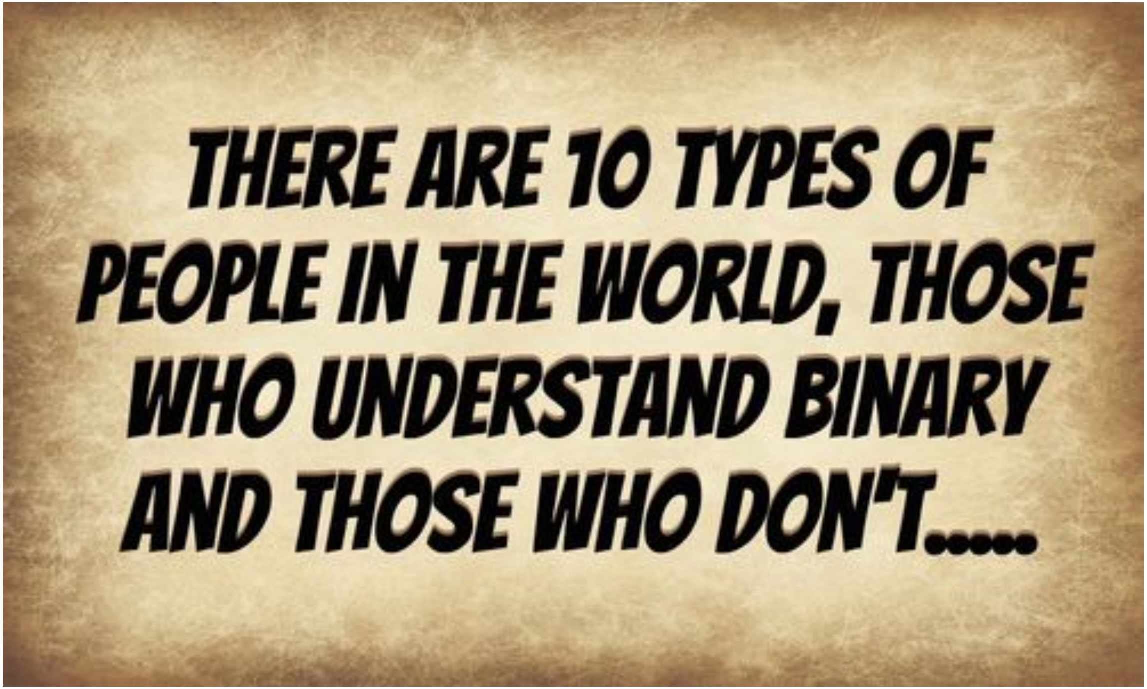
Number Representation and Operators


THERE ARE 10 TYPES OF
PEOPLE IN THE WORLD, THOSE
WHO UNDERSTAND BINARY
AND THOSE WHO DON'T.....

Decimal Numbering System

Ten symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Represent larger numbers as a sequence of digits

- Each digit is one of the available symbols

Example: 7061 in decimal (base 10)

- $7061_{10} = (7 \times 10^3) + (0 \times 10^2) + (6 \times 10^1) + (1 \times 10^0)$

Octal Numbering System

Eight symbols: 0, 1, 2, 3, 4, 5, 6, 7

- Notice that we no longer use 8 or 9

Base Comparison:

Base 10:	0,	1,	2,	3,	4,	5,	6,	7,	8,	9,	10,	11,	12…
Base 8:	0,	1,	2,	3,	4,	5,	6,	7,	10,	11,	12,	13,	14…

<u>Example</u>: What is 15_8 in base 10?

- $15_8 = (1 \times 8^1) + (5 \times 8^0) = 13_{10}$

<u>Example</u>: What is 7061_8 in base 10?

- $7061_8 = (7 \times 8^3) + (0 \times 8^2) + (6 \times 8^1) + (1 \times 8^0) = 3633_{10}$

Binary Numbering System

Two symbols: 0, 1

Convention: $2_{10} = 10_2 =$ 0b10

Base 10	Base 8	Base 2
0	0	0
1	1	1
2	2	10
3	3	11
4	4	100
5	5	101
6	6	110
7	7	111
8	10	1000
9	11	1001

Example: What is 0b110 in base 10?

- 0b110 $= 110_2 = (1 \times 2^2) + (1 \times 2^1) + (0 \times 1^0) = 6_{10}$

Hexadecimal Number System

Hexadecimal is base 16 (>10)

Sixteen Symbols:

- Ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
- Six letters: A, B, C, D, E, F

Convention: $16_{10} = 10_{16} =$ 0x10

Example: What is 0xA5 in base 10?

- 0xA5 $= A5_{16} = (10 \times 16^{1}) + (5 \times 16^{0}) = 165_{10}$

Question

Which of the following orderings is correct?

(A) 0b1010 < 0xC < 11

(B) 0xC < 0b1010 < 11

(C) 0b1010 < 11 < 0xC

(D) 11 < 0b1010 < 0xC

(E) 0xC < 11 < 0b1010

Question

Which of the following orderings is correct?

(A) 0b1010 < 0xC < 11

(B) 0xC < 0b1010 < 11

(C) <u>0b1010 < 11 < 0xC</u>

(D) 11 < 0b1010 < 0xC

(E) 0xC < 11 < 0b1010

0b1010 = (8 * 1) + (2 * 1) = 10

0xC = 12

10 < 11 < 12

Base Conversion

Converting to Base 10

Can convert from any base *to* base 10

- 110_2 = $(1 \times 2^{2}) + (1 \times 2^{1}) + (0 \times 2^{0}) = 6_{10}$
- $0xA5 = A5_{16}$ = $(10 \times 16^{1}) + (5 \times 16^{0}) = 165_{10}$

We learned to think in base 10, so this is fairly natural for us.

