Floating Point II, x86-64 Intro

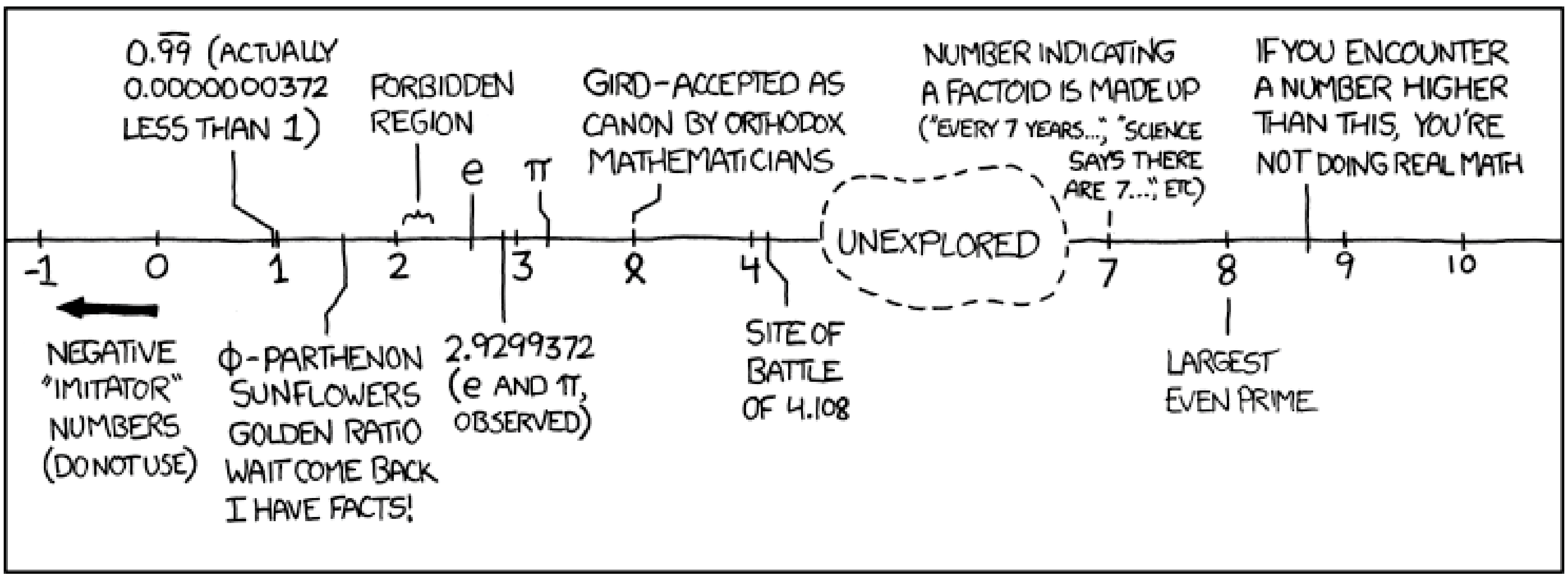
CSE 351 Autumn 2017

Instructor:

Justin Hsia

Teaching Assistants:

Lucas Wotton	Michael Zhang	Parker DeWilde	Ryan Wong
Sam Gehman	Sam Wolfson	Savanna Yee	Vinny Palaniappan



http://xkcd.com/899/

Administrivia

- Lab 1 due on Friday (10/13)
  - Submit `bits.c`, `pointer.c`, `lab1reflect.txt`
- Homework 2 due next Friday (10/20)
  - On Integers, Floating Point, and x86-64
- Section tomorrow on Integers and Floating Point
- Peer Instruction Questions are for your benefit!
  - TAs are scattered about as well to help

Floating point topics

- Fractional binary numbers
- IEEE floating-point standard
- **Floating-point operations and rounding**
- Floating-point in C

- There are many more details that we won't cover
  - It's a 58-page standard…

Floating Point Encoding Summary

Exponent	Mantissa	Meaning
0x00	0	± 0
0x00	non-zero	± denorm num
0x01 – 0xFE	anything	± norm num
0xFF	0	± ∞
0xFF	non-zero	NaN

smallest E
everything else
largest E

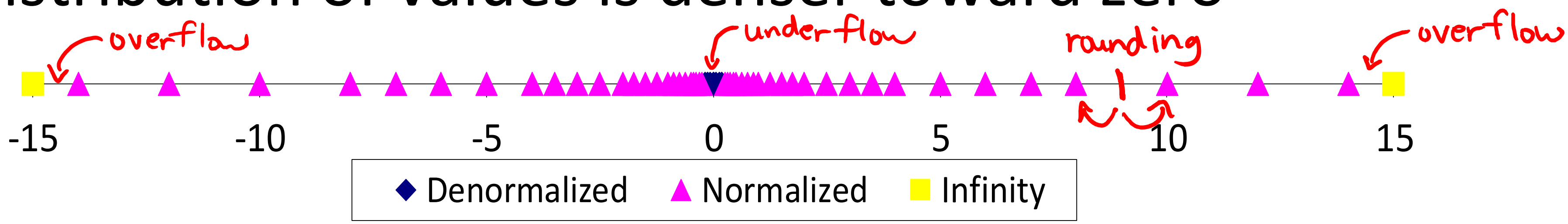
Distribution of Values

- What ranges are NOT representable?
  - Between largest norm and infinity **Overflow** (Exp too large)
  - Between zero and smallest denorm **Underflow** (Exp too small)
  - Between norm numbers? **Rounding**
- Given a FP number, what's the bit pattern of the next largest representable number?

