Data III & Integers I

CSE 351 Autumn 2017

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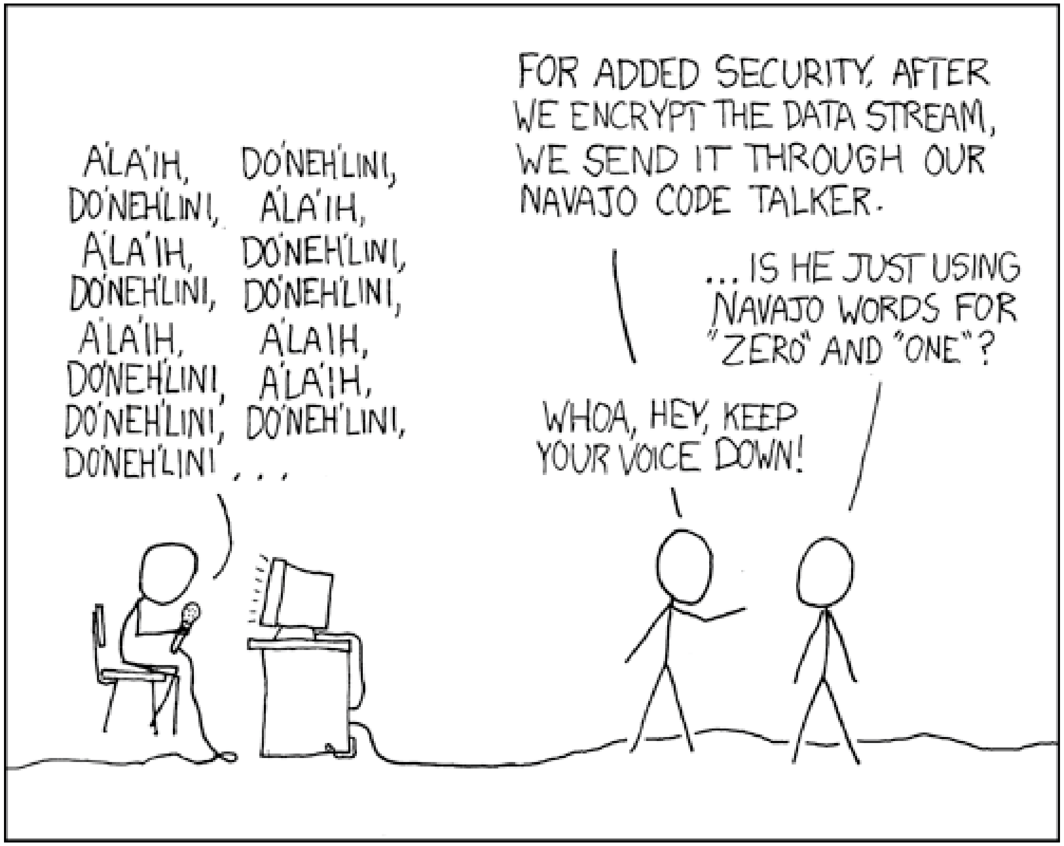


http://xkcd.com/257/

Null-Terminated Strings

6 chars 1 char 5 chars

- **Example:** "Donald Trump" stored as a 13-byte array

Decimal:	68	111	110	97	108	100	32	84	114	117	109	112	0
Hex:	0x44	0x6F	0x6E	0x61	0x6C	0x64	0x20	0x54	0x72	0x75	0x6D	0x70	0x00
Text:	D	o	n	a	l	d		T	r	u	m	p	\0

13 bytes total!

- Last character followed by a 0 byte (`'\0'`) (a.k.a. "null terminator")
  - Must take into account when allocating space in memory
  - Note that `'0'` ≠ `'\0'` (i.e. character 0 has non-zero value)
- How do we compute the length of a string?
  - Traverse array until null terminator encountered

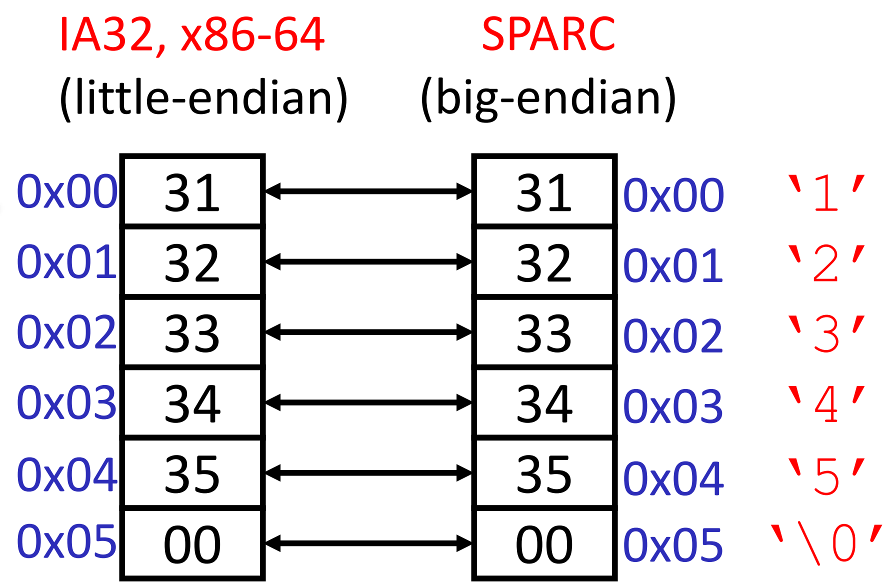
Endianness and Strings

C (char = 1 byte)

```
char s[6] = "12345";
```

String literal

0x31 = 49 decimal = ASCII ‘1’



- Byte ordering (endianness) is not an issue for 1-byte values
  - The whole array does not constitute a single value
  - Individual elements are values; chars are single bytes

