64 Intro

CSE 351 Spring 2018



http://xkcd.com/899/

Administrivia

- Lab 1 due Friday
  - Submit `bits.c`, `pointer.c`, `lab1reflect.txt`
- Homework 2 due following Tuesday
  - On Integers, Floating Point, and x86-64

Floating point topics

- Fractional binary numbers
- IEEE floating-point standard
- **Floating-point operations and rounding**
- Floating-point in C

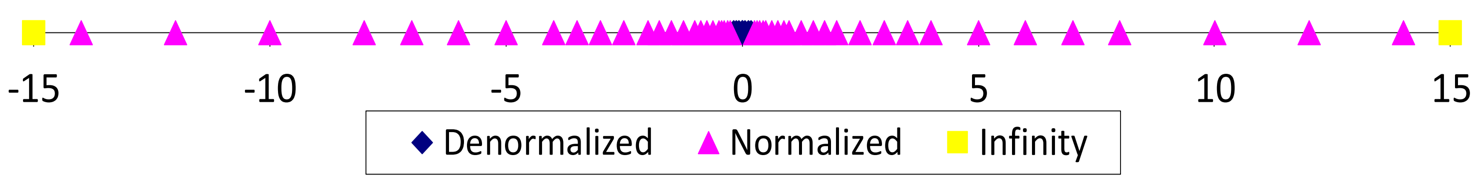
- There are many more details that we won't cover
  - It's a 58-page standard…

Floating Point Encoding Summary

E	M	Meaning
0x00	0	± 0
0x00	non-zero	± denorm num
0x01 – 0xFE	anything	± norm num
0xFF	0	± ∞
0xFF	non-zero	NaN

Distribution of Values

- What ranges are NOT representable?
  - Between largest norm and infinity **Overflow** (Exp too large)
  - Between zero and smallest denorm **Underflow** (Exp too small)
  - Between norm numbers? **Rounding**
- Given a FP number, what's the bit pattern of the next largest representable number?
  - What is this "step" when Exp = 0?
  - What is this "step" when Exp = 100?
- Distribution of values is denser toward zero



Floating Point Operations: Basic Idea

Value = $(-1)^{S} \times \text{Mantissa} \times 2^{\text{Exponent}}$

- $x +_f y = \text{Round}(x + y)$
- $x *_f y = \text{Round}(x * y)$

- Basic idea for floating point operations:
  - First, compute the exact result
  - Then *round* the result to make it fit into desired precision:
    - Possibly over/underflow if exponent outside of range
    - Possibly drop least-significant bits of mantissa to fit into M bit vector

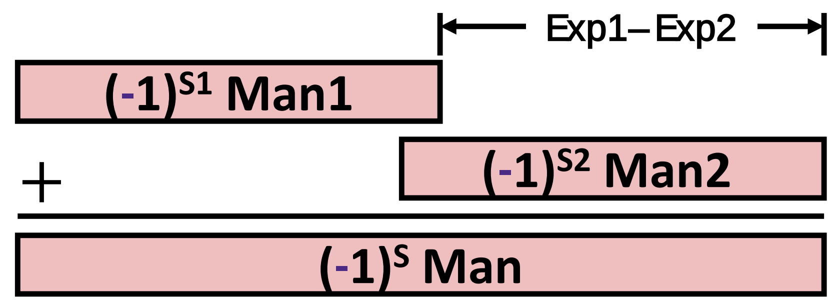
Floating Point Addition

Line up the binary points!

- $(-1)^{S1} \times Man1 \times 2^{Exp1} + (-1)^{S2} \times Man2 \times 2^{Exp2}$
  - Assume Exp1 > Exp2

```
  1.010*2^2         1.0100*2^2
+ 1.000*2^-1      + 0.0010*2^2
  ???               1.0110*2^2
```

- Exact Result: $(-1)^{S} \times Man \times 2^{Exp}$
  - Sign S, mantissa Man:
    - Result of signed align & add
  - Exponent E: E1



- Adjustments:
  - If Man ≥ 2, shift Man right, increment Exp
  - If Man < 1, shift Man left k positions, decrement Exp by k
  - Over/underflow if Exp out of range
  - Round Man to fit mantissa precision

Floating Point Multiplication

- $(-1)^{S1} \times Man1 \times 2^{Exp1} \times (-1)^{S2} \times Man2 \times 2^{Exp2}$

- Exact Result:
  - Sign S: S1 ^ S2
  - Mantissa Man: Man1 × Man2
  - Exponent Exp: Exp1 + Exp2

- Adjustments:
  - If Man ≥ 2, shift Man right, increment Exp
  - Over/underflow if Exp out of range
  - Round Man to fit mantissa precision

Mathematical Properties of FP Operations

- Exponent overflow yields +∞ or -∞
- Floats with value +∞, -∞, and NaN can be used in operations
  - Result usually still +∞, -∞, or NaN; but not always intuitive

- Floating point operations do not work like real math, due to *rounding* – any programmer using floats in any language *must* understand these issues!