  if M=0b0...00, then $2^{Exp} \times 1.0$

  if M=0b0...01, then $2^{Exp} \times (1+2^{-23})$

  diff = 2^{Exp-23}
  - What is this "step" when Exp = 0? 2^{-23}
  - What is this "step" when Exp = 100? 2^{77}
- Distribution of values is denser toward zero



Floating Point Operations: Basic Idea

Value = $(-1)^{S} \times \text{Mantissa} \times 2^{\text{Exponent}}$

S	E	M

- $x +_f y = \text{Round}(x + y)$
- $x *_f y = \text{Round}(x * y)$

- Basic idea for floating point operations:
  - First, compute the exact result
  - Then *round* the result to make it fit into desired precision:
    - Possibly over/underflow if exponent outside of range
    - Possibly drop least-significant bits of mantissa to fit into M bit vector

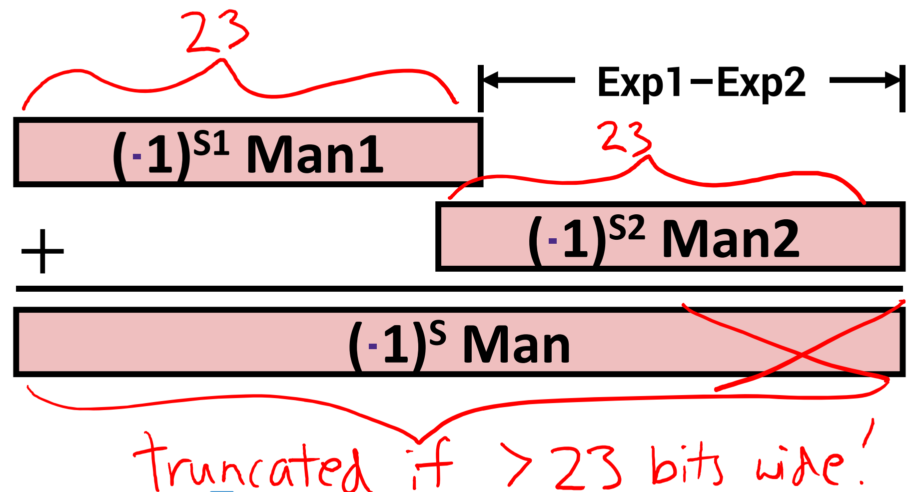
Floating Point Addition

Line up the binary points!

- $(-1)^{S1} \times Man1 \times 2^{Exp1} + (-1)^{S2} \times Man2 \times 2^{Exp2}$
  - Assume Exp1 > Exp2

```
   1.010*2^2          1.0100*2^2
 + 1.000*2^-1       + 0.0010*2^2
   ???                1.0110*2^2
```

- Exact Result: $(-1)^{S} \times Man \times 2^{Exp}$
  - Sign S, mantissa Man:
    - Result of signed align & add
  - Exponent E: E1



- Adjustments:
  - If Man ≥ 2, shift Man right, increment Exp
  - If Man < 1, shift Man left k positions, decrement Exp by k
  - Over/underflow if Exp out of range
  - Round Man to fit mantissa precision

Floating Point Multiplication

- $(-1)^{S1}\times \text{Man1}\times 2^{\text{Exp1}}$ × $(-1)^{S2}\times \text{Man2}\times 2^{\text{Exp2}}$

- Exact Result: $(-1)^{S}\times M\times 2^{E}$
  - Sign S: S1 ^ S2
  - Mantissa Man: Man1 × Man2
  - Exponent Exp: Exp1 + Exp2

- Adjustments:
  - If Man ≥ 2, shift Man right, increment Exp
  - Over/underflow if Exp out of range
  - Round Man to fit mantissa precision

Mathematical Properties of FP Operations

- Exponent overflow yields +∞ or -∞
- Floats with value +∞, -∞, and NaN can be used in operations
  - Result usually still +∞, -∞, or NaN; but not always intuitive
- Floating point operations do not work like real math, due to rounding
  - Not associative: `(3.14+1e100)–1e100 != 3.14+(1e100–1e100)`
    - **0** and **3.14**
  - Not distributive: `100*(0.1+0.2) != 100*0.1+100*0.2`
    - **30.000000000000003553** and **30**
  - Not cumulative
    - Repeatedly adding a very small number to a large one may do nothing

Floating point topics

- Fractional binary numbers
- IEEE floating-point standard
- Floating-point operations and rounding
- **Floating-point in C**

- There are many more details that we won't cover
  - It's a 58-page standard…

Floating Point in C

!!!