Examining Data Representations

- Code to print byte representation of data
  - Any data type can be treated as a *byte array* by **casting** it to `char`
  - C has unchecked casts !! DANGER !!

```
void show_bytes(char* start, int len) {
    int i;
    for (i = 0; i < len; i++)
        printf("%p\t0x%.2x\n", start+i, *(start+i));
    printf("\n");
}
```

format string

pointer arithmetic on char*

```
void show_int(int x) {
    show_bytes( (char *) &x, sizeof(int));
}
```

int*

4 bytes

"cast" (treat as)

Administrivia

- Homework 1 due tonight
- Lab 1 released
  - *This is considered the hardest assignment of the class by many students*
  - Some progress due Monday 10/9, Lab 1 due Friday 1/13

- Poll Everywhere: you can change your vote

Memory, Data, and Addressing

- Representing information as bits and bytes
- Organizing and addressing data in memory
- Manipulating data in memory using C
- **Boolean algebra and bit-level manipulations**

Boolean Algebra

- Developed by George Boole in 19th Century
  - Algebraic representation of logic (True → 1, False → 0)
  - AND: `A&B=1` when both A is 1 and B is 1
  - OR: `A|B=1` when either A is 1 or B is 1
  - XOR: `A^B=1` when either A is 1 or B is 1, but not both
  - NOT: `~A=1` when A is 0 and vice-versa
  - DeMorgan's Law: `~(A|B) = ~A & ~B`
    `~(A&B) = ~A | ~B`

AND B

	&	0	1
A	0	**0**	**0**
	1	**0**	**1**

OR

|	0	1
0	**0**	**1**
1	**1**	**1**

XOR

^	0	1
0	**0**	**1**
1	**1**	**0**

NOT

~	
0	**1**
1	**0**

General Boolean Algebras

- Operate on bit vectors
  - Operations applied bitwise
  - All of the properties of Boolean algebra apply

```
  01101001      01101001      01101001
& 01010101    | 01010101    ^ 01010101    ~ 01010101
  01000001      01111101      00111100      10101010
```

- Examples of useful operations:

$$x \,^\wedge x = 0$$

```
  01010101
^ 01010101
  00000000  ← creates 0
```

"sets to 1" $x \mid 1 = 1,$ $0 \mid 1 = 1$ $1 \mid 1 = 1$

"leaves as is" $x \mid 0 = x$ $0 \mid 0 = 0$ $1 \mid 0 = 1$

```
  01010101  ← data of interest
| 11110000  ← bit mask (specifically chosen)
  11110101
```

$\underbrace{1111}_{\text{set}}\underbrace{0101}_{\text{left as is}}$

Bit-Level Operations in C

- & (AND), | (OR), ^ (XOR), ~ (NOT)
  - View arguments as bit vectors, apply operations bitwise
  - Apply to any "integral" data type
    - `long`, `int`, `short`, `char`, `unsigned`
    - (8 B) (4 B) (2 B) (1 B)

(bit vector will be width of data type)

- Examples with `char a, b, c;`

C code | Internally | Result

```
a = (char) 0x41;    // 0x41->0b 0100 0001
b = ~a;             //       0b 1011 1110->0x BE
```

```
a = (char) 0x69;    // 0x69->0b 0110 1001
b = (char) 0x55;    // 0x55->0b 0101 0101
c = a & b;          //       0b 0100 0001->0x 41
```

```
a = (char) 0x41;    // 0x41->0b 0100 0001
b = a;              //       0b 0100 0001
c = a ^ b;          //       0b 0000 0000->0x 00
```

Contrast: Logic Operations

"bang"

- Logical operators in C: `&&` (AND), `||` (OR), `!` (NOT)
  - **0** is False, **anything nonzero** is True
  - **Always** return 0 or 1
  - Early termination (a.k.a. short-circuit evaluation) of `&&`, `||`
- Examples (`char` data type)

0b 1100 1100 0b 0011 0011

0x CC & 0x33 → 0x00

  - `!0x41 -> 0x00` (T, F)
  - `!0x00 -> 0x01` (F, T)
  - `!!0x41 -> 0x01` (T, T)
  - `0xCC && 0x33 -> 0x01`
  - `0x00 || 0x33 -> 0x01`

0x00 | 0x33 → 0x33

  - `p && *p++` ← read data at p, then move p
    - Avoids null pointer (0x0) access via *early termination*
    - Short for: `if (p) { *p++; }`

null pointer check

Roadmap

C:

```
car *c = malloc(sizeof(car));
c->miles = 100;
c->gals = 17;
float mpg = get_mpg(c);
free(c);
```

Java:

```
Car c = new Car();
c.setMiles(100);
c.setGals(17);
float mpg =
    c.getMPG();
```

Memory & data
Integers & floats
Machine code & C
x86 assembly
Procedures & stacks
Arrays & structs
Memory & caches
Processes
Virtual memory
Operating Systems

Assembly language:

```
get_mpg:
    pushq   %rbp
    movq    %rsp, %rbp
    ...
    popq    %rbp
    ret
```

Machine code:

```
0111010000011000
100011010000010000000010
1000100111000010
110000011111101000011111
```