Mathematical Properties of FP Operations

Rounding issue 1: No exact representation of some "fractions"

(1.0 / 3) * 3 != 1.0

Mathematical Properties of FP Operations

Rounding issue 2: Limited mantissa means lost precision

(3.14 + 1e100) – 1e100 == 0.0

Addition/subtraction no longer *associative*

3.14 + (1e100 – 1e100) == 3.14

Mathematical Properties of FP Operations

Lack of associativity can be worse with loops:

```
float x = huge_number;
for(i=0; i < large_number; i++)
    x += small_number;
```

vs.

```
float x = 0;
for(i=0; i < large_number; i++)
    x += small_number;
x += huge_number;
```

Mathematical Properties of FP Operations

Rounding issue 3: No distributivity either

```
printf("%.20f %.20f\n",
       100*(0.1+0.2),
       100*0.1 + 100*0.2);
```

30.00000000000000355271

30.00000000000000000000

Floating-point guidelines

!!!

- Assume possible small rounding at *every* operation
  - Can *compound* across many operations

- Also beware overflow (infinity) and underflow (zero)

- Never compare floats for equality (cf. rounding)
  - Compiler won't complain, but a very likely bug (!)
  - Ask if |e1-e2| is "small" for some "small" you care about

- This and preceding slides are the "key takeaways"
  - Justified by your understanding of the bit-representation and the trade-offs it is dealing with
  - Floats work fine for simple stuff, else hard to do mathematically correct things (cf. *numerical analysis*)

Floating point topics

- Fractional binary numbers
- IEEE floating-point standard
- Floating-point operations and rounding
- **Floating-point in C**

- There are many more details that we won't cover
  - It's a 58-page standard…

Floating Point in C

- C offers two (well, 3) levels of precision

`float`	`1.0f`	single precision (32-bit)
`double`	`1.0`	double precision (64-bit)
`long double`	`1.0L`	*("double double" or quadruple)* precision (64-128 bits)

- `#include <math.h>` to get `INFINITY` and `NAN` constants
- Equality (==) comparisons are allowed but shouldn't be used
  - Interesting tidbit: 0.0 == -0.0 despite different bits

Floating Point Conversions in C

!!!

- Casting between `int`, `float`, and `double` **changes** the bit representation
  - `int` → `float`
    - May be rounded (not enough bits in mantissa: 23)
    - Overflow impossible
  - `int` or `float` → `double`
    - Exact conversion (all 32-bit `int`s representable)
  - `long` → `double`
    - Depends on word size (32-bit is exact, 64-bit may be rounded)
  - `double` or `float` → `int`
    - Truncates fractional part (rounded toward zero)
    - "Not defined" when out of range or NaN: generally sets to `Tmin` (even if the value is a very big positive)

Floating Point and the Programmer

```
#include <stdio.h>

int main(int argc, char* argv[]) {
  float f1 = 1.0;
  float f2 = 0.0;
  int i;
  for (i = 0; i < 10; i++)
    f2 += 1.0/10.0;

  printf("0x%08x  0x%08x\n", *(int*)&f1, *(int*)&f2);
  printf("f1 = %10.9f\n", f1);
  printf("f2 = %10.9f\n\n", f2);

  f1 = 1E30;
  f2 = 1E-30;
  float f3 = f1 + f2;
  printf("f1 == f3? %s\n", f1 == f3 ? "yes" : "no" );

  return 0;
}
```

```
$ ./a.out
0x3f800000  0x3f800001
f1 = 1.000000000
f2 = 1.000000119

f1 == f3? yes
```

Floating Point Summary

- Floats also suffer from the fixed number of bits available to represent them
  - Can get overflow/underflow
  - "Gaps" produced in representable numbers means we can lose precision, unlike `int`s
    - Some "simple fractions" have no exact representation (*e.g.* 0.2)
    - "Every operation gets a slightly wrong result"
- Floating point arithmetic not associative or distributive
  - Mathematically equivalent ways of writing an expression may compute different results
- Never test floating point values for equality!
- Careful when converting between `int`s and `float`s!