- C offers two (well, 3) levels of precision

float	1.0f	single precision (32-bit)
double	1.0	double precision (64-bit)
long double	1.0L	*("double double" or quadruple)* precision (64-128 bits)

- #include <math.h> to get INFINITY and NAN constants
- Equality (==) comparisons between floating point numbers are tricky, and often return unexpected results, so just avoid them!

instead use abs(f1 - f2) < 2^{-20}

↑ some arbitrary threshold

Floating Point Conversions in C

!!!

- Casting between `int`, `float`, and `double` **changes** the bit representation
  - `int` → `float`
    - May be rounded (not enough bits in mantissa: 23)
    - Overflow impossible
  - `int` or `float` → `double`
    - Exact conversion (all 32-bit `int`s representable)
  - `long` → `double`
    - Depends on word size (32-bit is exact, 64-bit may be rounded)
  - `double` or `float` → `int`
    - Truncates fractional part (rounded toward zero)
    - “Not defined” when out of range or NaN: generally sets to `Tmin` (even if the value is a very big positive)

Floating Point and the Programmer

$1.0 \times 2^0 \rightarrow S=0,\ E=0111\ 1111,\ M=0...0$

f1 = 0b 0/011 1111 1/000 0000 0000 0000 0000 0000 = 0x3F800000

```
#include <stdio.h>

int main(int argc, char* argv[]) {
  float f1 = 1.0;   // specify float constant
  float f2 = 0.0;
  int i;
  for (i = 0; i < 10; i++)
    f2 += 1.0/10.0;
    // f2 should == 10 × 1/10 = 1
  printf("0x%08x  0x%08x\n", *(int*)&f1, *(int*)&f2);
  printf("f1 = %10.9f\n", f1);
  printf("f2 = %10.9f\n\n", f2);

  f1 = 1E30;    // 10^30
  f2 = 1E-30;   // 10^-30
  float f3 = f1 + f2;
  printf("f1 == f3? %s\n", f1 == f3 ? "yes" : "no" );
  // 10^30 == 10^30 + 10^-30
  return 0;
}
```

```
$ ./a.out
0x3f800000  0x3f800001   (f2)
f1 = 1.000000000
f2 = 1.000000119

f1 == f3? yes
```

Floating Point Summary

- Floats also suffer from the fixed number of bits available to represent them
  - Can get overflow/underflow
  - "Gaps" produced in representable numbers means we can lose precision, unlike `int`s
    - Some "simple fractions" have no exact representation (*e.g.* 0.2)
    - "Every operation gets a slightly wrong result"
- Floating point arithmetic not associative or distributive
  - Mathematically equivalent ways of writing an expression may compute different results
- Never test floating point values for equality!
- Careful when converting between `int`s and `float`s!

Number Representation Really Matters

- **1991:** Patriot missile targeting error
  - clock skew due to conversion from integer to floating point
- **1996:** Ariane 5 rocket exploded ($1 billion)
  - overflow converting 64-bit floating point to 16-bit integer
- **2000:** Y2K problem
  - limited (decimal) representation: overflow, wrap-around
- **2038:** Unix epoch rollover
  - Unix epoch = seconds since 12am, January 1, 1970
  - signed 32-bit integer representation rolls over to TMin in 2038
- **Other related bugs:**
  - 1982: Vancouver Stock Exchange 10% error in less than 2 years
  - 1994: Intel Pentium FDIV (floating point division) HW bug ($475 million)
  - 1997: USS Yorktown “smart” warship stranded: divide by zero
  - 1998: Mars Climate Orbiter crashed: unit mismatch ($193 million)

Roadmap

C:

```
car *c = malloc(sizeof(car));
c->miles = 100;
c->gals = 17;
float mpg = get_mpg(c);
free(c);
```

Java:

```
Car c = new Car();
c.setMiles(100);
c.setGals(17);
float mpg =
    c.getMPG();
```

Assembly language:

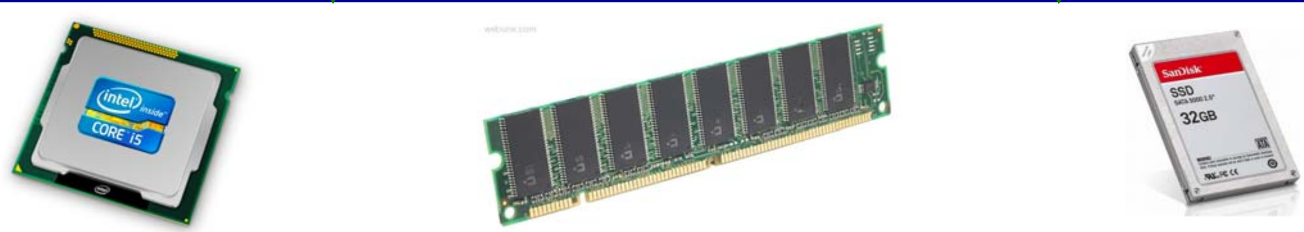
```
get_mpg:
    pushq   %rbp
    movq    %rsp, %rbp
    ...
    popq    %rbp
    ret
```