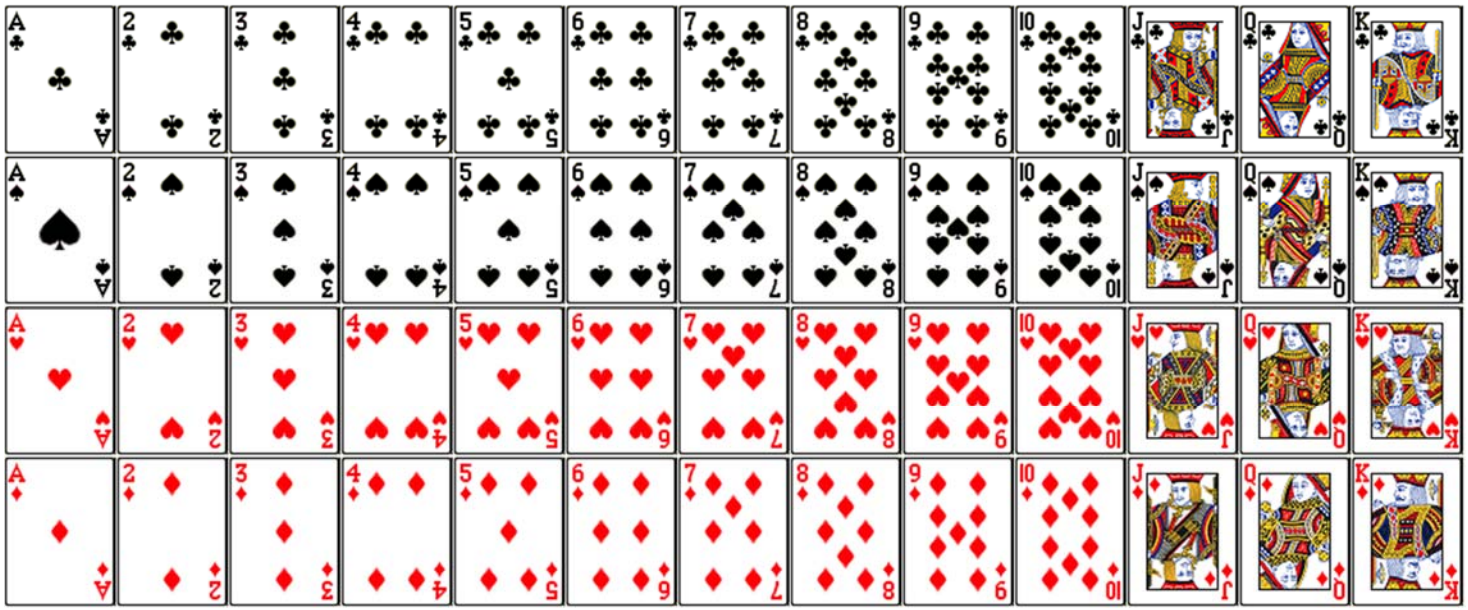
OS:

Windows 8 Mac

Computer system:

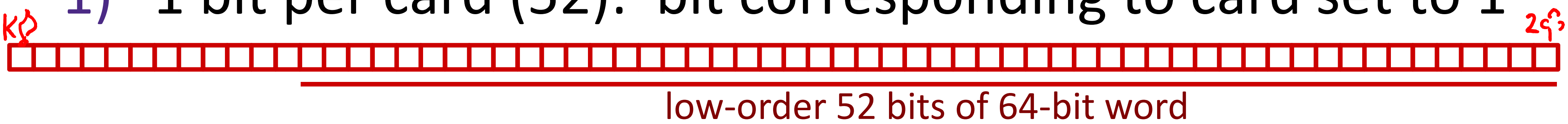
But before we get to integers….

- Encode a standard deck of playing cards
- 52 cards in 4 suits
  - How do we encode suits, face cards?
- What operations do we want to make easy to implement?
  - Which is the higher value card?
  - Are they the same suit?



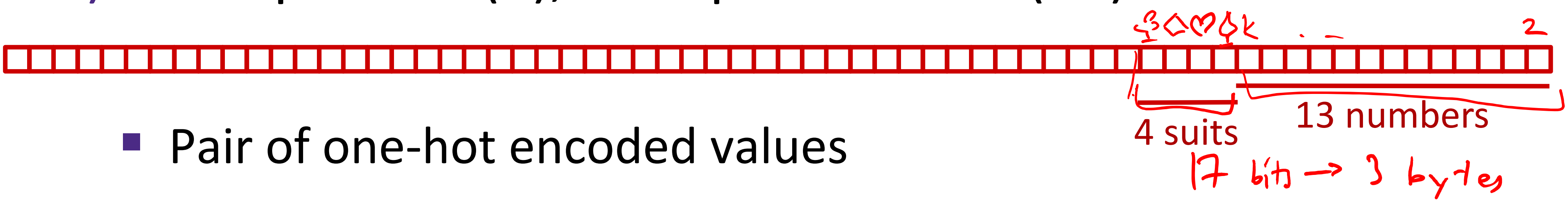
Two possible representations

1) 1 bit per card (52): bit corresponding to card set to 1



- "One-hot" encoding (similar to set notation)
- Drawbacks:
  - Hard to compare values and suits
  - Large number of bits required

2) 1 bit per suit (4), 1 bit per number (13): 2 bits set



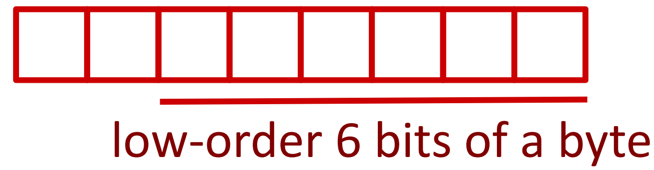
- Pair of one-hot encoded values
- Easier to compare suits and values, but still lots of bits used

❖ Can we do better?

Two better representations

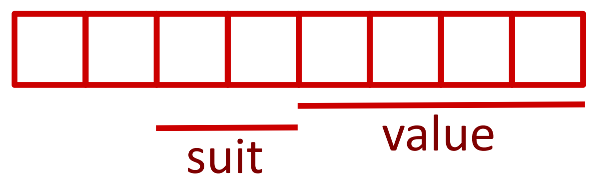
3) Binary encoding of all 52 cards – only 6 bits needed

- $2^6 = 64 \geq 52$



- Fits in one byte (smaller than one-hot encodings)
- How can we make value and suit comparisons easier?