Challenge: Convert into other bases (e.g. 2, 16)

Challenge Question

Convert 13_{10} to binary

Hints:

- $2^3 = 8$
- $2^2 = 4$
- $2^1 = 2$
- $2^0 = 1$

Converting from Decimal to Binary

Given a decimal number N:

- List increasing powers of 2 from right to left until ≥ N
- From left to right, ask is that (power of 2) ≤ N?
  - If **YES**, put a 1 below and subtract that power from N
  - If **NO**, put a 0 below and keep going

Example for 13:

~~13~~
~~5~~
~~1~~
0

$2^4=16$	$2^3=8$	$2^2=4$	$2^1=2$	$2^0=1$
0	1	1	0	1

Converting from Decimal to Base B

Given a decimal number N:

- List increasing powers of B from right to left until ≥ N
- From left to right, ask is that (power of B) ≤ N?
  - If **YES**, put *how many* of that power go into N and subtract from N
  - If **NO**, put a 0 and keep going

Example for 165 into hexadecimal (base 16):

~~165~~
~~5~~
0

$16^2=256$	$16^1=16$	$16^0=1$
0	**A (10)**	**5**

Converting Binary ↔ Hexadecimal

Hex → Binary

- Substitute hex digits, then drop leading zeros
- Example: 0x2D in binary
  - 0x2 is 0b0010, 0xD is 0b1101
  - Drop two leading zeros, answer is 0b101101

Binary → Hex

- Pad with leading zeros until multiple of 4, then substitute groups of 4
- Example: 0b101101
  - Pad to 0b 0010 1101
  - Substitute to get 0x2D

Base 10	Base 16	Base 2
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
10	A	1010
11	B	1011
12	C	1100
13	D	1101
14	E	1110
15	F	1111

Binary → Hex Practice

Convert 0b100110110101101

- How many digits? 15
- Pad: 0b 0100 1101 1010 1101
- Substitute: 0x4DAD

Base 10	Base 16	Base 2
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
10	A	1010
11	B	1011
12	C	1100
13	D	1101
14	E	1110
15	F	1111

Why are we learning this?

Why does all of this matter?

- Humans think about numbers in base 10, but…

  computers think about numbers in base 2

- How is it that computers can do all of the amazing things that they do?

  Binary encoding

Binary Encoding

Numerical Encoding

- AMAZING FACT: You can represent *anything* countable using numbers!
  - Need to agree on an encoding
  - Kind of like learning a new language

<u>Examples</u>:

- Decimal Numbers: 0→0b0, 1→0b1, 2→0b10, etc.
- English Letters: BJC→0x424A43, yay→0x796179
- Emoticons: 😃0x0, 😞0x1, 😎0x2, 😇0x3, 😈0x4, 🙋0x5

Binary Encoding

With N binary digits, how many things can you represent?

- Need N bits to represent n things, where $2^N \geq n$
- <u>Example</u>: 5 bits for alphabet because $2^5 = 32 > 26$

A binary digit is known as a bit

A group of 4 bits (1 hex digit) is called a nibble

A group of 8 bits (2 hex digits) is called a byte

bit→2 things, nibble→16 things, byte→256 things

So What's It Mean?

A sequence of bits can have many meanings!

Consider the hex sequence 0x4E6F21

Common interpretations include:

- The decimal number 5140257
- The characters "No!"
- The background color of this slide
- The real number 7.203034×10^{-39} [floating point]

It is up to the program/programmer to decide how to interpret the sequence of bits

Number Representation Recap

Humans think about numbers in decimal

Computers think about numbers in binary

Base conversion to go between

- Hex is more human-readable than binary

All information on a computer is in binary

- Nice because big difference between “high” and “low”

Binary encoding can represent *anything*!

- Program needs to know how to interpret bits

Operators Recap

- NOT: ~
  - This will flip all bits in the operand
- AND: &
  - This will perform a bitwise AND on every pair of bits
- OR: |
  - This will perform a bitwise OR on every pair of bits
- XOR: ^
  - This will perform a bitwise XOR on every pair of bits
- SHIFT: <<, >>
  - This will shift the bits right or left
    - logical vs. arithmetic

Operators Recap

- NOT: !
  - Evaluates the entire operand, rather than each bit
  - Produces a 1 if == 0, produces 0 if nonzero
- AND: &&
  - Produces 1 if both operands are nonzero
- OR: ||
  - Produces 1 if either operand is nonzero

Lab 1

- Worksheet in class
- Tips:
  - Work on 8-bit versions first, then scale your solution to work with 32-bit inputs
  - Save intermediate results in variables for clarity
  - Shifting by more than 31 bits is **UNDEFINED**. This will NOT yield 0

Examples

Create 0xFFFFFFFF using only one operator

- Limited to constants from 0x00 to 0xFF
- Naïve approach:

0xFF + (0xFF << 8) + (0xFF << 16) ...

- Better approach:

~0x00 = 0xFFFFFFFF

Examples

Replace the leftmost byte of a 32-bit integer with 0xAB

- Let our integer be x
- First, we want to create a mask for the lower 24 bits
  - `~(0xFF << 24)` will do that using just two operators
- (x & mask) will zero out the leftmost 8 bits
- Now, we want to OR in 0xAB to those zeroed-out bits
- Final result:
  - `(x & mask) | (0xAB << 24)`
- Total operators: 5