Machine code:

```
0111010000011000
100011010000010000000010
1000100111000010
110000011111101000011111
```

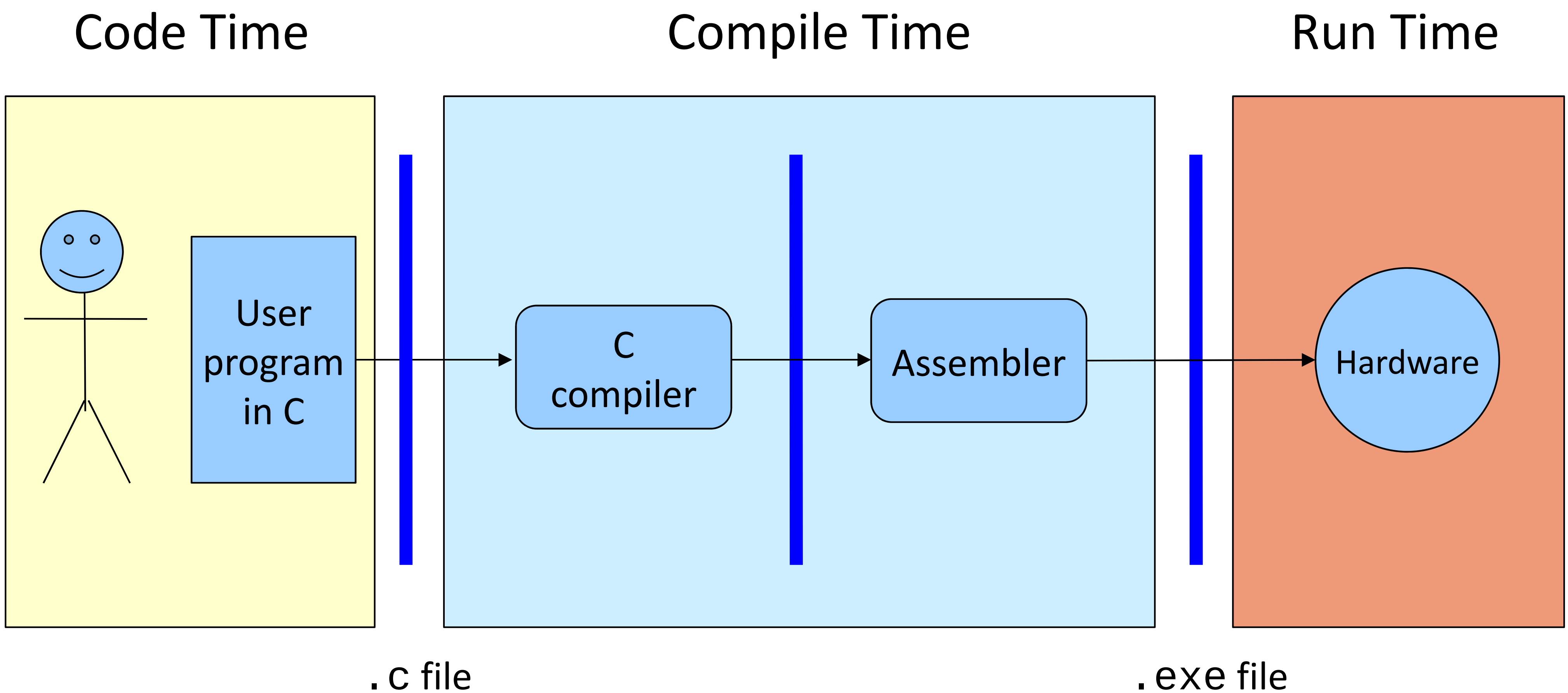
OS:

Computer system:



Memory & data
Integers & floats
x86 assembly
Procedures & stacks
Executables
Arrays & structs
Memory & caches
Processes
Virtual memory
Memory allocation
Java vs. C

Translation



What makes programs run fast(er)?