4) Separate binary encodings of suit (2 bits) and value (4 bits)



- Also fits in one byte, and easy to do comparisons

K	Q	J	. . .	3	2	A
1101	1100	1011	. . .	0011	0010	0001

♣	00
♦	01
♥	10
♠	11

Compare Card Suits

mask: a bit vector designed to achieve a desired behavior when used with a bitwise operator on another bit vector *v*.
Here we turns all *but* the bits of interest in *v* to 0.

```
char hand[5];          // represents a 5-card hand
char card1, card2;     // two cards to compare
card1 = hand[0];
card2 = hand[1];
...
if ( sameSuitP(card1, card2) ) { ... }
```

text substitution

```
#define SUIT_MASK  0x30

int sameSuitP(char card1, char card2) {
  return (!((card1 & SUIT_MASK) ^ (card2 & SUIT_MASK)));
  //return (card1 & SUIT_MASK) == (card2 & SUIT_MASK);
}
```

returns int

equivalent

SUIT_MASK = 0x30 =

0	0	1	1	0	0	0	0

suit (keep) value (discard)

$x \& 0 = 0$

$x \& 1 = x$

Compare Card **Suits**

mask: a bit vector designed to achieve a desired behavior when used with a bitwise operator on another bit vector *v*.
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```
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```
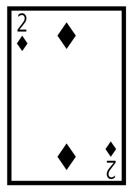
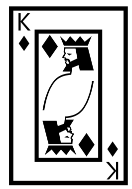




card1: 0 0 0 1 0 0 1 0

1 &

SUIT_MASK: 0 0 1 1 0 0 0 0

=

0 0 0 1 0 0 0 0

card2: 0 0 0 1 1 1 0 1

& 1

SUIT_MASK: 0 0 1 1 0 0 0 0

=

0 0 0 1 0 0 0 0

bitwise

x & 0 = 0

x & 1 = x

2 ^

0 0 0 0 0 0 0 0

3 !

logical

0 0 0 0 0 0 0 1

! (x^y) equivalent to x==y

Compare Card Values

mask: a bit vector designed to achieve a desired behavior when used with a bitwise operator on another bit vector *v*.

```
char hand[5];          // represents a 5-card hand
char card1, card2;     // two cards to compare
card1 = hand[0];
card2 = hand[1];
...
if ( greaterValue(card1, card2) ) { ... }
```

```
#define VALUE_MASK  0x0F

int greaterValue(char card1, char card2) {
  return ((unsigned int)(card1 & VALUE_MASK) >
          (unsigned int)(card2 & VALUE_MASK));
}
```

VALUE_MASK = 0x0F =

0	0	0	0	1	1	1	1

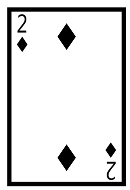
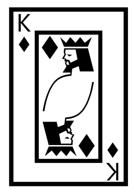
suit (discard)

value (keep)

Compare Card Values

mask: a bit vector designed to achieve a desired behavior when used with a bitwise operator on another bit vector *v*.

```
#define VALUE_MASK  0x0F

int greaterValue(char card1, char card2) {
  return ((unsigned int)(card1 & VALUE_MASK) >
          (unsigned int)(card2 & VALUE_MASK));
}
```

0	0	1	0	0	0	1	0

&

0	0	0	0	1	1	1	1

=

0	0	0	0	0	0	1	0

VALUE_MASK

0	0	1	0	1	1	0	1

&

0	0	0	0	1	1	1	1

=

0	0	0	0	1	1	0	1

$2_{10} > 13_{10}$

0 (false)

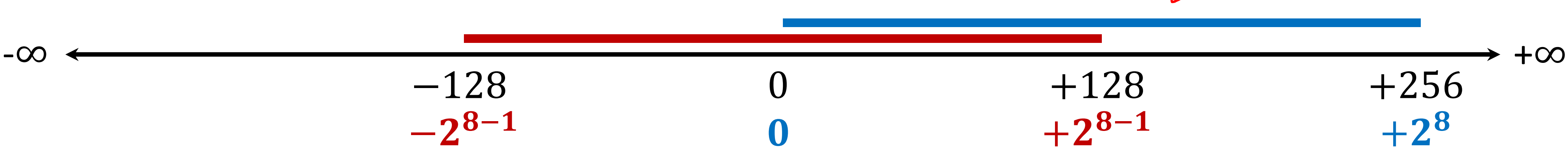
Integers

- **Binary representation of integers**
  - **Unsigned and signed**
  - Casting in C
- Consequences of finite width representation
  - Overflow, sign extension
- Shifting and arithmetic operations

Encoding Integers

- The hardware (and C) supports two flavors of integers
  - *unsigned* – only the non-negatives
  - *signed* – both negatives and non-negatives
- Cannot represent all integers with *w* bits
  - Only 2^w distinct bit patterns
  - Unsigned values: $0 \ldots 2^w-1$
  - Signed values: $-2^{w-1} \ldots (2^{w-1}-1)$
- **Example:** 8-bit integers (*e.g.* `char`)

same widths, just shifted



Unsigned Integers

- Unsigned values follow the standard base 2 system
  - $b_7b_6b_5b_4b_3b_2b_1b_0 = b_72^7 + b_62^6 + \cdots + b_12^1 + b_02^0$
- Add and subtract using the normal "carry" and "borrow" rules, just in binary

$$\begin{array}{r} 63 \\ +\underline{\quad 8} \\ 71 \end{array} \qquad \begin{array}{r} \scriptstyle 1\,1\,1\quad\quad\; \\ 00111111 \\ +\underline{00001000} \\ 01000111 \end{array}$$

$\leftarrow$ x, 6 1's in a row

$x+1 = 0b1\,000\,000$
$= 2^6$

$x = 2^6 - 1$

- Useful formula: $2^{N-1} + 2^{N-2} + \ldots + 2 + 1 = 2^N - 1$
  - *i.e.* N ones in a row $= 2^N - 1$
- How would you make *signed* integers?