Number Representation Really Matters

- **1991:** Patriot missile targeting error
  - clock skew due to conversion from integer to floating point
- **1996:** Ariane 5 rocket exploded ($1 billion)
  - overflow converting 64-bit floating point to 16-bit integer
- **2000:** Y2K problem
  - limited (decimal) representation: overflow, wrap-around
- **2038:** Unix epoch rollover
  - Unix epoch = seconds since 12am, January 1, 1970
  - signed 32-bit integer representation rolls over to TMin in 2038
- **Other related bugs:**
  - 1982: Vancouver Stock Exchange 10% error in less than 2 years
  - 1994: Intel Pentium FDIV (floating point division) HW bug ($475 million)
  - 1997: USS Yorktown “smart” warship stranded: divide by zero
  - 1998: Mars Climate Orbiter crashed: unit mismatch ($193 million)

Roadmap

C:

```
car *c = malloc(sizeof(car));
c->miles = 100;
c->gals = 17;
float mpg = get_mpg(c);
free(c);
```

Java:

```
Car c = new Car();
c.setMiles(100);
c.setGals(17);
float mpg =
    c.getMPG();
```

Memory & data
Integers & floats
x86 assembly
Procedures & stacks
Executables
Arrays & structs
Memory & caches
Processes
Virtual memory
Memory allocation
Java vs. C

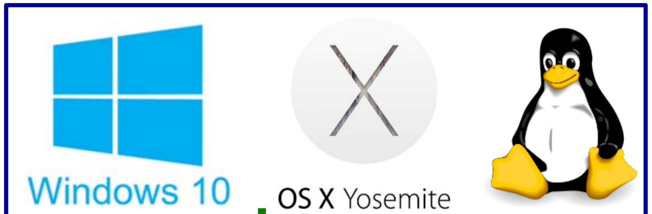
Assembly language:

```
get_mpg:
    pushq   %rbp
    movq    %rsp, %rbp
    ...
    popq    %rbp
    ret
```

Machine code:

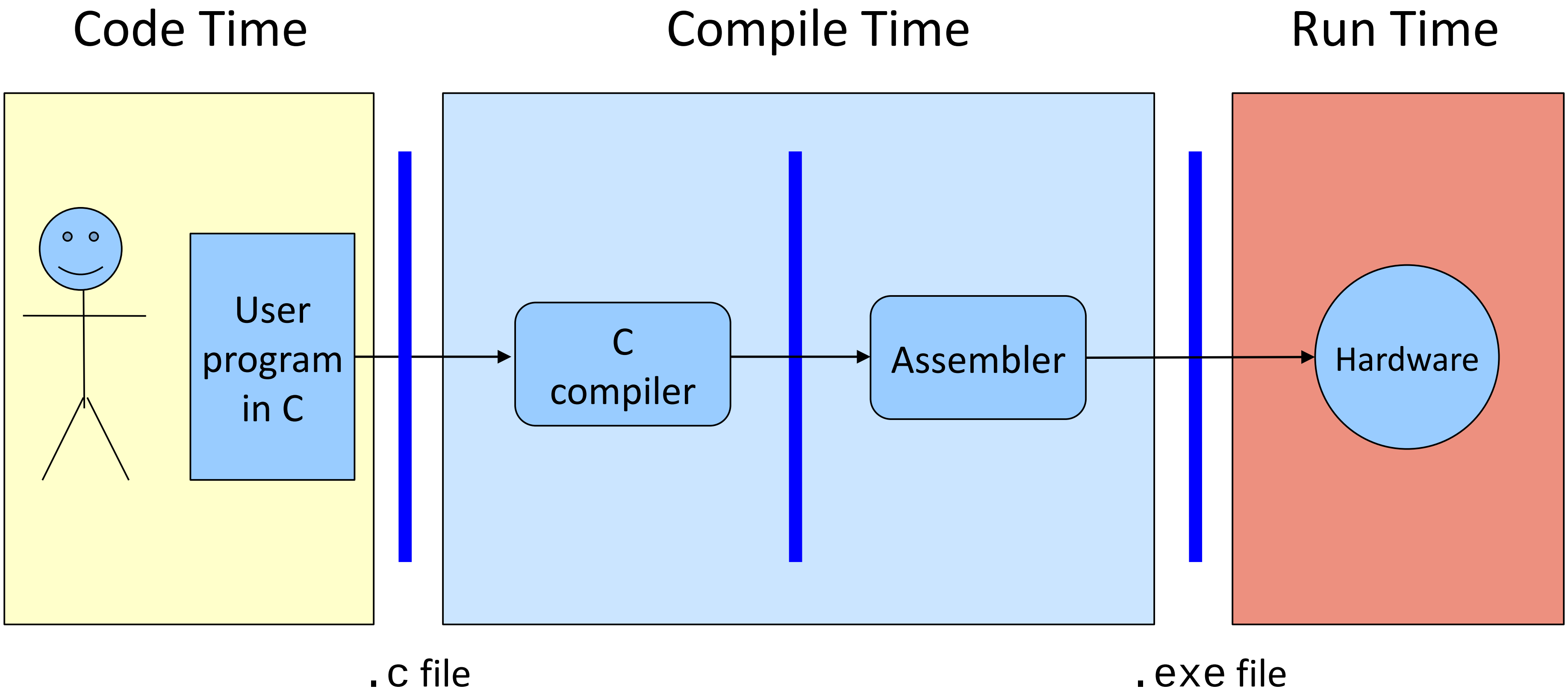
```
0111010000011000
100011010000010000000010
1000100111000010
110000011111101000011111
```