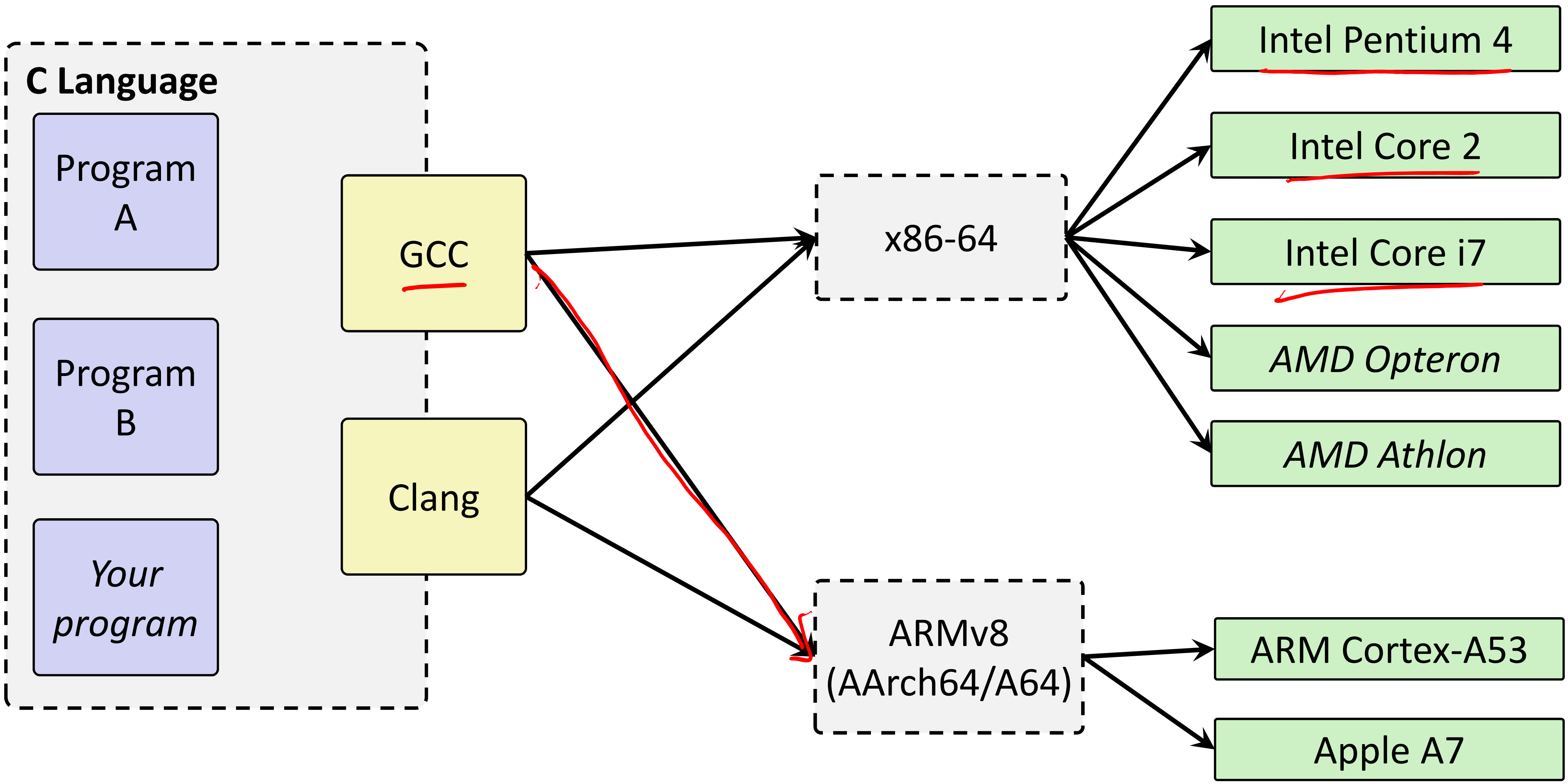
HW Interface Affects Performance

Source code
Different applications or algorithms

Compiler
Perform optimizations, generate instructions

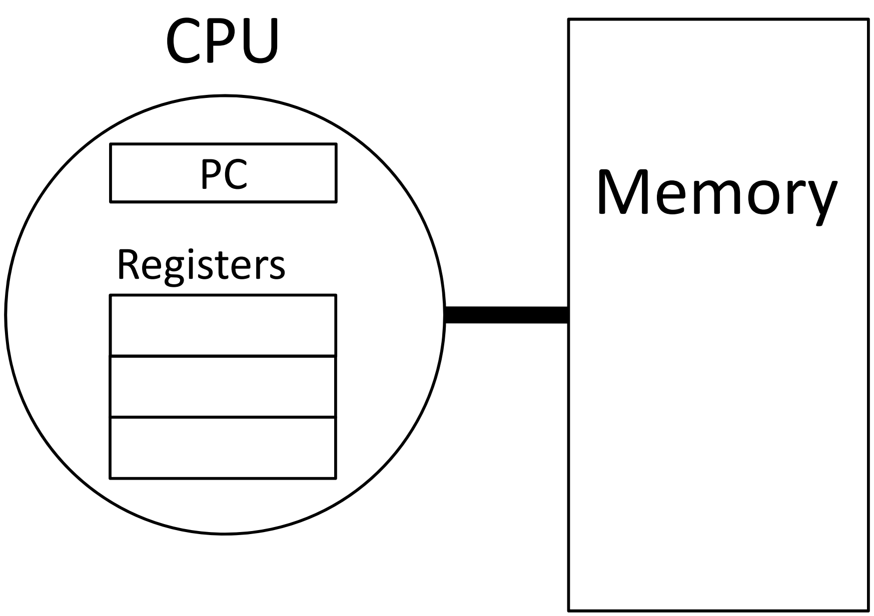
Architecture
Instruction set

Hardware
Different implementations



Instruction Set Architectures

- The ISA defines:
  - The system's state (*e.g.* registers, memory, program counter)
  - The instructions the CPU can execute
  - The effect that each of these instructions will have on the system state



Instruction Set Philosophies

- *Complex Instruction Set Computing* (CISC): Add more and more elaborate and specialized instructions as needed
  - Lots of tools for programmers to use, but hardware must be able to handle all instructions
  - x86-64 is CISC, but only a small subset of instructions encountered with Linux programs
- *Reduced Instruction Set Computing* (RISC): Keep instruction set small and regular
  - Easier to build fast hardware
  - Let software do the complicated operations by composing simpler ones

General ISA Design Decisions

- Instructions
  - What instructions are available? What do they do?