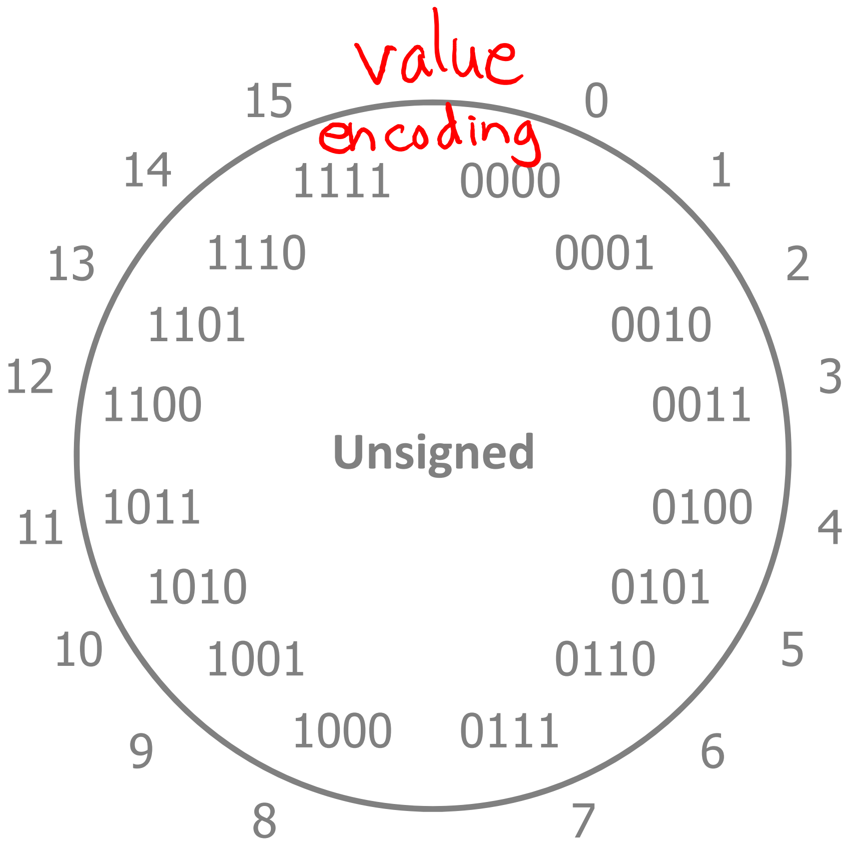
Sign and Magnitude

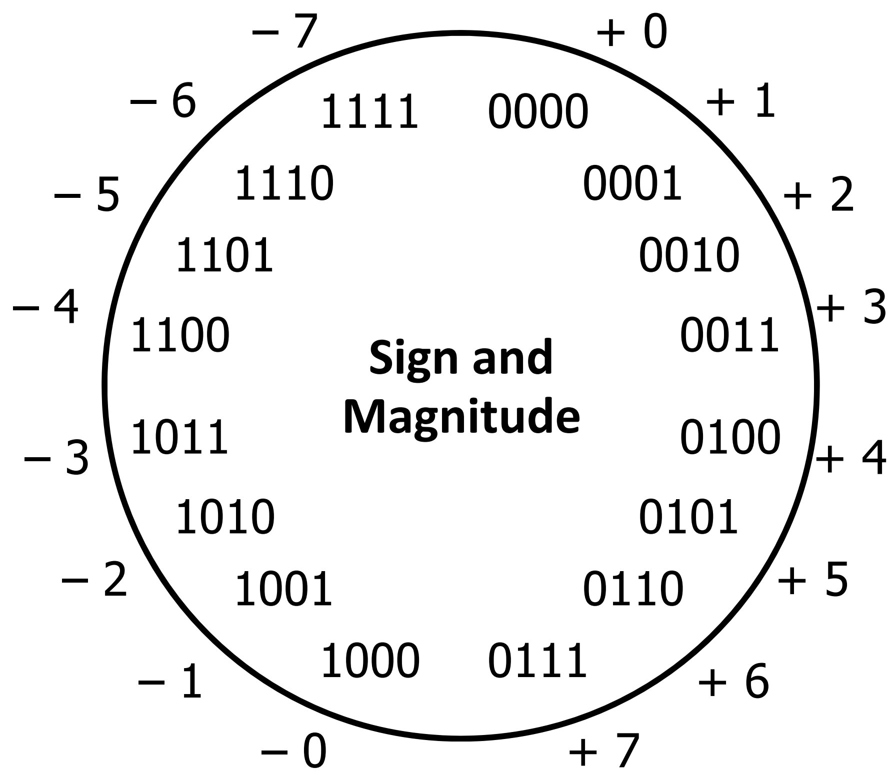
Most Significant Bit

- Designate the high-order bit (MSB) as the "sign bit"
  - `sign=0`: positive numbers; `sign=1`: negative numbers
- Benefits:
  - Using MSB as sign bit matches positive numbers with unsigned unsigned: 0b 0010 = 2^1 = 2 ; sign + mag: 0b 0010 = $+2^1$ = 2 ☑
  - All zeros encoding is still = 0
- Examples (8 bits):
  - 0x00 = 00000000_2 is non-negative, because the sign bit is 0
  - 0x7F = 01111111_2 is non-negative ($+127_{10}$)
  - 0x85 = $1\underbrace{0000101}_{2^2+2^0=5}{}_2$ is negative (-5_{10})
  - 0x80 = 10000000_2 is negative... zero???