OS:



Computer system:

Translation



What makes programs run fast(er)?

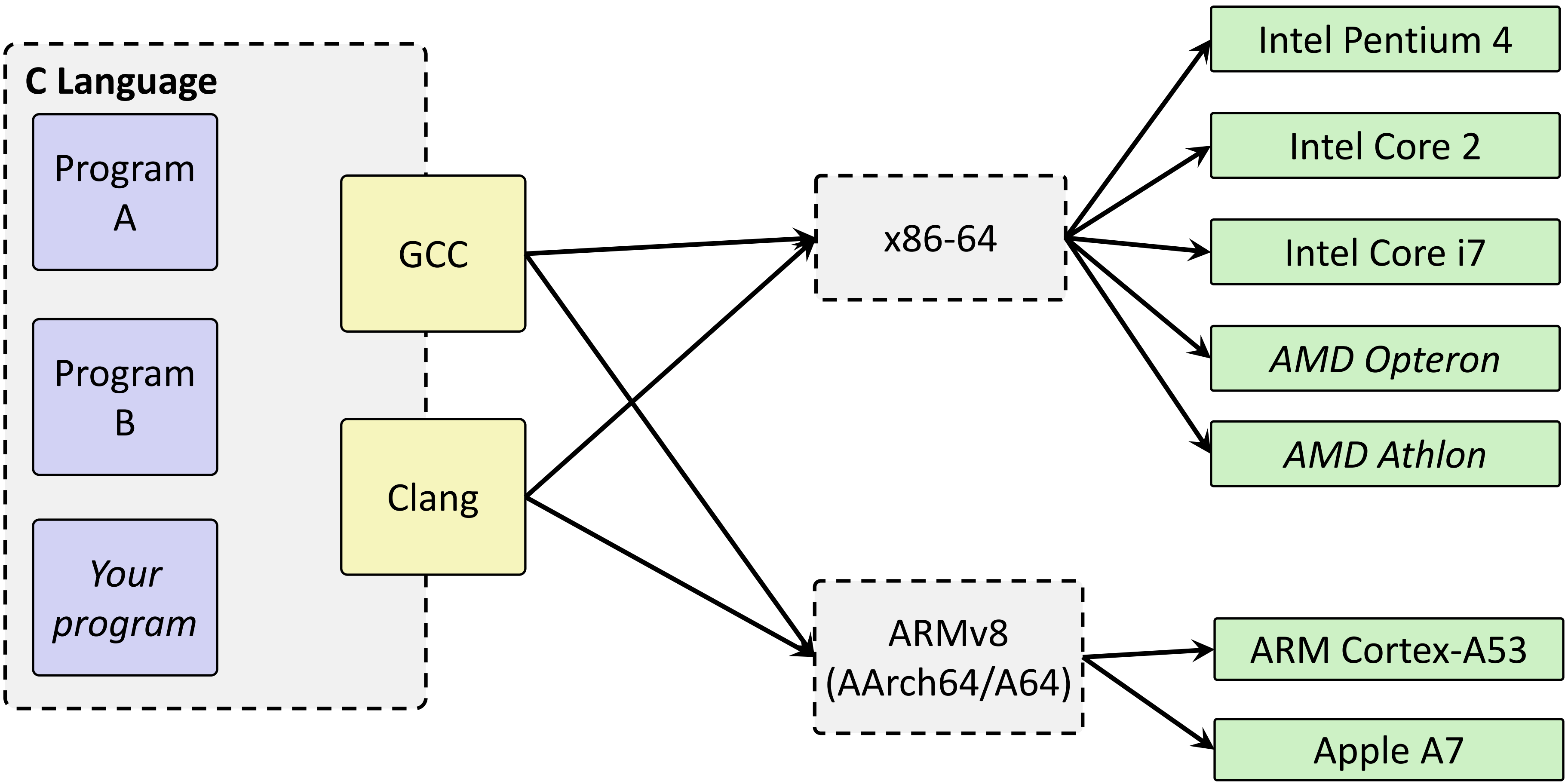
HW Interface Affects Performance

Source code
Different applications or algorithms

Compiler
Perform optimizations, generate instructions

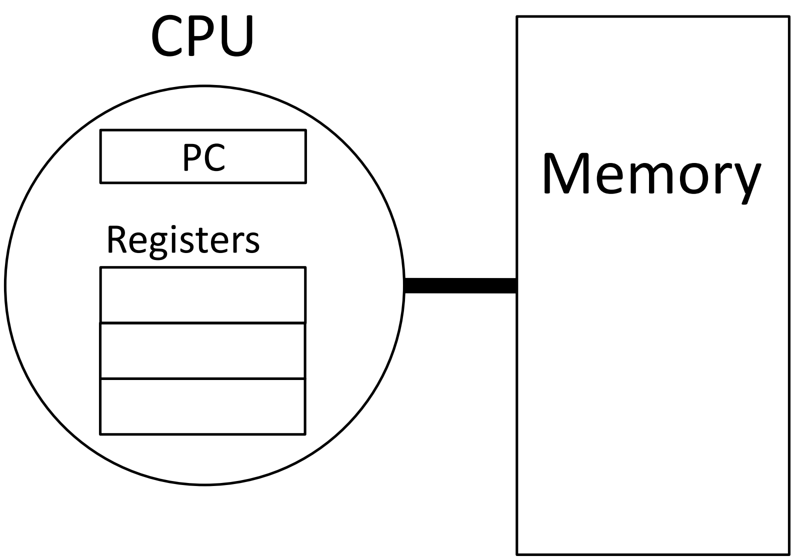
Architecture
Instruction set

Hardware
Different implementations



Instruction Set Architectures

- The ISA defines:
  - The system's state (*e.g.* registers, memory, program counter)
  - The instructions the CPU can execute
  - The effect that each of these instructions will have on the system state



Instruction Set Philosophies

- *Complex Instruction Set Computing* (CISC): Add more and more elaborate and specialized instructions as needed
  - Lots of tools for programmers/compilers to use, but hardware must be able to handle all instructions
  - x86-64 is CISC, but only a small subset of instructions encountered with Linux programs
- *Reduced Instruction Set Computing* (RISC): Keep instruction set small and regular
  - Easier to build fast hardware
  - Let software do the complicated operations by composing simpler ones

General ISA Design Decisions

- Instructions
  - What instructions are available? What do they do?
  - How are they encoded?
- Registers
  - How many registers are there?
  - How wide are they?
- Memory
  - How do you specify a memory location?