  - How are they encoded? instructions are data! binary encoding
- Registers
  - How many registers are there?
  - How wide are they? word size
- Memory
  - How do you specify a memory location? address → word size different ways to specify an address

Mainstream ISAs

x86

Designer	Intel, AMD
Bits	16-bit, 32-bit and 64-bit
Introduced	1978 (16-bit), 1985 (32-bit), 2003 (64-bit)
Design	CISC
Type	Register-memory
Encoding	Variable (1 to 15 bytes)
Endianness	Little

Macbooks & PCs
(Core i3, i5, i7, M)
x86-64 Instruction Set

ARM architectures

Designer	ARM Holdings
Bits	32-bit, 64-bit
Introduced	1985; 31 years ago
Design	RISC
Type	Register-Register
Encoding	AArch64/A64 and AArch32/A32 use 32-bit instructions, T32 (Thumb-2) uses mixed 16- and 32-bit instructions. ARMv7 user-space compatibility[1]
Endianness	Bi (little as default)

Smartphone-like devices
(iPhone, iPad, Raspberry Pi)
ARM Instruction Set

MIPS

Designer	MIPS Technologies, Inc.
Bits	64-bit (32→64)
Introduced	1981; 35 years ago
Design	RISC
Type	Register-Register
Encoding	Fixed
Endianness	Bi

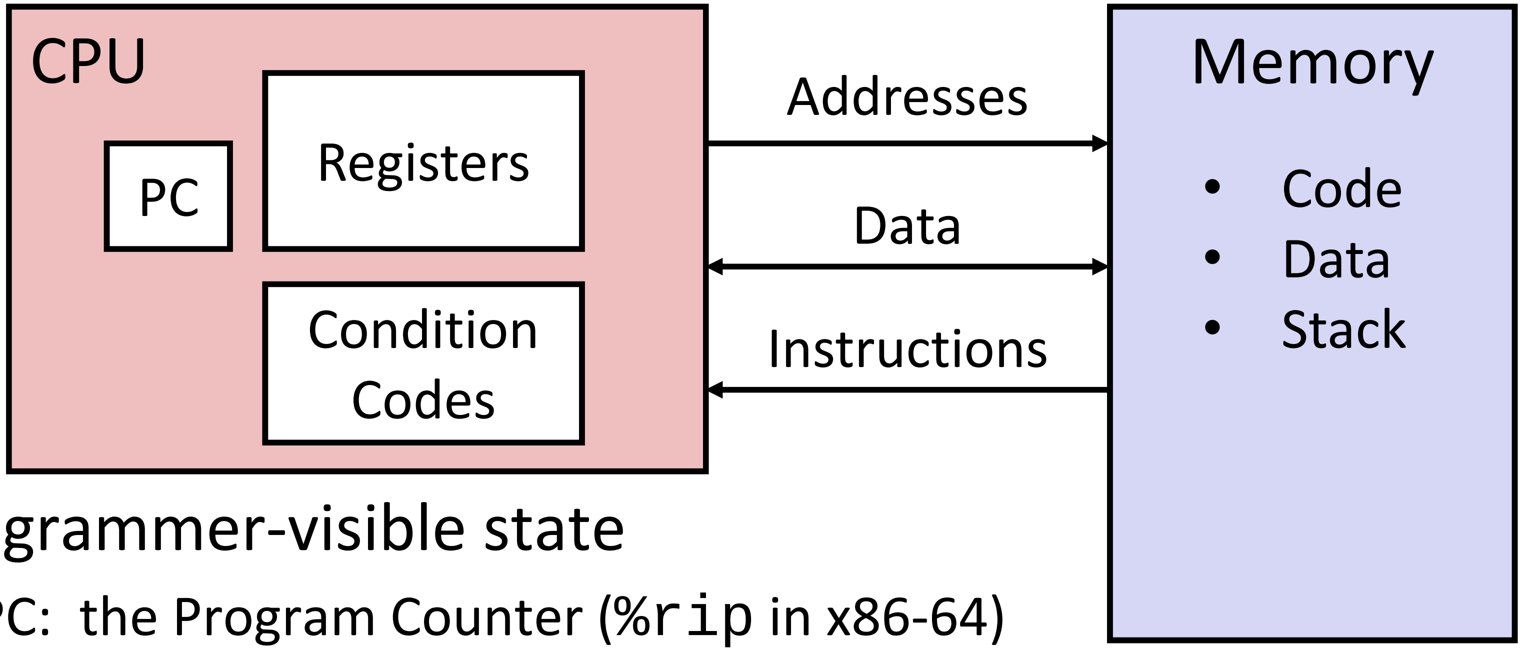
Digital home & networking equipment
(Blu-ray, PlayStation 2)
MIPS Instruction Set