Sign and Magnitude

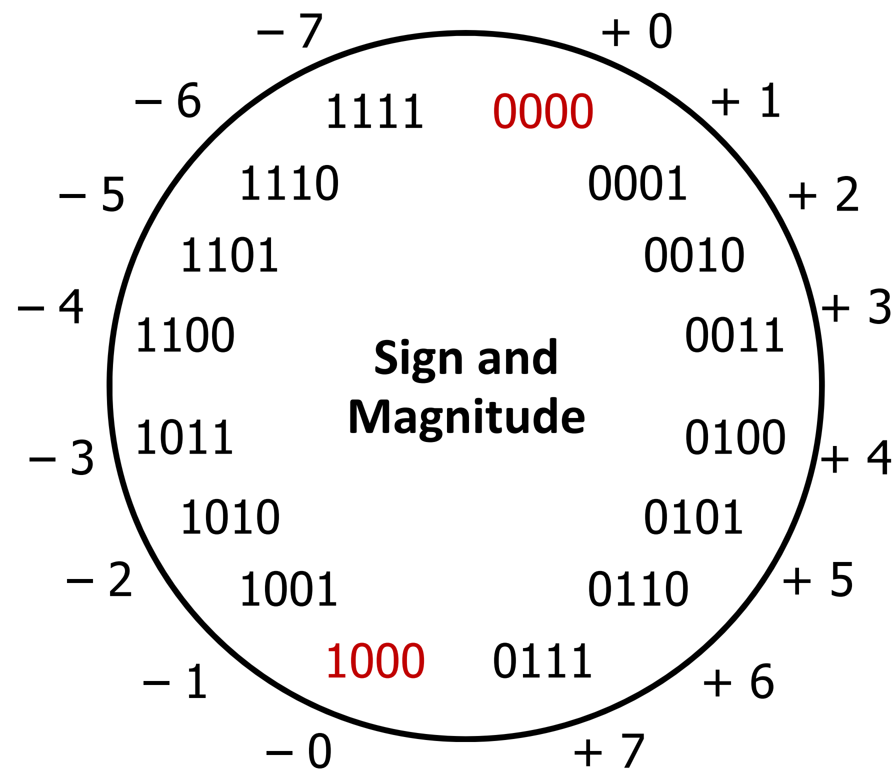
- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks?





Sign and Magnitude

- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks:
  - Two representations of 0 (bad for checking equality)



Sign and Magnitude

- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks:
  - Two representations of 0 (bad for checking equality)
  - Arithmetic is cumbersome
    - Example: `4-3 != 4+(-3)`