Mainstream ISAs

x86

Designer	Intel, AMD
Bits	16-bit, 32-bit and 64-bit
Introduced	1978 (16-bit), 1985 (32-bit), 2003 (64-bit)
Design	CISC
Type	Register-memory
Encoding	Variable (1 to 15 bytes)
Endianness	Little

Macbooks & PCs
(Core i3, i5, i7, M)
x86-64 Instruction Set

ARM architectures

Designer	ARM Holdings
Bits	32-bit, 64-bit
Introduced	1985; 31 years ago
Design	RISC
Type	Register-Register
Encoding	AArch64/A64 and AArch32/A32 use 32-bit instructions, T32 (Thumb-2) uses mixed 16- and 32-bit instructions. ARMv7 user-space compatibility[1]
Endianness	Bi (little as default)

Smartphone-like devices
(iPhone, iPad, Raspberry Pi)
ARM Instruction Set

MIPS

Designer	MIPS Technologies, Inc.
Bits	64-bit (32→64)
Introduced	1981; 35 years ago
Design	RISC
Type	Register-Register
Encoding	Fixed
Endianness	Bi

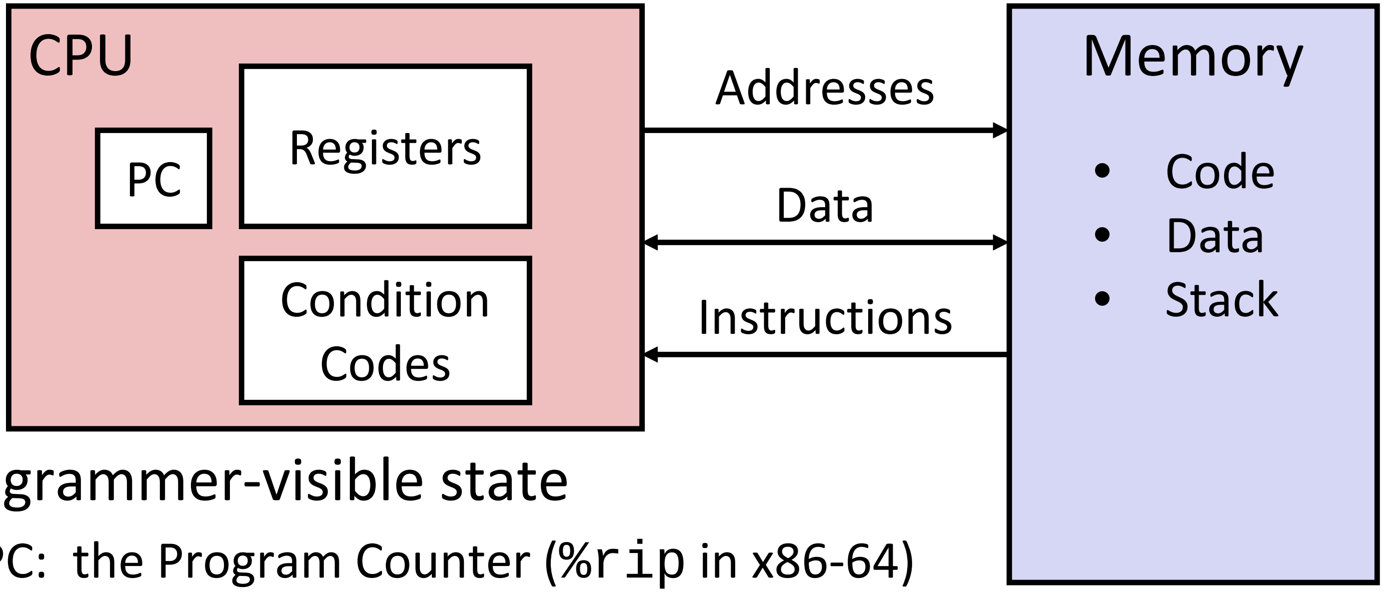
Digital home & networking equipment
(Blu-ray, PlayStation 2)
MIPS Instruction Set

Definitions

- **Architecture (ISA):** The parts of a processor design that one needs to understand to write assembly code
  - "What is directly visible to software"
- **Microarchitecture:** Implementation of the architecture
  - CSE/EE 469, 470

- Are the following part of the architecture?
  - Number of registers?
  - How about CPU frequency?
  - Cache size? Memory size?

Assembly Programmer's View



- Programmer-visible state
  - PC: the Program Counter (`%rip` in x86-64)
    - Address of next instruction
  - Named registers
    - Together in "register file"
    - Heavily used program data
  - Condition codes
    - Store status information about most recent arithmetic operation
    - Used for conditional branching

- Memory
  - Byte-addressable array
  - Code and user data
  - Includes *the Stack* (for supporting procedures)

x86-64 Assembly "Data Types"

- Integral data of 1, 2, 4, or 8 bytes
  - Data values
  - Addresses (untyped pointers)
- Floating point data of 4, 8, 10 or 2x8 or 4x4 or 8x2
  - Different registers for those (*e.g.* `%xmm1`, `%ymm2`)
  - Come from *extensions to x86* (SSE, AVX, …)

  Not covered In 351
- No aggregate types such as arrays or structures
  - Just contiguously allocated bytes in memory
- Two common syntaxes
  - "AT&T": used by our course, slides, textbook, gnu tools, …
  - "Intel": used by Intel documentation, Intel tools, …
  - Must know which you're reading

What is a Register?