Definitions

- **Architecture (ISA):** The parts of a processor design that one needs to understand to write assembly code
  - "What is directly visible to software"
- **Microarchitecture:** Implementation of the architecture
  - CSE/EE 469, 470

- Are the following part of the architecture?
  - Number of registers? Yes
  - How about CPU frequency? No
  - Cache size? Memory size? No – modular

Assembly Programmer's View



- Programmer-visible state
  - PC: the Program Counter (`%rip` in x86-64)
    - Address of next instruction
  - Named registers
    - Together in "register file"
    - Heavily used program data
  - Condition codes
    - Store status information about most recent arithmetic operation
    - Used for conditional branching

- Memory
  - Byte-addressable array
  - Code and user data
  - Includes *the Stack* (for supporting procedures)

x86-64 Assembly "Data Types"

- Integral data of 1, 2, 4, or 8 bytes
  - Data values
  - Addresses (untyped pointers)
- Floating point data of 4, 8, 10 or 2x8 or 4x4 or 8x2
  - Different registers for those (*e.g.* %xmm1, %ymm2)
  - Come from *extensions to x86* (SSE, AVX, …)

  Not covered In 351
- No aggregate types such as arrays or structures
  - Just contiguously allocated bytes in memory
- Two common syntaxes
  - ✓ "AT&T": used by our course, slides, textbook, gnu tools, …
  - ✗ "Intel": used by Intel documentation, Intel tools, …
  - Must know which you're reading

What is a Register?

- A location in the CPU that stores a small amount of data, which can be accessed very quickly (once every clock cycle)

- Registers have *names*, not *addresses*
  - In assembly, they start with % (*e.g.* `%rsi`)

- Registers are at the heart of assembly programming
  - They are a precious commodity in all architectures, but *especially* x86