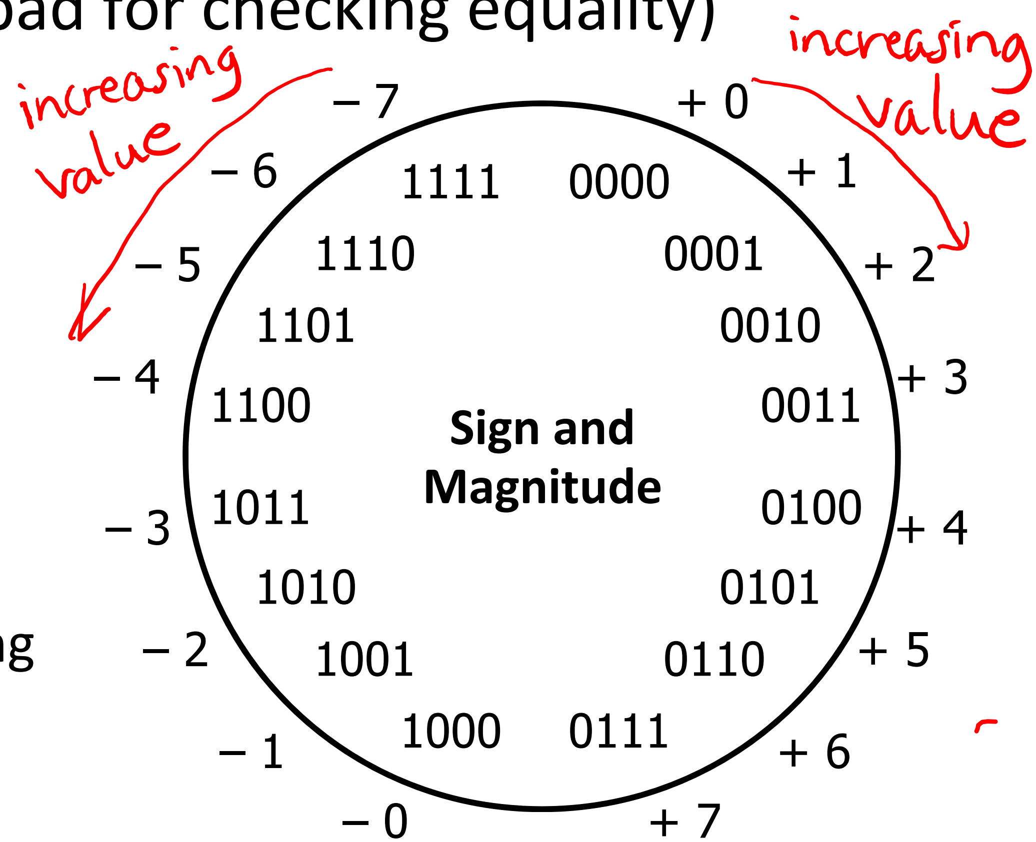
4	0100
– 3	– 0011
1	0001

✓

4	0100
+ –3	+ 1011
–7	1111

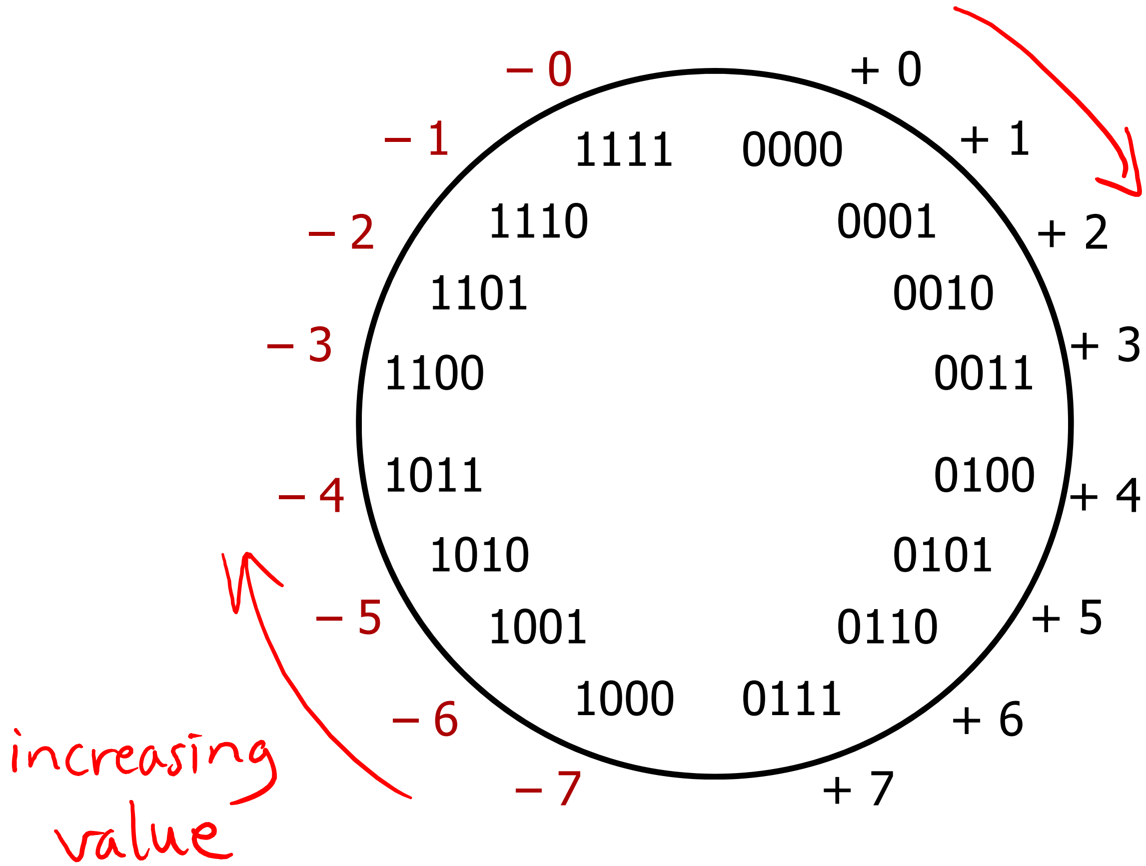
✗

    - Negatives “increment” in wrong direction!



Two's Complement

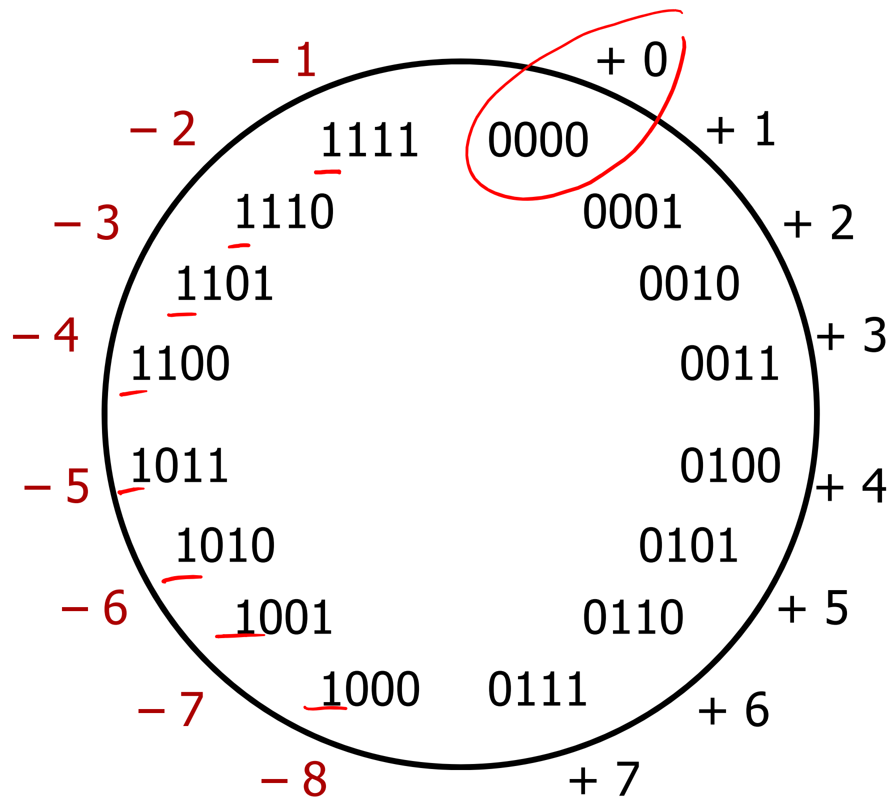
- Let's fix these problems:
  1) "Flip" negative encodings so incrementing works



Two’s Complement

- Let’s fix these problems:
  1) “Flip” negative encodings so incrementing works
  2) “Shift” negative numbers to eliminate –0

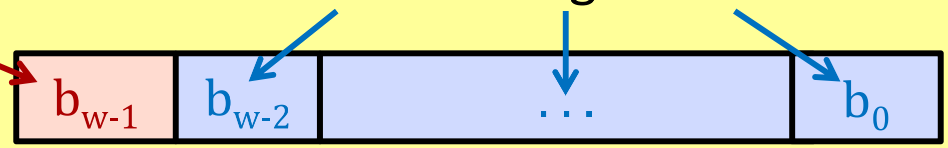
- MSB *still* indicates sign!
  - This is why we represent one more negative than positive number (-2^{N-1} to $2^{N-1}-1$)



Two's Complement Negatives

- Accomplished with one neat mathematical trick!