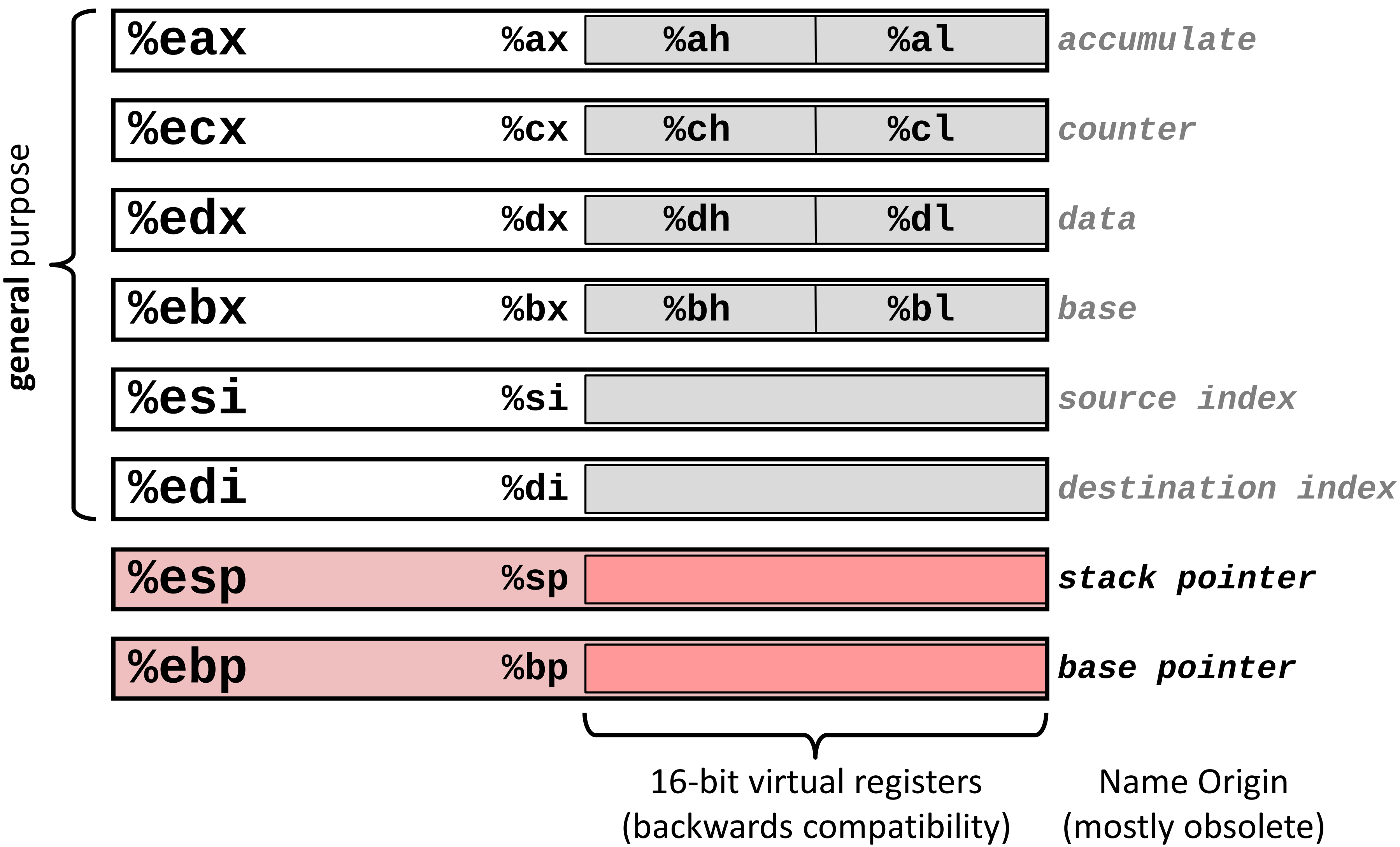
- A location in the CPU that stores a small amount of data, which can be accessed very quickly (once every clock cycle)
- Registers have *names*, not *addresses*
  - In assembly, they start with % (*e.g.* `%rsi`)
- Registers are at the heart of assembly programming
  - They are a precious commodity in all architectures, but *especially* x86