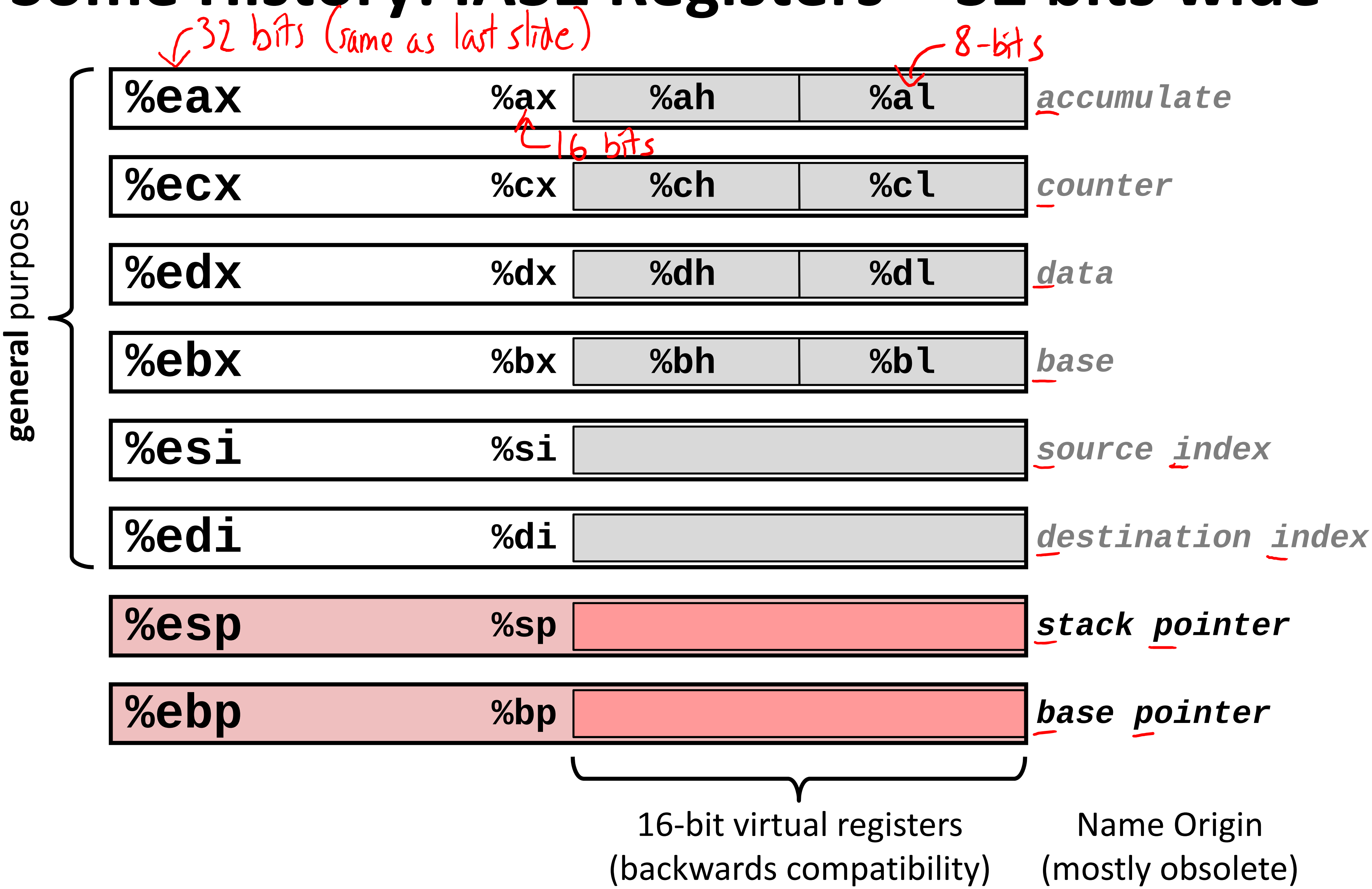
only 16!

x86-64 Integer Registers – 64 bits wide

64 bits	32 bits	64 bits	32 bits
%rax	%eax	%r8	%r8d
%rbx	%ebx	%r9	%r9d
%rcx	%ecx	%r10	%r10d
%rdx	%edx	%r11	%r11d
%rsi	%esi	%r12	%r12d
%rdi	%edi	%r13	%r13d
%rsp	%esp	%r14	%r14d
%rbp	%ebp	%r15	%r15d

- Can reference low-order 4 bytes (also low-order 2 & 1 bytes)

Some History: IA32 Registers – 32 bits wide



Memory vs. Registers

Memory	vs.	Registers
Addresses	**vs.**	Names
`0x7FFFD024C3DC`		`%rdi`
Big	**vs.**	Small
~ 8 GiB		(16 x 8 B) = 128 B
Slow	**vs.**	Fast
~50-100 ns		sub-nanosecond timescale
Dynamic	**vs.**	Static
Can “grow” as needed while program runs		fixed number in hardware

Three Basic Kinds of Instructions

1) Transfer data between memory and register
   - *Load* data from memory into register
     - `%reg` = Mem[address]
   - *Store* register data into memory
     - Mem[address] = `%reg`

Remember: Memory is indexed just like an array of bytes!

2) Perform arithmetic operation on register or memory data
   - `c = a + b;     z = x << y;     i = h & g;`

3) Control flow: what instruction to execute next
   - Unconditional jumps to/from procedures
   - Conditional branches

Operand types

- ***Immediate:*** Constant integer data
  - Examples: **`$0x400`**, **`$-533`** *(hex, decimal)*
  - Like C literal, but prefixed with **`'$'`**
  - Encoded with 1, 2, 4, or 8 bytes *depending on the instruction*
- ***Register:*** 1 of 16 integer registers
  - Examples: **`%rax`**, **`%r13`**
  - But **`%rsp`** reserved for special use *(stack pointer)*
  - Others have special uses for particular instructions
- ***Memory:*** Consecutive bytes of memory at a computed address
  - Simplest example: **`(%rax)`** ← *take data in %rax, treat as address, pull data at that address*
  - Various other "address modes"

```
%rax
%rcx
%rdx
%rbx
%rsi
%rdi
%rsp
%rbp

%rN
```

%rsp: stack pointer

%rN: r8 - r15

Summary

- Converting between integral and floating point data types *does* change the bits
  - Floating point rounding is a HUGE issue!
    - Limited mantissa bits cause inaccurate representations
    - Floating point arithmetic is NOT associative or distributive
- x86-64 is a complex instruction set computing (CISC) architecture
- **Registers** are named locations in the CPU for holding and manipulating data
  - x86-64 uses 16 64-bit wide registers
- Assembly operands include immediates, registers, and data at specified memory locations