b_{w-1} has weight -2^{w-1}, other bits have usual weights $+2^{i}$

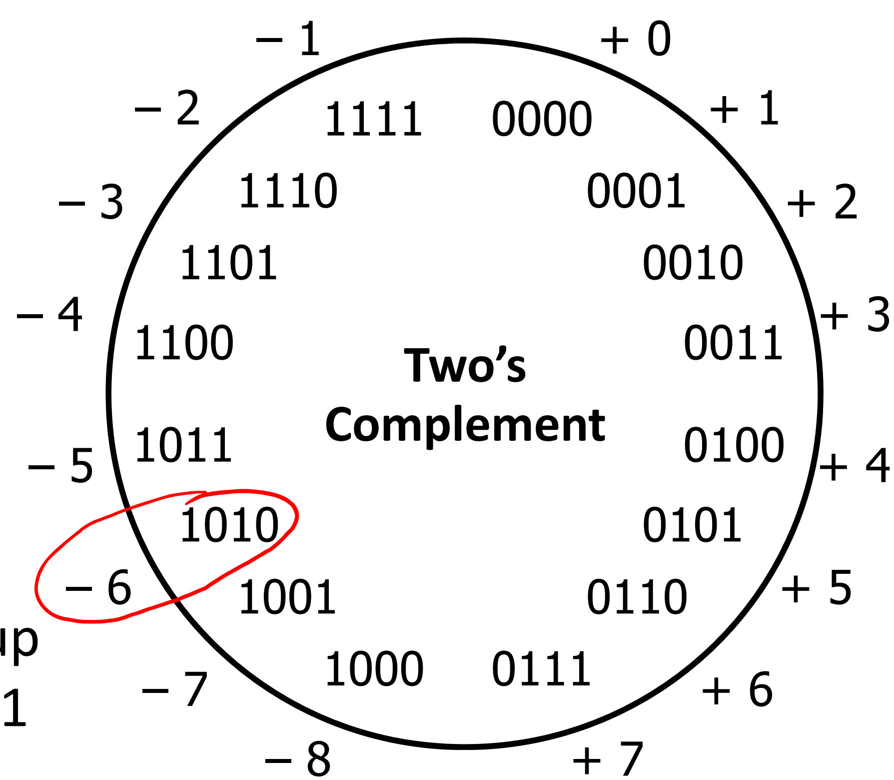


- 4-bit Examples:
  - 1010_2 unsigned:
    $1*2^3+0*2^2+1*2^1+0*2^0 =$ **10**
  - 1010_2 two's complement:
    $-1*2^3+0*2^2+1*2^1+0*2^0 =$ **–6**
- -1 represented as:

$1111_2 = -2^3 + (2^3 - 1)$

3 ones in a row

  - MSB makes it super negative, add up all the other bits to get back up to -1



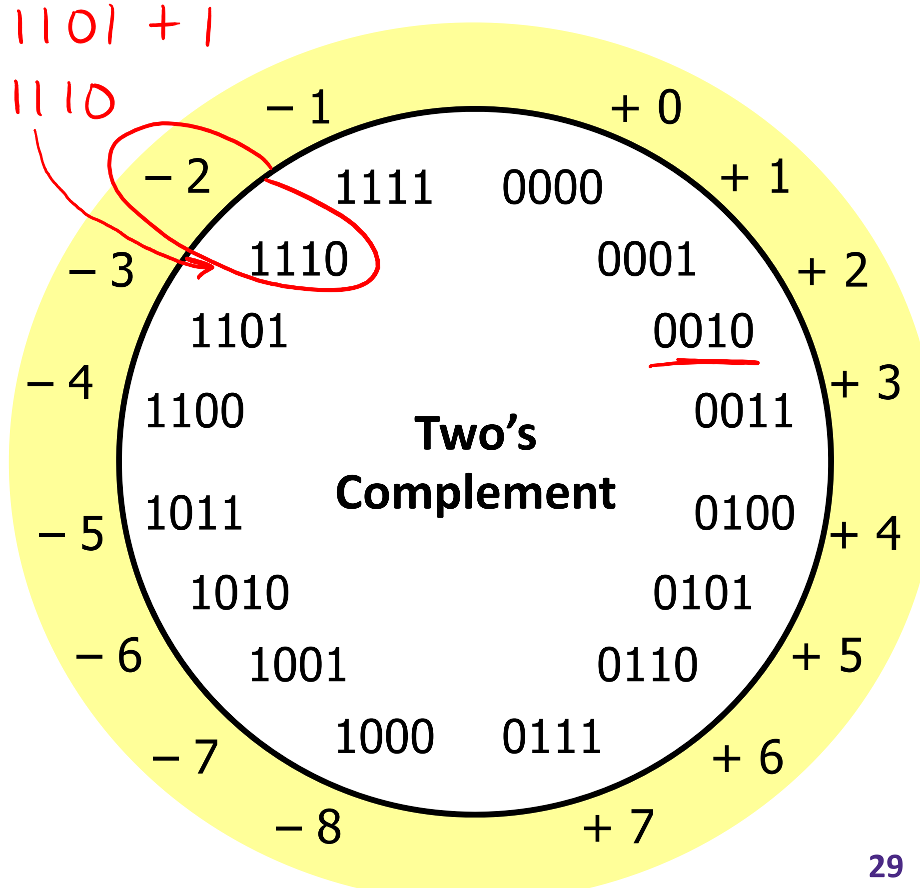
Why Two's Complement is So Great

- Roughly same number of (+) and (–) numbers
- Positive number encodings match unsigned
- Single zero
- All zeros encoding = 0

2 = 0b0010
-2 = 0b1101 + 1
= 0b1110

- Simple negation procedure:
  - Get negative representation of any integer by taking bitwise complement and then adding one!

    (~x + 1 == -x)



Peer Instruction Question

- Take the 4-bit number encoding `x = 0b1011`
- Which of the following numbers is NOT a valid interpretation of `x` using any of the number representation schemes discussed today?
  - Unsigned, Sign and Magnitude, Two's Complement
  - Vote at http://PollEv.com/justinh

A. -4

B. -5

C. 11

D. -3

E. We're lost…

Summary

- Bit-level operators allow for fine-grained manipulations of data
  - Bitwise AND (`&`), OR (`|`), and NOT (`~`) different than logical AND (`&&`), OR (`||`), and NOT (`!`)
  - Especially useful with bit masks
- Choice of *encoding scheme* is important
  - Tradeoffs based on size requirements and desired operations
- Integers represented using unsigned and two's complement representations
  - Limited by fixed bit width
  - We'll examine arithmetic operations next lecture