x86-64 Integer Registers – 64 bits wide

%rax	%eax	%r8	%r8d
%rbx	%ebx	%r9	%r9d
%rcx	%ecx	%r10	%r10d
%rdx	%edx	%r11	%r11d
%rsi	%esi	%r12	%r12d
%rdi	%edi	%r13	%r13d
%rsp	%esp	%r14	%r14d
%rbp	%ebp	%r15	%r15d

- Can reference low-order 4 bytes (also low-order 2 & 1 bytes)

Some History: IA32 Registers – 32 bits wide



Memory vs. Registers

Memory		Registers
Addresses	vs.	Names
`0x7FFFD024C3DC`		`%rdi`
Big	vs.	Small
~ 8 GiB		(16 x 8 B) = 128 B
Slow	vs.	Fast
~50-100 ns		sub-nanosecond timescale
Dynamic	vs.	Static
Can "grow" as needed while program runs		fixed number in hardware

Three Basic Kinds of Instructions

1) Transfer data between memory and register
   - *Load* data from memory into register
     - `%reg` = Mem[address]
   - *Store* register data into memory
     - Mem[address] = `%reg`

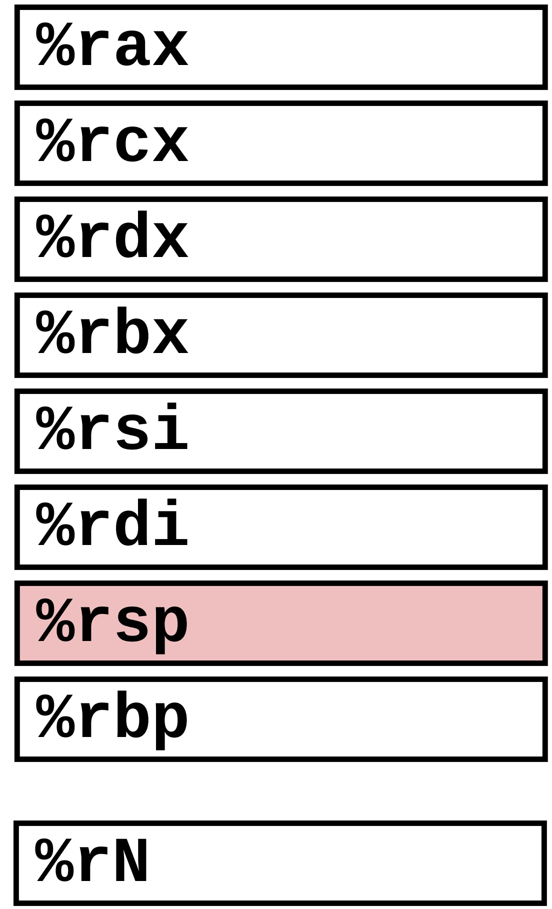
Remember: Memory is indexed just like an array of bytes!

2) Perform arithmetic operation on register or memory data
   - `c = a + b;     z = x << y;     i = h & g;`

3) Control flow: what instruction to execute next
   - Unconditional jumps to/from procedures
   - Conditional branches

Operand types

- ***Immediate:*** Constant integer data
  - Examples: **`$0x400`**, **`$-533`**
  - Like C literal, but prefixed with **‘$’**
  - Encoded with 1, 2, 4, or 8 bytes *depending on the instruction*
- ***Register:*** 1 of 16 integer registers
  - Examples: **`%rax`**, **`%r13`**
  - But **`%rsp`** reserved for special use
  - Others have special uses for particular instructions
- ***Memory:*** Consecutive bytes of memory at a computed address
  - Simplest example: **`(%rax)`**
  - Various other “address modes”



Summary

- Converting between integral and floating point data types *does* change the bits
  - Floating point rounding is a HUGE issue!
    - Limited mantissa bits cause inaccurate representations
    - Floating point arithmetic is NOT associative or distributive
- x86-64 is a complex instruction set computing (CISC) architecture
- **Registers** are named locations in the CPU for holding and manipulating data
  - x86-64 uses 16 64-bit wide registers
- Assembly operands include immediates, registers, and data at specified memory locations